THE PRINCIPLE OF THE ACTIVE JMC SCATTERER

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ABSTRACT

The principle of formulating the JMC method to produce secondary sources that function as active scatterers on a hypothetical scattering surface is established, to be applied, e.g., in concert halls. The examination is based on the modified JMC method, to ensure that the logic does not lead to the need of changing the primary sources. The actively reflecting plane serves as an example of the JMC formulation for the active scatterer. The solution for the actively reflecting plane works on the local control principle: each reflecting subarea needs information of the primary field only at that subarea. The solution can also apply to piecemeal planar surfaces and to smooth convex surfaces, approximately.

1. INTRODUCTION

The JMC method is suitable for formulating the problem of active noise control with the general system theory. More generally, the method applies to the reshaping of acoustic or any other fields [1], [2], [3], to wave reconstruction [3], and to wave propagation problems [4], [5]. Its name originates from the first three pioneers of the method: **J**essel, **M**angiante, and **C**anévet [2] (the JMC group). Uosukainen presented the modified JMC method [6]. The modified JMC method differs from the original one so that in the former the primary sources are not changed in any case.

The purpose of this paper is to establish, at a general level and especially applied to acoustic fields, the principle of formulating the JMC method to produce secondary sources that function as active scatterers on a hypothetical scattering surface. As an example, an actively reflecting plane is introduced. The examination is based on the modified JMC method, to ensure that the logic does not lead to the need of changing the primary sources. A more detailed presentation has been given by Uosukainen [7].

2. MODIFIED JMC METHOD

In the original situation there is a deterministic field (of any type) in which linear operator $\mathbf{L}$ (typically a differential operator) connects sources $\boldsymbol{S}$ and field $\boldsymbol{F}$ via

$$\mathbf{L}\boldsymbol{F} = \boldsymbol{S} \ . \qquad (1)$$

Instead of field $\boldsymbol{F}$, field $\boldsymbol{F}'$ is desired, which can be obtained from the original field using operator $\mathbf{M}$ as

$$\mathbf{M}\boldsymbol{F} = \boldsymbol{F}' \ . \qquad (2)$$

In the original JMC method, operator $\mathbf{M}$ also weights the original sources to sources $\boldsymbol{S}'$. In the modified JMC method, the original sources always remain unchanged. Both in the original and modified JMC method, there is a need for additional sources $\boldsymbol{S}''$ such that field equation (1) for the modified field is valid. The field equation of desired field $\boldsymbol{F}'$ with the original sources unchanged is

$$\mathbf{L}\boldsymbol{F}' = \boldsymbol{S} + \boldsymbol{S}'' \ . \tag{3}$$

The expression above, together with equations (1) and (2), yield for the secondary sources in the modified JMC method

$$\boldsymbol{S}'' = \mathbf{L}\boldsymbol{F}' - \boldsymbol{S} = \mathbf{L}\mathbf{M}\boldsymbol{F} - \mathbf{L}\boldsymbol{F} = \mathbf{M}'\boldsymbol{F} \ , \tag{4}$$

where

$$\mathbf{M}' = \mathbf{L}(\mathbf{M} - \mathbf{I}) \ , \tag{5}$$

where **I** is the identity operator.

3. JMC FORMULATION OF THE ACTIVE SCATTERER

3.1. General formulation

A hypothetical scattering obstacle with its boundary surface *A* is defined according to Figure 1.

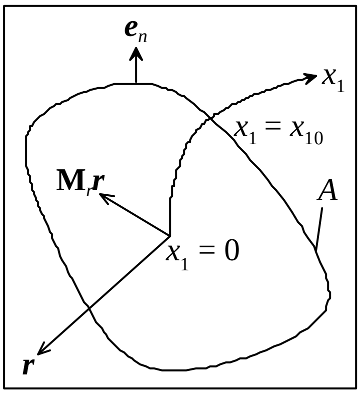

Figure 1. *A hypothetical scattering obstacle.*

The modified field is assumed to be the sum of original field $\boldsymbol{F}$ and some extra field $\boldsymbol{F}_s$ (scattered field)

$$\begin{aligned} \boldsymbol{F}' &= \boldsymbol{F} + \boldsymbol{F}_s \\ \boldsymbol{F}_s(\boldsymbol{r}) &= \mathbf{M}_s\boldsymbol{F}(\mathbf{M}_r\boldsymbol{r}) \ , \end{aligned} \tag{6}$$

where $\boldsymbol{r}$ is a spatial coordinate vector, and $\mathbf{M}_s$ and $\mathbf{M}_r$ are operators. It is supposed that operator $\mathbf{M}_r$ maps vector $\boldsymbol{r}$ on the other side of surface *A*, i.e.,

$$\begin{aligned} &\mathbf{M}_r\boldsymbol{r} \text{ is inside } A \text{ if } \boldsymbol{r} \text{ is outside } A \\ &\mathbf{M}_r\boldsymbol{r} \text{ is outside } A \text{ if } \boldsymbol{r} \text{ is inside } A \\ &\mathbf{M}_r\boldsymbol{r} = \boldsymbol{r} \ , \text{ if } \boldsymbol{r} \text{ is at } A \ , \end{aligned} \tag{7}$$

see Figure 1. It is further supposed that extra field $\boldsymbol{F}_s$ vanishes inside *A* and obeys the homogeneous field equation outside *A*,

$$\begin{aligned} \boldsymbol{F}_s &= 0 \ \text{ inside } A \\ \mathbf{L}\boldsymbol{F}_s &= 0 \ \text{ outside } A \ . \end{aligned} \tag{8}$$

The latter formula, together with equations (1) and (3), implicates that the only possible place for the secondary sources are on surface *A*.

The fact that the modified field generally obeys equation (2) yields

$$\mathbf{M} = \mathbf{I} + \mathbf{M}'_s \ , \tag{9}$$

where operator $\mathbf{M}_s'$ operates so that

$$\mathbf{M}'_s \boldsymbol{F}(\boldsymbol{r}) = \mathbf{M}_s \boldsymbol{F}(\mathbf{M}_r \boldsymbol{r}) \ . \tag{10}$$

The secondary sources, according to the modified JMC method, are now as stated in equation (4), where, according to equations (5) and (9),

$$\mathbf{M}' = \mathbf{L}(\mathbf{M} - \mathbf{I}) = \mathbf{L}\mathbf{M}'_s \ , \tag{11}$$

so

$$\boldsymbol{S}''(\boldsymbol{r}) = \mathbf{L}\mathbf{M}'_s \boldsymbol{F}(\boldsymbol{r}) = \mathbf{L}\mathbf{M}_s \boldsymbol{F}(\mathbf{M}_r \boldsymbol{r}) \ . \tag{12}$$

According to equations (6) and (8), operator $\mathbf{M}_s$ has to be of the form

$$\mathbf{M}_s = \mathbf{M}_{s0} \varepsilon(x_1 - x_{10}) \ , \tag{13}$$

where $\mathbf{M}_{s0}$ is a continuous function of spatial coordinates, $\varepsilon(x_1 - x_{10})$ is a step function, and where it is supposed that boundary A is formed of a constant x_1 surface $x_1 = x_{10}$, see Figure 1. The secondary sources on A are due to the discontinuity of $\mathbf{M}_s$ at $x_1 = x_{10}$. Equation (8) can be written outside A, utilizing equations (6) and (8), as

$$\mathbf{L}\boldsymbol{F}_s(\boldsymbol{r}) = \mathbf{L}\mathbf{M}_s \boldsymbol{F}(\mathbf{M}_r \boldsymbol{r}) = \mathbf{L}\mathbf{M}_{s0} \boldsymbol{F}(\mathbf{M}_r \boldsymbol{r}) = 0 \ \text{outside } A \ . \tag{14}$$

Due to the continuity of $\mathbf{M}_{s0}$, this must hold also at A. Because $\boldsymbol{F}_s$ vanishes inside A, according to equation (8), the equation above is valid everywhere, i.e.,

$$\mathbf{L}\mathbf{M}_{s0} \boldsymbol{F}(\mathbf{M}_r \boldsymbol{r}) = 0 \ . \tag{15}$$

Now the secondary sources are, according to equations (12), (13), and (15),

$$\begin{aligned} \boldsymbol{S}''(\boldsymbol{r}) &= \mathbf{L}\big(\mathbf{M}_{s0}\varepsilon(x_1 - x_{10})\boldsymbol{F}(\mathbf{M}_r\boldsymbol{r})\big) = \mathbf{L}\big(\mathbf{M}_{s0}\boldsymbol{F}(\mathbf{M}_r\boldsymbol{r})\big)\varepsilon(x_1 - x_{10}) + \mathbf{L}\big(\varepsilon(x_1 - x_{10})\big)\mathbf{M}_{s0}\boldsymbol{F}(\mathbf{M}_r\boldsymbol{r}) \\ &= \mathbf{L}\big(\varepsilon(x_1 - x_{10})\big)\mathbf{M}_{s0}\boldsymbol{F}(\boldsymbol{r}) \ \text{at } A \ . \end{aligned} \tag{16}$$

The final general solution above depends on the original field at A, operator $\mathbf{M}_{s0}$, and the field operator operating on the step function at A.

3.2. Application to acoustic fields

In acoustic fields in flowless and homogenous ideal fluids field $\boldsymbol{F}$, sources $\boldsymbol{S}$, and operator $\mathbf{L}$ connecting them are

$$\begin{aligned} \mathbf{L} &= \begin{bmatrix} Q_0 \dfrac{\partial}{\partial t} & \nabla \cdot \\ \nabla & \rho_0 \dfrac{\partial}{\partial t} \end{bmatrix} \\ \boldsymbol{F} &= \begin{bmatrix} p \\ \boldsymbol{u} \end{bmatrix} \\ \boldsymbol{S} &= \begin{bmatrix} q \\ \boldsymbol{f} \end{bmatrix} , \end{aligned} \tag{17}$$

where t is time, Q_0 and ρ_0 are the compressibility and the density of the unperturbed fluid, p and $\boldsymbol{u}$ are the sound pressure and the particle velocity of the acoustic field, and q and $\boldsymbol{f}$ are the monopole and dipole distributions per unit volume.

Operator **L** operating on the step function yields now

$$\mathbf{L}\left(\varepsilon(x_1 - x_{10})\right) = \begin{bmatrix} 0 & \nabla\varepsilon(x_1 - x_{10})\cdot \\ \nabla\varepsilon(x_1 - x_{10}) & 0 \end{bmatrix} = \delta(x_1 - x_{10})\begin{bmatrix} 0 & \boldsymbol{e}_n \cdot \\ \boldsymbol{e}_n & 0 \end{bmatrix} , \qquad (18)$$

where $\delta(x_1 - x_{10})$ is the Dirac delta function and $\boldsymbol{e}_n$ is a unit outward normal vector on surface A, see Figure 1. The secondary sources are now, according to equations (16), (17), and (18),

$$\boldsymbol{S}'' = \begin{bmatrix} q' \\ \boldsymbol{f}'' \end{bmatrix} = \delta(x_1 - x_{10})\begin{bmatrix} 0 & \boldsymbol{e}_n \cdot \\ \boldsymbol{e}_n & 0 \end{bmatrix}\mathbf{M}_{s0}\begin{bmatrix} p \\ \boldsymbol{u} \end{bmatrix} = \delta(x_1 - x_{10})\begin{bmatrix} \mathbf{M}_{su}\boldsymbol{u}\cdot\boldsymbol{e}_n \\ \mathbf{M}_{sp} p\boldsymbol{e}_n \end{bmatrix} = \delta(x_1 - x_{10})\begin{bmatrix} \mathbf{M}_{su}\boldsymbol{u} \\ \mathbf{M}_{sp} p\mathsf{I} \end{bmatrix}\cdot\boldsymbol{e}_n , \qquad (19)$$

where operator $\mathbf{M}_{s0}$ has been divided into two operators, $\mathbf{M}_{sp}$ operating on the sound pressure, and $\mathbf{M}_{su}$ operating on the particle velocity

$$\mathbf{M}_{s0}\begin{bmatrix} p \\ \boldsymbol{u} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{sp} p \\ \mathbf{M}_{su}\boldsymbol{u} \end{bmatrix} , \qquad (20)$$

and where I is identic dyadic ($\mathsf{I} \cdot \boldsymbol{a} = \boldsymbol{a} \cdot \mathsf{I} = \boldsymbol{a}$). Integrating expression (19) with respect to x_1 yields surface secondary source distribution $\boldsymbol{S}_s''$ on A as

$$\boldsymbol{S}_s'' = \begin{bmatrix} q_s'' \\ \boldsymbol{f}_s'' \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{su}\boldsymbol{u}\cdot\boldsymbol{e}_n \\ \mathbf{M}_{sp} p\boldsymbol{e}_n \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{su}\boldsymbol{u} \\ \mathbf{M}_{sp} p\mathsf{I} \end{bmatrix}\cdot\boldsymbol{e}_n \text{ at } A . \qquad (21)$$

The solution above for the acoustic fields depends on the original sound pressure and the normal component of the original particle velocity at A, and operator $\mathbf{M}_{s0}$.

3.3. Reflecting plane

Dyadic K producing the reflection transformation of the original field with respect to the plane $x = 0$ can be presented as [8]

$$\mathsf{K} = \mathsf{I} - 2\mathbf{e}_x\mathbf{e}_x = -\mathbf{e}_x\mathbf{e}_x + \mathbf{e}_y\mathbf{e}_y + \mathbf{e}_z\mathbf{e}_z , \qquad (22)$$

where $\mathbf{e}_x$ is a unit vector in x-direction (normal to the reflecting plane), see Figure 2. The dyadic of the reflection transformation inverts the normal component (with respect to the reflecting plane) of the vector as opposite without changing the other components in any way. The reflection transformation operates on both the actual field vectors and coordinate vector $\boldsymbol{r}$, see Figure 2. The transformed field may be interpreted to be caused by a mirror image of the original source with respect to the surface. The strength of the mirror image and its distance from the reflecting surface are equal to those of the original source.

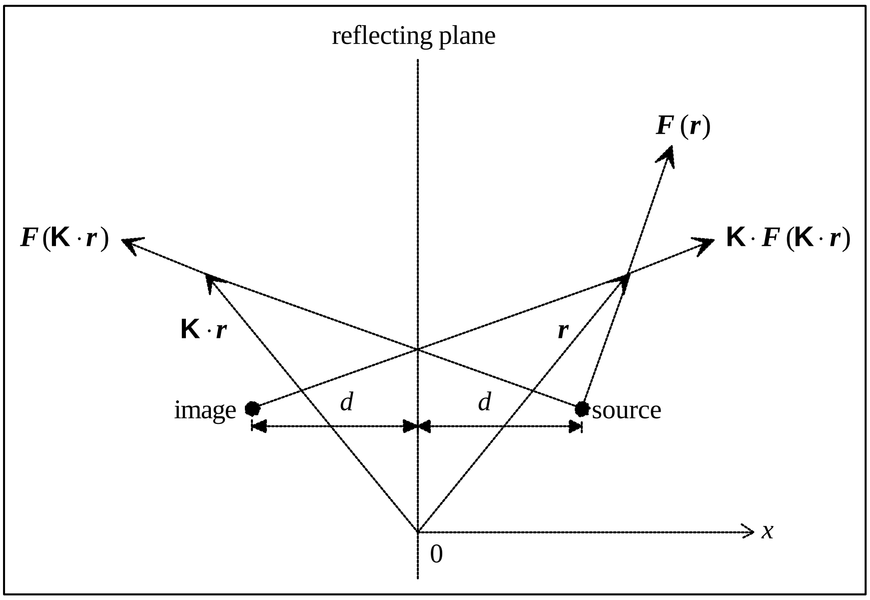

Figure 2. *The effect of the reflection transformation to field vector **F** and co-ordinate vector **r**.*

If the reflecting surface is not ideal, the amplitude of the reflected field is smaller than that of the original field on the reflecting surface. The reflection may also change the phase of the field. This can be taken into account with complex reflection coefficient R. The reflection coefficient must be properly chosen to ensure that the reflected field satisfies the homogeneous field equation in the half space $x > 0$. One possibility is to use a reflection coefficient independent of the angle of incidence. With a reflection coefficient chosen as stated above, the reflected acoustic fields (subscript r) obey

$$\begin{aligned} p_r(\boldsymbol{r}) &= Rp(\mathbf{K}\cdot\boldsymbol{r}) \\ \boldsymbol{u}_r(\boldsymbol{r}) &= R\mathbf{K}\cdot\boldsymbol{u}(\mathbf{K}\cdot\boldsymbol{r})\ . \end{aligned} \tag{23}$$

According to the presentation of the spatial variable in the reflection transformation, the propagation direction with respect to the normal of the plate is changed into the opposite, remaining original in lateral directions.

Operators $\mathbf{M}_s$ and $\mathbf{M}_r$, defined in equations (6), (7), (13), and (20), are now

$$\begin{aligned} \mathbf{M}_s &= \mathbf{M}_{s0}\varepsilon(x) \\ \mathbf{M}_{s0} &= \begin{bmatrix} \mathbf{M}_{sp} \\ \mathbf{M}_{su} \end{bmatrix} = R\begin{bmatrix} 1 \\ \mathbf{K}\cdot \end{bmatrix} \\ \mathbf{M}_r &= \mathbf{K}\cdot\ . \end{aligned} \tag{24}$$

The secondary source densities, according to equations (21), (22), and (24), are now

$$\boldsymbol{S}_s'' = \begin{bmatrix} q_s'' \\ \boldsymbol{f}_s'' \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{su}\boldsymbol{u} \\ \mathbf{M}_{sp}p\mathbf{I} \end{bmatrix}\cdot\boldsymbol{e}_x = R\begin{bmatrix} \mathbf{K}\cdot\boldsymbol{u} \\ p\mathbf{I} \end{bmatrix}\cdot\boldsymbol{e}_x = R\begin{bmatrix} -\boldsymbol{u}\cdot\boldsymbol{e}_x \\ p\boldsymbol{e}_x \end{bmatrix} \text{ at } x = 0\ . \tag{25}$$

The solution above works on the local control principle: the secondary source strengths at any point on A depend on the original fields only at the same point.

The planar secondary source expressions in the equation above approximately applies to piecemeal planar surfaces and even to smooth convex surfaces. In those cases the unit vector in x-direction has to be replaced with a unit vector normal to the surface.

4. CONCLUSIONS

The principle of formulating the JMC method to produce secondary sources that function as active scatterers on a hypothetical scattering surface was established. The principle can be applied, e.g., in the active control of the acoustical properties of concert halls. The examination was based on the modified JMC method, to ensure that the logic does not lead to the need of changing the primary sources. As an example of the scatterer, an actively reflecting plane was introduced. The solution works on the local control principle: each reflecting subarea needs information of the primary field only at that subarea. The solution can also apply to piecemeal planar surfaces and to smooth convex surfaces.

5. REFERENCES

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