Simulating guitar distortion circuits by wave digital and Kirchhoff domain methods

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Electronics are Musical Instruments

- Oscillators generate sound
- Amplifiers and filters modify sound
  - Dynamic Range Compressors (DRC), EQ, Reverb, Phaser/Flanger, Chorus, Voltage Controller Filter (VCF)
- Spectral palette for musicians





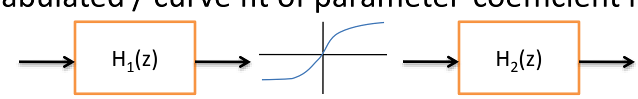
"Virtual Analog"

- Field of music DSP
- Reproduce effects of analog circuits
  - Parametric linear filters
  - Nonlinear distortion

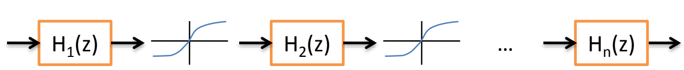
- Preservation of vintage musical effect circuits
- Flexibility – computer based studio

Survey of existing methods

- Filters and static nonlinearity
  - Doidic et al. (1998), Schattschneider and Zölzer (1999), Abel and Berners (2005), Fernandez-Cid et al. (1999)
  - Tabulated / curve fit of parameter-coefficient map



- Digital emulation of signal path (cascade of filters and nonlinearities)
  - Kuroki (1998), Möller et al (2002), Karjalainen et al (2006), etc

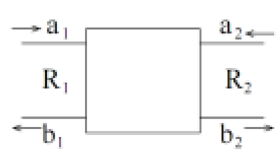


Numerical Simulation of ODE systems for Audio Effects

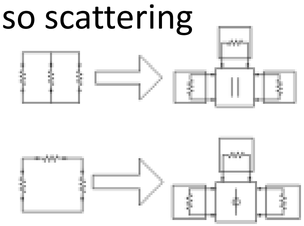
- Huovilainen (DAFx 2004, 2005) : Nonlinear Moog, modulation effects
- Yeh, et al. (DAFx 2007) : Diode clipper simulation
- Sarti and Tubaro (1999) : Nonlinear wave digital filters
- De Sanctis, et al. (DAFx 2003) : Automatic synthesis of WDFs
- Karjalainen and Pakarinen (2006) : WDF common cathode circuit
- Borin et al. (2000) : Eliminating delay free loops (K-method)
- Fontana, et al. (DAFx 2004) : Nonlinear filter networks

Wave Digital Filter Principles

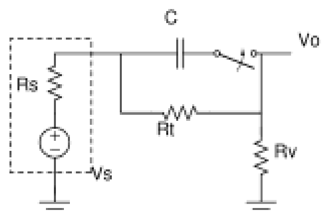
- Change of variable from voltage, current to waves *a*, *b*, and port impedance *R*

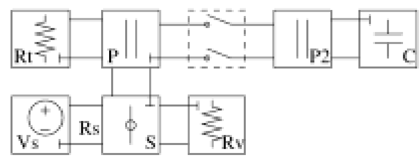


- Circuit elements become scattering junctions
- Interconnection of elements are also scattering junctions (Adaptors)
  - N-port parallel and series junctions are O(*N*)
  - Generic N-port scattering junction is O(N^2)

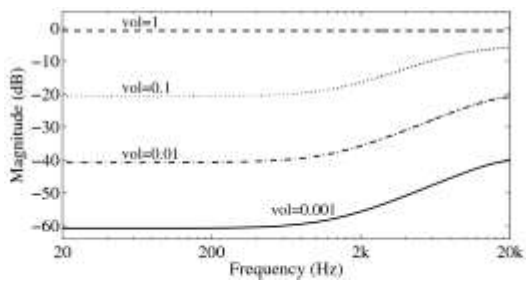


WDF Bright Switch

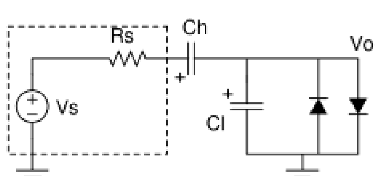


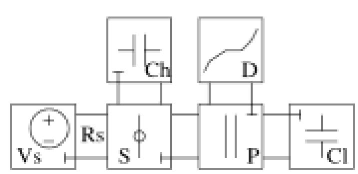


- Map circuit elements to WDF elements
- Parallel and series connections: efficient implementation
- Numerically robust and allows smooth parameter changes



WDF Diode Clipper

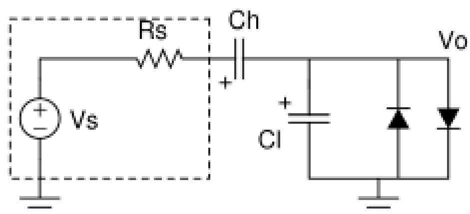




- Nonlinear element at top of WDF tree.
- Solve nonlinear equation for *b* = f(*a*)

$$2I_s \sinh\left(\frac{a+b}{2V_t}\right) - \frac{a-b}{2R_p} = 0$$

Multivariate state and nonlinearities



- Consider diode clipper with high pass capacitor
- Seek a systematic way to solve nonlinear ODE

SSMN – State Space with Memoryless Nonlinearity (K-Method)

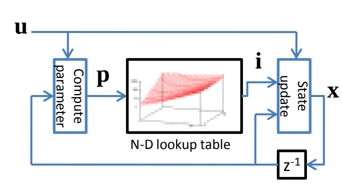
- State $\mathbf{x}$ is capacitor voltages, inductor currents
- Inputs $\mathbf{u}$ are independent sources
- Nonlinear vector $\mathbf{i}$: nonlinear voltage controlled current sources – diodes, transistors
- Vector of controlling voltages $\mathbf{v}$

- Discretize system time-derivative by integration formula (BE) and solve for $\mathbf{x}$[n]

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{Ci}$$

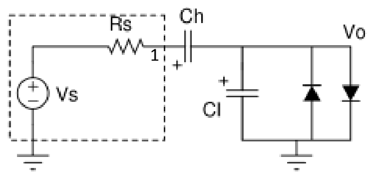
$$\mathbf{i} = \mathbf{f}(\mathbf{v})$$

$$\mathbf{v} = \mathbf{Dx} + \mathbf{Eu} + \mathbf{Fi}$$



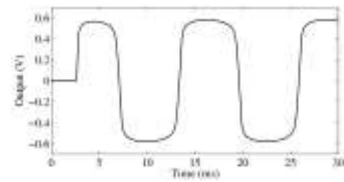
- Solution derives a memoryless nonlinearity
- Parameter $\mathbf{p}$ is linear combination of $\mathbf{u}$ and $\mathbf{x}$

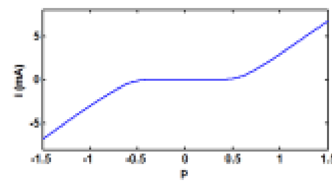
Diode clipper revisited



$$\mathbf{x} = \begin{bmatrix} V_{Ch} \\ V_{Cl} \end{bmatrix} \qquad \mathbf{u} = [V_s]$$

$$\mathbf{i} = \begin{bmatrix} I_s(\exp(V_o/V_t)-1) \\ -I_s(\exp(-V_o/V_t)-1) \end{bmatrix}$$



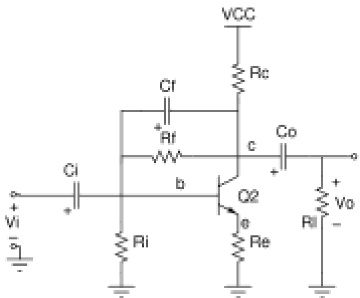


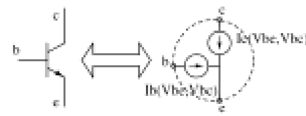
- Solution to implicit nonlinear mapping from **p** -> **i** can be tabulated

Comparison of WDF and SSMN Diode Clippers

- Results are identical in MATLAB
- Assuming nonlinearity in algorithm is precomputed:
- WDF
  - Parallel/series scattering junctions
    - 4 multiplies
    - 8 adds
- SSMN
  - Matrix-vector multiplies
    - 13 multiplies
    - 12 adds

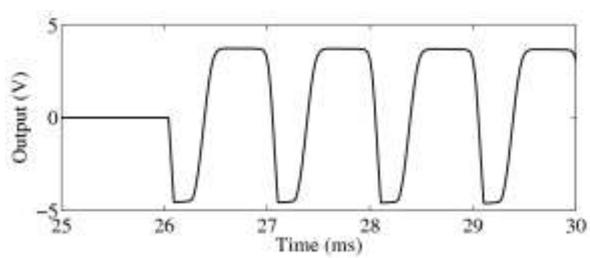
SSMN common-emitter amplifier



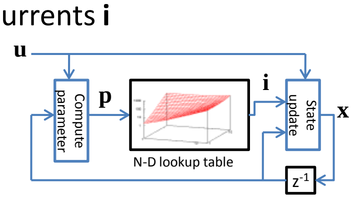


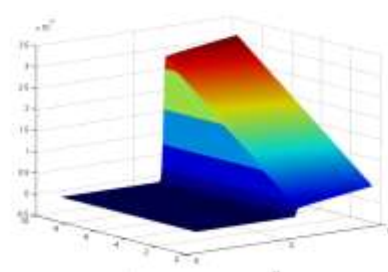
$$\mathbf{x} = \begin{bmatrix} V_{Cf} \\ V_{Ci} \\ V_{Co} \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} V_i \\ V_{CC} \end{bmatrix} \qquad \mathbf{i} = \begin{bmatrix} I_b(V_{be}, V_{bc}) \\ I_c(V_{be}, V_{bc}) \end{bmatrix}$$

Common-emitter results

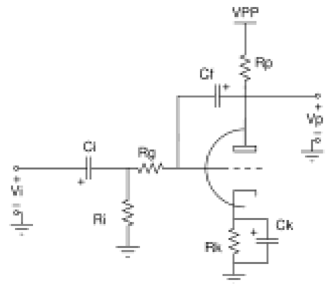


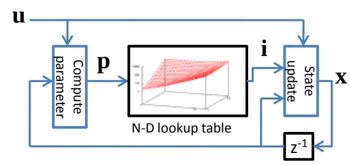
- 2D nonlinear mapping from parameter vector **p** to device currents **i**



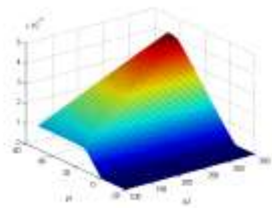


SSMN common-cathode tube preamp





$$\mathbf{x} = \begin{bmatrix} V_{Ci} \\ V_{Cf} \\ V_{Ck} \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} V_i \\ V_{PP} \end{bmatrix} \qquad \mathbf{i} = \begin{bmatrix} I_g(V_{gk}, V_{pk}) \\ I_p(V_{gk}, V_{pk}) \end{bmatrix}$$

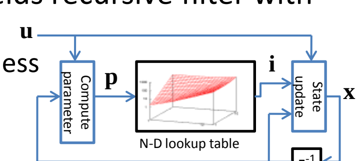


- 2D nonlinear mapping from parameter vector **p** to device currents **i**

CCRMA

Numerical methods for distortion effects extend prior work in musical acoustics.

- Explored application of WDF and SSMN to guitar distortion circuits
- WDF
  - Efficient and robust for special cases
  - Hard to apply in general situation: subject of ongoing research
- SSMN
  - Procedure to map circuits to SSMN formulation
  - Matrix-vector operations can be fast
  - Resulting static nonlinearity depends on sampling rate
- Numerical approximation of ODE yields recursive filter with static nonlinearity
  - Resulting nonlinearity is still memoryless
  - Memory is entirely in state vector



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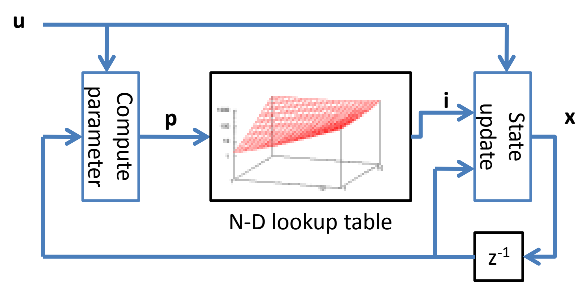

DAFx 2008
TKK, Espoo, Finland