Evaluating parameters of time-varying sinusoids by demodulation

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☆ What problem do we solve?

- Error of sinusoidal parameter estimation due to parameter variations.

☆ How?

- A demodulation method to cancel parameter variations, and
- A variable-window-size method to limit the harm done by parameter variations.

☆ Where does it start?

$$\hat{f} = \frac{\sum_{l=0}^{N-2} \eta_l(\hat{f}) \int_0^1 f(l+t)dt}{\sum_{l=0}^{N-2} \eta_l(\hat{f})},$$

$$\eta_l(g) = \sum_{n=l+1}^{N-1} \sum_{m=0}^{l} (n-m) w_m^2 w_n^2 a_n a_m \operatorname{sinc} 2\int_m^n \left(f(t) - g\right)dt$$

- "Error compensation in modeling time-varying sinusoids", DAFx'06.

☆ then?

$$\hat{f} - f_0 \sim D\tau^2, \qquad \kappa = \frac{\int_{-1}^{1}\int_{-1}^{1} w^2(m) w^2(n) \left(n^4 + m^4\right) dm dn}{3\int_{-1}^{1}\int_{-1}^{1} w^2(m) w^2(n) \left(n^2 - m^2\right)^2 dm dn}$$

$$D = \kappa f_2 + a_1 f_1,$$

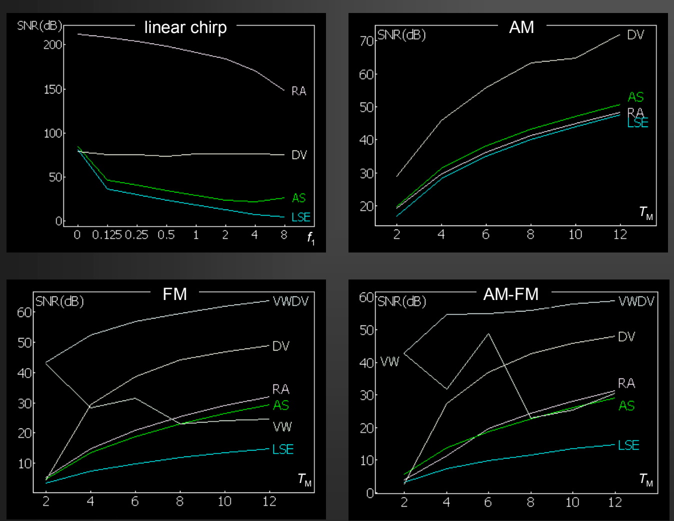
☆ Therefore to reduce the error, we

- cut down parameter dynamics before evaluating;
- use small window sizes when necessary.

☆ Test results

4 signal classes: linear chirp, AM, FM, AM-FM

Test criterion: synthesis SNR

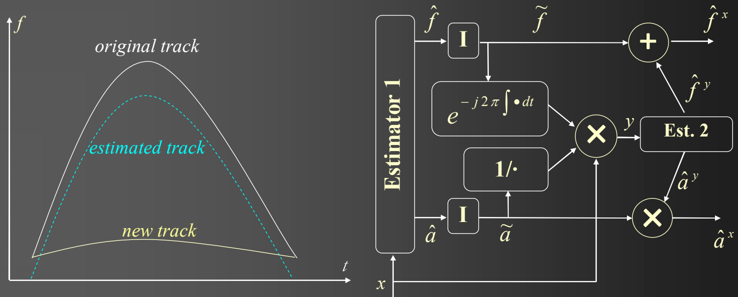


☆ Demodulation

$$y_n = \frac{x_n}{\tilde{a}_n} e^{-j2\pi \int_0^n \tilde{f}(t)dt}, \quad \hat{f}^x = \tilde{f} + \hat{f}^y, \quad \hat{a}^x = \tilde{a} \cdot \hat{a}^y, \quad \hat{\varphi}^x = \hat{\varphi}^y + 2\pi \int \tilde{f}dt$$

4 steps of demodulation method:

1) create an approximate sinusoid to x, 2) demodulate x using that approximate as y; 3) evaluate parameters of y; 4) update the approximate estimates using the new ones.



☆ Variable window size

3 steps of variable-window-size method:

1) find parameter variation speed D from primitive estimates; 2) reduce window size τ if $D\tau^2$ is above a preset value; 3) reestimate parameters using the new window size.

☆ Other tested methods

Reassignment:

$$\hat{t} = -\operatorname{Im}\frac{\partial X_k / \partial\omega}{X_k}, \quad \hat{f} = \frac{k}{N} + \operatorname{Im}\frac{\partial X_k / \partial t}{2\pi X_k},$$

$$\frac{\partial\hat{f}}{\partial\hat{t}} = \frac{1}{2\pi} \frac{\operatorname{Im}\frac{\partial^2 X_k / \partial t^2}{X_k} - \operatorname{Im}\frac{\left(\partial X_k / \partial t\right)^2}{X_k^{\,2}}}{-\operatorname{Im}\frac{\partial^2 X_k / \partial t \partial\omega}{X_k} + \operatorname{Im}\frac{\left(\partial X_k / \partial\omega\right)\left(\partial X_k / \partial t\right)}{X_k^{\,2}}}$$

Abe-Smith:

$$p = -\frac{\pi A''(\hat{f})}{A''^2(\hat{f}) + \Phi''^2(\hat{f})}$$

$$\alpha = -2p\Phi'(f), \beta = p\frac{\Phi''(\hat{f})}{A''(\hat{f})}$$

$$\hat{\hat{f}} = \hat{f} - \alpha\beta / 2\pi p$$

☆ Conclusion

-Demodulation method removes error due to low or medium signal dynamics;

-Variable-window-size method removes error due to large signal dynamics;

-These two method can be combined directly.

-These methods are free from side-effects common to model-fitting methods.

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