Generalization of the Derivative Analysis Method to Non-Stationary Sinusoidal Modeling

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Outline

Introduction

sinusoidal modeling

sound signal represented as a sum of sinusoids
controlled in amplitude and frequency (or phase)

(short-term) stationarity hypothesis

amplitude and frequency parameters considered as constant
within one (short-time) analysis frame

→ numerous (STFT-based) analysis methods…

- parabolic interpolation [Smith & Serra (ICMC 1987)]
- spectral reassignment [Auger & Flandrin (IEEE Trans. SP 1995)]
- derivative algorithm [Desainte-Catherine & Marchand (JAES 2000)]

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(one step further in the Taylor expansion of these parameters)

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- **derivative algorithm**

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Sinusoidal Modeling

[McAulay & Quatieri (IEEE Trans. ASSP 1986)]
[Serra & Smith (Computer Music Journal 1990)]

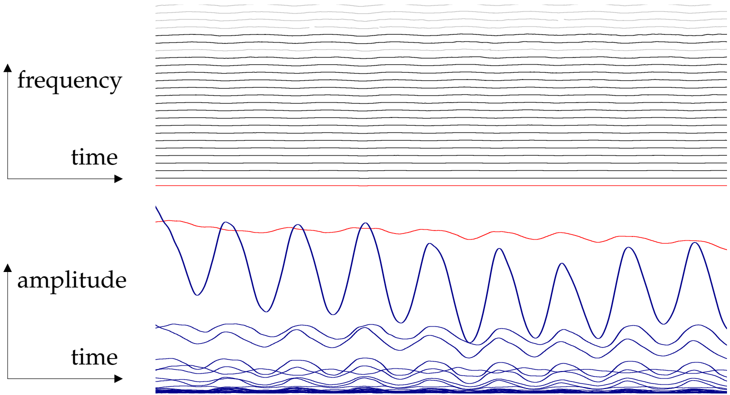
The (analytic) audio signal s is given by:

$$s(t) = \sum_{p=1}^{P} a_p(t) \ \exp(\phi_p(t)) \quad \text{with} \quad \frac{d\phi_p}{dt}(t) = \omega_p(t)$$

where P is the number of **partials**.

The functions a_p, ω_p, and ϕ_p are the instantaneous amplitude, frequency, and phase of the p^{th} partial, respectively.

Trajectories of the Partials



Frequencies and amplitudes, as functions of time,
of the partials of an alto saxophone sound, during ≈ 1.5s

Non-Stationary Case

For one partial ($P = 1$), for one frame (centered on time $t = 0$):

$$s(t) = \exp\left(\underbrace{(\lambda_0 + \mu_0 t)}_{\lambda(t)=\log(a(t))} + j \underbrace{\left(\phi_0 + \omega_0 t + \frac{\psi_0}{2} t^2\right)}_{\phi(t)} \right)$$

→ How to estimate the instantaneous parameters (at $t = 0$)?

- amplitude $\exp(\lambda_0) = a_0$
- **amplitude modulation** μ_0
- phase ϕ_0
- frequency ω_0
- **frequency modulation** ψ_0

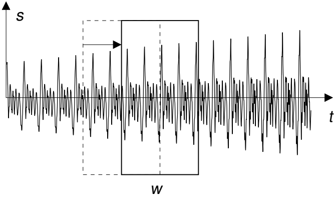
(**NB:** the stationary case is when $\mu_0 = \psi_0 = 0$)

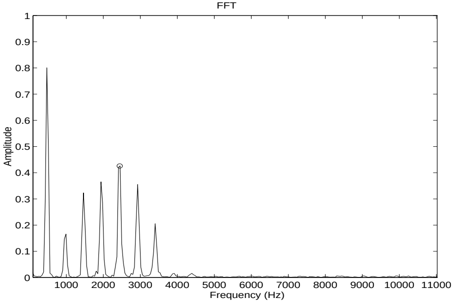
Outline

Pick Picking

Short-Term Fourier Transform

$$S_w(t,\omega) = \int_{-\infty}^{+\infty} s(\tau)w(\tau - t)\exp\left(-j\omega(\tau - t)\right)\, d\tau$$





using local maxima *m* of short-term magnitude spectrum

Analysis Window *w* (*e.g.* Hann window)

w with finite time support and **band-limited in frequency**:
for one peak corresponding to one specific partial,
the influence of the other partials can be neglected
(in the general case when $P > 1$)

$$S_w(0, \omega) = \underbrace{a_0 e^{j\phi_0}}_{s_0} \cdot \Gamma_w(\omega_0 - \omega, \mu_0, \psi_0) \quad \text{where}$$

$$\Gamma_w(\omega, \mu_0, \psi_0) = \int_{-\infty}^{+\infty} w(t) \exp\left(\mu_0 t + j\left(\omega t + \frac{\psi_0}{2} t^2\right)\right) dt$$

(**NB:** in the stationary case where $\Gamma_w(\omega_0 - \omega, 0, 0) = W(\omega - \omega_0)$,
the peak corresponds to the spectrum W of the analysis window
centered on frequency ω_0 and scaled by the complex amplitude s_0)

Reassignment Method

uses the derivatives of the window

$$\hat{\omega}_0 = \hat{\omega}(0, \omega_m) \quad \text{where} \quad \hat{\omega}(t, \omega) = \omega - \underbrace{\Im\left(\frac{S_{w'}(t, \omega)}{S_w(t, \omega)}\right)}_{-\Delta_\omega}$$

$$\hat{\mu}_0 = \hat{\mu}(0, \omega_m) \quad \text{where} \quad \hat{\mu}(t, \omega) = -\Re\left(\frac{S_{w'}(t, \omega)}{S_w(t, \omega)}\right)$$

$$\hat{\psi}_0 = \hat{\psi}(0, \omega_m) \quad \text{where} \quad \hat{\psi} = \frac{\Im\left(\frac{S_{w''}}{S_w}\right) - \Im\left(\left(\frac{S_{w'}}{S_w}\right)^2\right)}{\Re\left(\frac{S_{tw} S_{w'}}{{S_w}^2}\right) - \Re\left(\frac{S_{tw'}}{S_w}\right)}$$

$$\text{finally} \quad \hat{a}_0 = \left|\frac{S_w(\omega_m)}{\Gamma_w(\Delta_\omega, \hat{\mu}_0, \hat{\psi}_0)}\right| \quad \text{and} \quad \hat{\phi}_0 = \angle\left(\frac{S_w(\omega_m)}{\Gamma_w(\Delta_\omega, \hat{\mu}_0, \hat{\psi}_0)}\right)$$

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finally $\hat{a}_0 = \left|\frac{S_w(\omega_m)}{\Gamma_w(\Delta_\omega, \hat{\mu}_0, \hat{\psi}_0)}\right|$ and $\hat{\phi}_0 = \angle\left(\frac{S_w(\omega_m)}{\Gamma_w(\Delta_\omega, \hat{\mu}_0, \hat{\psi}_0)}\right)$

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Outline

Frequency $\hat{\omega}_0$ and Amplitude Modulation $\hat{\mu}_0$

uses the derivatives of the signal

(the derivative of an exponential is an exponential. . .)

$$s'(t) = (\mu_0 + j(\omega_0 + \psi_0 t)) \cdot s(t)$$

$j\psi_0 t$ is an odd function $\implies$ its spectrum is real. . .

$$\hat{\omega}_0 = \Im\left(\frac{S'_w}{S_w}(\omega_m)\right)$$

moreover, its spectrum is null at frequency zero. . .

$$\hat{\mu}_0 = \Re\left(\frac{S'_w}{S_w}(\hat{\omega}_0)\right)$$

(**NB:** in theory, equivalent to spectral reassignment estimators)

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Frequency Modulation $\hat{\psi}_0$

with the second derivative...

$$s''(t) = ({\mu_0}^2 - {\omega_0}^2 - 2\omega_0\psi_0 t - {\psi_0}^2 t^2) + j(\psi_0 + 2\mu_0\omega_0 + 2\mu_0\psi_0 t) \cdot s(t)$$

using the same kind of properties...

$$\hat{\psi}_0 = \Im\left(\frac{S''_w}{S_w}(\hat{\omega}_0)\right) - 2\hat{\mu}_0\hat{\omega}_0.$$

Amplitude $\hat{a}_0$ and Phase $\hat{\phi}_0$

finally

$$\hat{a}_0 = \left| \frac{S_w(\hat{\omega}_0)}{\Gamma_w(0, \hat{\mu}_0, \hat{\psi}_0)} \right|$$

$$\hat{\phi}_0 = \angle \left(\frac{S_w(\hat{\omega}_0)}{\Gamma_w(0, \hat{\mu}_0, \hat{\psi}_0)} \right)$$

Discrete Derivative

practical problem:
How to get the derivatives s' from the (discrete-time) signal s?

$$s'(t) = \lim_{\epsilon \to 0} \frac{s(t+\epsilon) - s(t)}{\epsilon}$$

- a bad idea: approximate it by the difference ($\epsilon = 1/F_s$)
- a good idea: use the ideal differentiator filter…

$$h[n] = F_s \frac{(-1)^n}{n} \text{ for } n \neq 0, \text{ and } h[0] = 0$$

…windowed by the Hann window (of length 1023)

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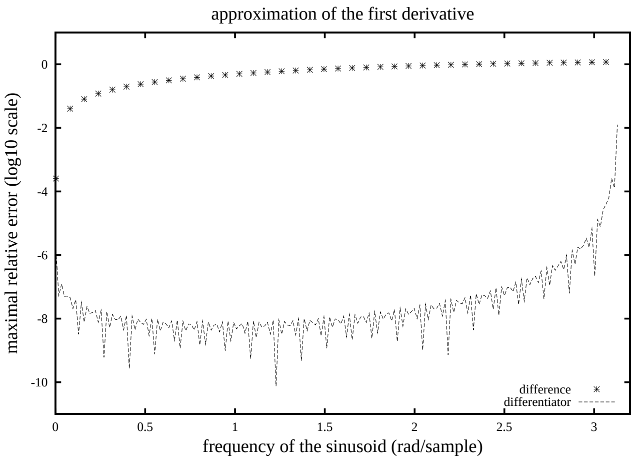


(high frequencies – above 3/4 Nyquist – are problematic)

Outline

Experiments

3 methods tested:

- reassignment (R) (the champion)
- 2 flavors of the derivative (D) (the challenger)
  - TD: theoretic derivative (derivative known analytically)
  - ED: estimated derivative (with the differentiator filter h)

(frame size $N = 511$, sampling frequency $F_s = 44100$Hz)

→ estimation precision for each parameter,
compared to the Cramér-Rao Bound (CRB)
(the best performance achievable by an unbiased estimator),
in presence of Gaussian white noise with various SNRs;

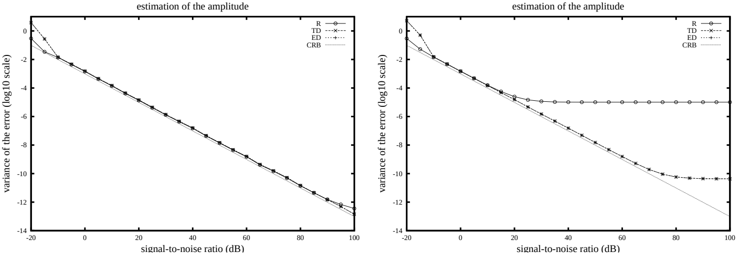
Experiments

with 5 parameters to test...

- ω_0: 99 frequencies linearly distributed in $(0, 3F_s/8)$Hz,
- ϕ_0: 9 phases linearly distributed in the $(-\pi, +\pi)$ interval,
- μ_0: either 0 (stationary case) or in $[-100, +100]$ (AM),
- ψ_0: either 0 (stationary case) or in $[-10000, +10000]$ (FM),
- amplitude a_0 set to 1.

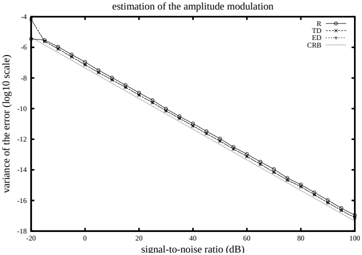
(conditions similar to [Betser *et al.* (IEEE Trans. SP 2008)], where the reassignment performs best, at least regarding frequency estimation)

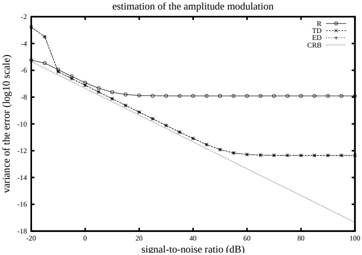
Results: Amplitude $\hat{a}_0$



D performs better in the non-stationary case

Results: Amplitude Modulation $\hat{\mu}_0$





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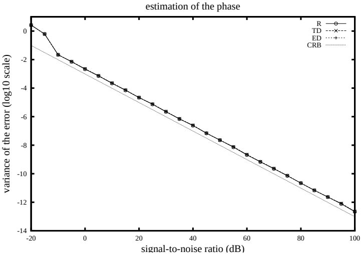
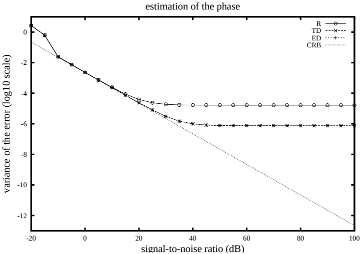
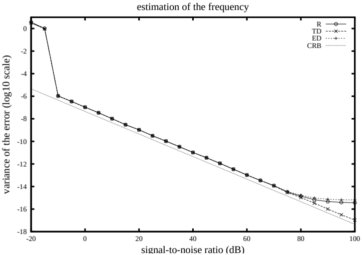
Results: Phase $\hat{\phi}_0$

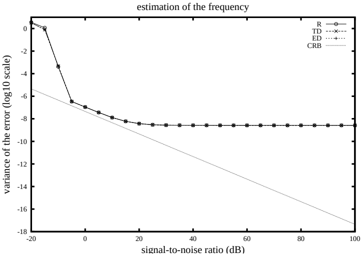




D performs better in the non-stationary case

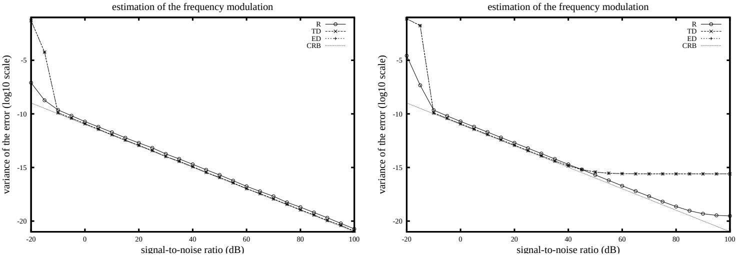
Results: Frequency $\hat{\omega}_0$





although R and ED perform equally, TD indicates that
ED can beat R in the stationary case, with a better derivative

Results: Frequency Modulation $\hat{\psi}_0$



R performs better in the non-stationary case

Conclusion

Summary:

- the derivative method is generalized to the non-stationary case,
- computing the discrete derivative is not a problem anymore,
- the derivative method outperforms the reassignment method in all cases except for the estimation of the frequency modulation.

Future Work:

- understand why the reassignment method is better in this case,
- study the behavior of the methods in more complex AM/FM conditions (such as sinusoidal tremolo/vibrato),
- propose a very fast algorithm for the new method. . .