Modeling of long and complex responses using Kautz filters and time-domain partitions

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A Transcription of the Title

**Complex responses** – long duration, high sample rate, complex time-frequency characteristics: distributed onset, high mode/resonance density, variable decay rates – in our case, measured room impulse responses

**Kautz filters** – a special class of pole-zero filters, originating from rational orthonormal basis representations – favorable properties due to orthogonality and the transversal (allpass) structure

**Time-domain partitions** – splitting the modeling task into manageable portions, a time-domain approach using 1) interlaced (polyphase) decomposition or 2) sequential segmentation of the target response

### Motivation and premises

- The need for transfer function models for complicated systems such as room impulse responses
- Traditional choices: finite or infinite impulse response (FIR/IIR) linear time-invariant (LTI) filters using conventional structures and design methods
- Harsh reality:
  - FIR: high filter orders are required due to the nature of the target responses – a practical burden
  - IIR: in addition or consequently, potential stability issues – or simply, impossible to construct – also a principled problem
- Kautz filters: provide efficient high-order IIR filters for many purposes with considerably less computational complexity compared to FIR filters
- For really tough cases, such as concert hall impulse responses at high sample rates/full bandwidth: various subband techniques, partial artificial reproduction and/or the proposed time-domain partitions

### Kautz filters in a nutshell

The generic form of a Kautz filter is given by the transfer function

\[ H(z) = \sum_{i=0}^{N} w_i G_i(z) \]

and it is specified by two sets of parameters:

- Poles \( \{ z_i \}_{i=0}^{M} \), \( |z_i| < 1 \), define the orthonormal (Kautz) functions \( G_i(z) \), \( i = 0, \ldots, N \) – and the filter order
- Tap-output weights \( \{ w_i \}_{i=0}^{N} \) – in our case, orthogonal (Kautz-Fourier) expansion/projection coefficients

The recurrent form of the basis functions \( G_i(z) \) result in the depicted transversal structure:

The [Kautz filter](#) – for complex conjugate pole pairs, a modified real Kautz filter structure is used. More familiar special cases of the Kautz filter:

- For \( z_i \equiv 0 \) it degenerates to an FIR filter
- For \( z_i \equiv a, -1 < a < 1 \), it is a Laguerre filter

### Sales talk – why Kautz filters?

- Efficient pole-zero modeling using FIR filter-like structure and design
- Kautz filter techniques using our pole generation method – challenges conventional IIR filter design
- Unconditionally stable, good numerical properties due to the transversal (allpass) structure
- Trivial model reduction/modification schemes due to orthogonality

### Basic design steps

- Generate candidate pole sets (using our BU-method) w.r.t. the target response and desired model orders
- Possible variants (of the BU-method) using combined warping and frequency-zooming techniques
- Evaluate the Kautz-Fourier weights, inspect the result
- If needed, modify the pole set: prune, tune, attach, cluster – re-evaluate the weights, construct the model

### Example I: Kautz models for a measured room impulse response (sample rate 22050 Hz, 8192 samples, 371 ms):

- A straightforward demonstration of the BU-method
- Kautz models, orders 60–340, in steps of 40
- Intermediate Bark-scale warping \((\lambda \approx 0.65)\) is used to emphasize low-frequencies (warped BU-method)

### Utilizing two time-domain partitions

- Impossible dimensions: for example, 100 000 samples and more than 1000 relevant resonant frequencies
- Solutions: subband techniques, artificial reproduction
- Alternative or additional methods: following partitions

### Polyphase Decomposition

The \( M \)-th order polyphase decomposition of an impulse response \( h(n) \), \( n = 0, \ldots, N M \), is given by

\[ H(z) = \sum_{k=0}^{M-1} z^{-k} H_k(z^{MN}), \quad H_k(z) = \sum_{n=0}^{N-1} h(Mn + k) z^{-n} \]

The component FIR filters \( H_k(z) \) are then simply approximated by Kautz filters:

- Design the Kautz sub-filters w.r.t. \( h_k(n) \)
- The construction rely on “waveform matching”
- The required sub-filter order \( N_k \) is typically \( N/10 \)

### Example II: a measured room impulse response (44.1 kHz, 32768 s.), approximated by an \( 61 \times 60 \)th order Kautz model:

- The implied sub-signal length is 512 samples
- A fixed sub-filter order 60 – could be optimized
- The normalized square error is 0.0024

### Sequential Segmentation

A given target response may also be partitioned into successive segments corresponding to the transfer function

\[ H(z) = \sum_{k=0}^{N-1} h_k(n) z^{-n} \]

or schematically as below \((d = t_0)\) is the initial delay:

The component FIR filters \( H_k(z) \) are then once more approximated by Kautz filters, taking into account the possible IIR leakage:

- Choose the partition, isolate the target response
- Construct the Kautz model of a desired/sufficient order
- Subtract the possible “overflow” of the Kautz model responses when forming following target responses

### Example III: the early part of a concert hall impulse response modeled by a chosen partition of Kautz models:

- Partition into relevant events – early reflections?
- A low-order model – exact match by increasing order
- “Parametric” control of the early response?

### Conclusions and Remarks

- Kautz filter techniques are proposed as a genuine alternative to traditional FIR/IIR filter design
- Two simple time-domain partitions are used for unwrapping long and complex responses
- Just an idea – many open aspects, and possibilities?