# New Aspects on Nonlinear Power Amplifier Modeling in Radio Communication System Simulations

Mauri Honkanen and Sven-Gustav Häggman

Institute of Radio Communications (IRC) Helsinki University of Technology, Communications Laboratory P.O.Box 3000, FIN-02015 HUT, Finland E-mail: Mauri.Honkanen@hut.fi, sgh@hut.fi

## ABSTRACT

A new semi-physical behavioral amplifier model to characterize the AM/AM conversion of a bipolar solid-state high power amplifier in a mobile phone is introduced. The functional model is able to copy the low-voltage features of a real amplifier causing crossover distortion. Hence it gives an improved description of the nonlinearity of Class AB amplifiers widely used in mobile phones employing a linear modulation scheme. The AM/AM conversion model is accompanied by an AM/PM conversion model function to complete the amplifier model. The model is utilized in mobile communication system studies to investigate increase of common channel and adjacent channel interference due to the nonlinear behavior of the amplifier. The new model is compared to the existing widely used functional models, and it is found to give an excellent match to the two-tone performance of a measured amplifier.

#### **I. INTRODUCTION**

Even though today's European mobile systems like GSM and DECT use modulation schemes with constant envelope [1], there is a certain interest towards linear modulation formats like  $\pi/4$ -DQPSK, which is already in use in Northern America and Japan [2]. The linear modulation schemes provide the system with higher spectrum efficiency, but they also mean that the modulated signal has a nonconstant envelope. Unfortunately, signals with amplitude variations are subject to spectrum regeneration and inband interference from nonlinearities in the transmit systems, causing adjacent channel and common-channel interference, respectively.

Traditionally, the required high power amplifier (HPA) linearity is guaranteed by backing off the amplifier, i.e. the operation point of the amplifier is backed off from the saturation level to the more linear region of the amplifier characteristic. However, by doing so, the efficiency of the amplifier is decreased significantly, since the amplifier is operating most efficiently at or near the saturation level. In mobile phones the situation is most severe, because the final amplifier is the most power consuming part of a mobile telephone, and low efficiency of the amplifier results in frequent battery recharging. A certain amount of back-off is definitely needed to ensure that the distortion is reasonable. Nevertheless, in case of Class AB amplifiers commonly used

in mobile phones, it must be noted that there are also other nonlinear features in the HPA transfer characteristics in addition to the saturation.

Because adjacent channel interference levels have a significant impact on the overall system capacity, system specification work and comparison of different modulation schemes require an adequate HPA modeling to be used in the system simulations. In theory, nonlinear power amplifiers used in communication systems can be described completely by Volterra series expansion [3]. Nevertheless, that approach is far too complicated to be utilized in a simulation, where the nonlinear HPA block is typically just a small part of a complex system. A high-level power amplifier model is required to produce a correct enough output waveform as a function of an input waveform. For simulation purposes the nonlinearity is often represented as a look-up table which contains measured input-output relations of the device. However, the look-up table approach necessitates utilization of interpolation and extrapolation algorithms which might lead to unexpected behavior in the saturation region or generate discontinuities to the HPA transfer characteristic curve. Therefore an approach involving simple functional relationships with some physical basis was chosen to describe the amplifier. With that approach the characteristics are known everywhere, even beyond the normal region of interest.

The new bipolar amplifier model introduced here experiences an exponential behavior in the envelope transfer characteristics at low input voltages. This feature distinguishes it from previously presented functional nonlinearity models [3, 4]. The introduction of an exponential characteristic in a HPA model has been found to have quite a significant impact when the intermodulation behavior of a real HPA is attempted to model as closely as possible.

#### **II. MODEL DERIVATION**

Even though there is also some phase distortion present in a real nonlinear high power amplifier, the amplitude distortion is regarded as the dominating counterpart. Thus more emphasis has been put on the accurate amplitude distortion modeling. The model represents bipolar amplifiers, since it was thought that the same functional relationship is not capable of describing both bipolar and FET amplifiers adequately because of the fundamental differences between these transistor types. Essentially, the base-emitter junction of a bipolar transistor is nonlinear generating an exponential input nonlinearity. Hence, if the base-emitter voltage is increased, the base current grows exponentially. By assumption the ratio between the collector current and the base current is constant at all input voltages and the output voltage is measured over the load resistance through which the collector current flows. This relationship is easily described by the familiar diode expression

$$V''_{out,1}(V_{in}) = e^{kV_{in}} - 1$$
(1)

where k is a parameter which determines the steepness of the exponential curve and  $V_{in}$  is the input voltage. However, as the input voltage grows the exponential behavior is strongly linearized by the feedback present in the amplifier configuration, e.g. feedback provided by the external emitter resistance. Thus a part of the exponential curve is replaced by a linear portion with a nominal amplification v:

$$V_{out,2}''(V_{in}) = v \cdot V_{in} + b , \qquad (2)$$

The transition from the exponential part to the linear part happens at the point where the derivative of the exponential curve is equal to the nominal amplification, which ensures that the transition is continuous. The transition point is given by

$$V_{in,tr} = \frac{1}{k} \ln \frac{v}{k}.$$
 (3)

and the value for b in Eq. 2 is given by

$$b = e^{k \cdot V_{in,tr}} - 1 - v \cdot V_{in,tr} \,. \tag{4}$$

The parameter  $V_b$  makes it possible to adjust the zero input voltage location on the amplification curve so that real power amplifiers biased to different operation classes, e.g. class A, AB, B and C, can be modeled. The zero voltage location on the curve is moved to the right by an amount determined by  $V_b$ . Thus, the bigger  $V_b$  value, the more linear amplification one achieves. In a way the  $V_b$  parameter is analog to the real amplifier biasing. Now the biased input voltage is

$$V_{in,biased} = V_b + V_{in} \tag{5}$$

and the corresponding output zero voltage shift is achieved correctly by subtracting

$$V_{out,zero} = e^{k \cdot V_b} - 1 \tag{6}$$

from the output voltage values obtained from Eq. 1 and Eq. 2. Thus, whether the absolute value of a biased input waveform sample is smaller or bigger than  $V_{in,tr}$  given by Eq. 3, the corresponding output voltage sample is obtained from

$$V'_{out}(V_{in}) = e^{k(V_{in}+V_b)} - 1 - V_{out,zero} , \qquad V_{in} + V_b \le V_{in,tr}$$
(7)  
=  $e^{k \cdot V_b} (e^{k \cdot V_{in}} - 1)$ 

$$V'_{out}(V_{in}) = v \cdot (V_{in} + V_b) + b - V_{out,zero} = v \cdot (V_{in} + V_b) + b - e^{k \cdot V_b} + 1, \quad V_{in} + V_b > V_{in,tr} \quad (8)$$

Because amplification and the maximum output voltage are limited by the finite power supply to the amplifier, the output starts to saturate at a certain input voltage level. This model utilizes Rapp's model [4] to produce smooth saturation to the envelope transfer characteristic, and hence the final output voltage is given by

$$V_{out}(V_{in}) = \frac{V'_{out}}{\left(1 + \left[\left(\frac{V'_{out}}{A_0}\right)^2\right]^p\right]^{\frac{1}{2p}}}$$
(9)

where  $p \ge 0$  and  $A_0 \ge 0$ . In the equation,  $V'_{out}$  is given by either Eq. 7 or Eq. 8,  $A_0$  is the limiting output amplitude and p is the smoothness parameter defining the transition smoothness from the linear region to the saturated region.



Figure 1: Envelope transfer characteristics of a limiter, Rapp's amplifier model [4] and the proposed model.

Normally phase distortion is more dependent on the particular amplifier design than amplitude distortion. Here it is assumed that when enough power is applied to the input of the amplifier the transistors in the amplifier start to conduct. Then a clear stepwise change in the phase shift is observed. The magnitude of the step relies on the biasing of the amplifier. When input power is further increased, the phase shift curve begins to descend. This behavior is quite closely modeled with a mathematical expression

$$\Delta \phi (V_{in}) = \frac{b}{1 + e^{-c(V_{in} - a)}} \cdot \frac{1}{p_1 \cdot V_{in} + p_2}, \qquad (10)$$

where *a* is the cut-off input voltage where the transistors start to conduct, *b* is the switch-on step magnitude in degrees, *c* determines the steepness of the step-wise change, and  $p_1$  and  $p_2$  are parameters adjusting the descending slope to an appropriate one. The minimum phase shift given by the expression above is zero degrees. However, this is insignificant from the system point of view for only the range of the phase shift is what matters; not the absolute phase shift values.

Due to the bandpass nature of the signals of interest, an equivalent lowpass system approach can be employed. Hence, the complex output envelope of the amplifier model is given by

$$\widetilde{y}(t) = V_{out} \left[ A(t) \right] e^{j \left\{ \varphi(t) + \Delta \phi[A(t)] \right\}}$$
(11)

where A(t) and  $\varphi(t)$  are the instantaneous amplitude and phase of the input signal, respectively,  $V_{out}(\cdot)$  is the AM/AM distortion function obtained from Eq. 9, and  $\Delta \phi(\cdot)$  is the AM/PM distortion function given by Eq. 10.

#### **III. MODEL VERIFICATION**

The parameters of the proposed model were optimized with the golden section method [5] in order to fit the gain and phase shift provided by the model to the measured gain and phase shift of a class AB mobile phone amplifier.



Figure 2: The CW gains and phase shifts given by the proposed amplifier model and the measured amplifier.

The model capability is verified by means of a two-tone test which is the most widely used approach in RF power amplifier nonlinearity characterization. Two sinusoidal signals with equal amplitude and a frequency separation of a few kilohertz constitute a waveform which is applied to the nonlinear device. Because of the nonlinearity, intermodulation products of the applied tones can be found at noncommensurate frequencies. The two-tone test gives the intermodulation rejection (or intermodulation distortion, IMD) which is a single figure of merit at a given operation point. The two-tone IMD is expressed in dBc, meaning decibels below either of the two carriers. In theory, when the input power level is increased, the third-order power level has a 3:1 slope compared to the fundamental component. However, the 3:1 slope of the third-order product power level is not followed in the region where the amplifier begins to compress. On the contrary, there might even appear a local notch in the third-order power level. Hence, the intermodulation behavior of the amplifier near the compression cannot be predicted based on the measurements at low power levels.



Figure 3: Two-tone test results of the limiter HPA model.

The simulated two-tone test results of the limiter model and Rapp's model are presented in Fig. 3 and Fig. 4, together with the measured intermodulation results of a bipolar mobile phone amplifier. Generally, when the power of the sinusoidals is increased, more distortion and intermodulation power are generated because of the increasing influence of the saturation. However, according to the measurements, there is some distortion present also at the low power levels. It can be easily seen that neither the limiter model nor Rapp's model are able to characterize this low power distortion. Additionally, an inherent feature of a bipolar amplifier is a notch in the third-order intermodulation power near the saturation. The models mentioned above are incapable of simulating this behavior.

It is important to observe that, quite opposite to the functional models discussed above, the third-order and fifthorder intermodulation products of the proposed model are very close to the measured ones. There is some deviation in the saturated region, but that is quite expectable. The seventh-order intermodulation product is not modeled as well as the lower ones, but, on the other hand, its power is much smaller than that of the lower ones and thus its significance is of less importance. The most important observation is that the proposed model is capable of characterizing the notch of the third-order intermodulation product near the saturation. This phenomenon indicates that at a certain power level the fifth-order intermodulation product might even be stronger than the third-order, as illustrated in Fig. 5.



Figure 4: Two-tone test results of Rapp's HPA model.



Figure 5: Two-tone test results of the proposed HPA model.

#### IV. MODEL COMPARISON WITH QPSK SIGNAL

The effect of the exponential behavior at low power levels is studied by comparing the simulation results given by the proposed model, ideal limiter model and Rapp's model in a QPSK transmitter-receiver configuration shown in Fig. 6. The parameters for the proposed new model are the same as those used in the two-tone test simulations. The signal is ideally root-raised cosine filtered with a roll-off factor  $\alpha = 0.35$  and 20 samples per symbol is taken in order to reveal intermodulation products accurately enough. The output of the transmitter amplifier to be modeled is backed off by 3 dB, where the output back-off (OBO) is defined by

$$OBO = 10 \log_{10} \frac{A_0^2}{A_{out,avg}^2},$$
 (12)

where  $A_0$  is the limiting output amplitude and  $A_{out,avg}$  is the average amplifier output amplitude.



Figure 6: System configuration in adjacent channel interference simulations.



Figure 7: Simulated power spectral density of QPSK with the proposed nonlinear amplifier model.

The assumption that the amplitude distortion is the dominating counterpart in the nonlinear distortion in case of quite linear class AB amplifiers is clearly sustained by Fig. 7 where the spectral regrowth of QPSK signal contributed by the phase distortion is negligible compared to that caused by the amplitude distortion. Thus, the emphasis put on the accurate AM/AM distortion modeling is justified. The power spectral densities in Fig. 8 demonstrate that the exponential behavior has a significant effect on the spectral regrowth. An accurate modeling of two-tone test intermodulation products results in significantly higher fifth-order IMD levels than those suggested by Rapp's model and the limiter. Clearly, due to the ideal nature of a limiter, spectral regrowth proposed by the limiter model is notably small at reasonable back-off values, and hence the capability of limiter to model a mobile phone amplifier is questionable.



Figure 8: Simulated power spectral density of root-raised cosine filtered QPSK with different power amplifier models.



Figure 9: Simulated bit error ratios as a function of the first and second adjacent channel interference level.

The common channel interference caused by the amplifier nonlinearity is quite insignificant in case of QPSK signal, for the modulation scheme is quite robust. Nevertheless, adjacent channel interference is strongly dependent on the type of amplifier model used, as suggested by Fig. 8. With a channel separation of  $\Delta f = (1+\alpha)R_s$ , the effect of pure adjacent channel interference without a channel model or coding is studied. In case of the proposed model, it is assumed that the receiver is able to estimate the average phase distortion caused by the nonlinear amplifier. Fig. 9 reveals how strong first and second adjacent channels affect the bit error rate performance. The signal-to-interference ratio is defined by

$$SIR = 10\log_{10}\frac{P_s}{P_{adj}},$$
 (13)

where  $P_s$  is the power of the channel under study and  $P_{adj}$  is the power of the adjacent channel.

#### **V. CONCLUSIONS**

We have shown that a minor exponential behavior in the transfer characteristics of a bipolar power amplifier has a remarkable impact on the spectral regrowth, especially on the ratio of third and fifth-order intermodulation products. This exponential behavior is inherent to bipolar amplifiers because of the basic physics of the device and hence it has to been taken into consideration in order to reliably model adjacent channel interference. Functional models presented in the literature [3, 4] are clearly incapable of modeling this characteristic, and hence they are adequate only when Class A amplifiers seldom used in mobile phones are modeled. The new behavioral HPA model described in this paper seems to model very well the characteristics of a bipolar power amplifier, at least in case of a two-tone test. Thus, it is a clear improvement in the area of a functional power amplifier modeling. However, even though the two-tone test is a favored measure of nonlinearity, it must be noted that the amplitude and frequency distribution of a two-tone test signal differ quite significantly from that of a digitally modulated signal. Hence, spectral regrowth measurements with modulated signals are needed to further verify the viability of the proposed model.

#### ACKNOWLEDGMENTS

The work was financed by Technology Development Center, Nokia, Telecom Finland and Helsinki Telephone Company. The authors would like to thank Ossi Pöllänen, Esko Järvinen and Eero Nikula from Nokia Research Center for the fruitful discussions and valuable comments.

### REFERENCES

- W. C. Y. Lee, Mobile Cellular Telecommunications: Analog and Digital Systems, 2nd ed., New York: McGraw-Hill, 1995, ch. 15, pp. 463-550.
- [2] J. S. Kenney and A. Leke, "Power Amplifier Spectral Regrowth for Digital Cellular and PCS Applications", Microwave Journal, vol. 38, no. 10, pp. 74-92, October 1995.
- [3] M. C. Jeruchim, P. Balaban and K. S. Shanmugan, Simulation of Communication Systems, New York: Plenum Press, 1992, ch. 2, pp. 141-178.
- [4] C. Rapp, "Effects of HPA-Nonlinearity on a 4-DPSK/OFDM-Signal for a Digital Sound Broadcasting System", in Proceedings of the Second European Conference on Satellite Communications, Liege, Belgium, October 22-24, 1991, pp. 179-184.
- [5] M. S. Bazaraa, H. D. Sherali and C. M. Betty, Nonlinear Programming, Theory and Algorithms, New York: Wiley, 1993, ch. 8, pp. 270-272.