# Binomial array as a multistate phase diversity antenna 

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#### Abstract

- If two in-phase plane waves arrive at an antenna from different directions, the resulting signal is not necessarily the coherent sum of the two incoming signals. This is due to the fact that the antenna radiation pattern is actually a complex quantity containing amplitude and phase properties. Thus, it can be stated that antenna phase characteristics may vary as a function of direction. In free space the antenna phase pattern doesn't have any role in radio communications in contrast to mobile communications in an urban microcell where the radio channel is very complicated due to multipath propagation. Array antennas offer possibilities to control its phase properties in transmission and reception, which offers a simple technique to improve the combination of received multipath signal components. Binomial arrays are one possibility to implement similar directive beams which, however, have different phase patterns compared to each other. A set of these beams could be used in a discretely optimizing receiver system much in a same way as other diversity techniques are used. A four element array and results from a microcell radio channel environment simulation are presented as a demonstration of this technique.


Keywords- Mobile communications, antenna diversity.

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## I. Introduction

Urban areas are problematic for cellular radio networks because of numerous reasons. One considerable difficulty in a microcellular environment, where cell sizes are small and antennas for that reason must be kept below rooftops, is the fast-fading phenomenon. The complexity of the radio wave propagation environment allows several propagation paths to co-exist and this constitutes in the wide angular spread of the received signal components. In addition to the vast spreading of the received signal directions, the different signal components also are in different phases with respect to each other. Therefore, the received signal can be completely faded, if all these signal components are destructively summed in the receiver.

System performance degradation due to the interference and fading can be reduced with antenna and receiver technologies by using antenna diversity i.e. choice between multiple received antenna signals, which should preferably be as uncorrelated as possible. Many such arrangements have already been introduced in the literature, like space-, polarization-, jitter-, switched beam- and frequency diversity schemes [1] - [4]. Although most of these diversity techniques offer profitable gain compared to a single antenna reception, all of them have their disadvantages, too. For instance, switched beams can make the network planning in a microcellular environment much more difficult, since it is desirable to keep the interference from other cells as low as possible. Space diversity is often considered unappealing because it often requires more room for antenna structures than the other methods. The diversity arrangement proposed in this text can best be compared with space diversity methods. Both of these antenna arrangements, space diversity and the method described here, provide system performance gain by altering the phase differences in which the incoming multipath radio wave components are combined at the receiving antenna.

Array antennas have properties, which allow different phase patterns to exist, even if an antenna beam, i.e. the amplitude pattern, itself stays the same. The ambiguity of the phase pattern of an antenna array provides a possibility to optimize its phase properties without any affect on the radiation beam of the array. This enables one to combine more constructively the multipath signal components, which arrive to the receiver from many directions with different phases. For example, if two dominant rays approaching the receiving array cancel each other due to the opposite phases, the cancellation of the received signal can be prevented by having an antenna phase pattern, which also has opposite phases towards the incident waves.

Hence a destructive interference can be transformed to a constructive one by making an appropriate change in the phase pattern of the receiving antenna.

## II. Theory

Antenna arrays are constructed by antenna elements, which are similar and only displaced from each other without rotation. If this condition is valid, the effect of the array to the overall radiation pattern can be analyzed separately by the use of array factor $A F$. In case of a linearly aligned array, where all the antenna elements are equally spaced, the array factor of the antenna can be expressed by using z -transformation in the following polynomial form [5]

$$
\begin{equation*}
A F=A \prod_{n=1}^{N-1}\left(z-z_{n}\right)=\sum_{n=1}^{N}\left(A_{n} z^{n-1}\right) \tag{1}
\end{equation*}
$$

where $z=e^{-j k d u}$ is the z -transformation variable, $k$ is the wavenumber, $d$ is the element spacing and $u=\cos \alpha$ is the direction cosine, where $\alpha$ is the angle between the array axis and the observation direction. The resulting antenna pattern is fixed, if either the z -transformation plane nulls, $z_{n}$, or antenna element excitations, $A_{n}$, are known. $N$ is the number of antenna elements in the array.

The array factor is either a sum or a product of complex quantities and therefore it must be considered as a complex valued quantity. As the complex argument of the array factor varies as a function of direction, the antenna forms a complex sum of multipath signals arriving to the receiver.

The direction dependence of the phase pattern of an antenna doesn't have any importance if only one single propagation path exists between the receiver and the transmitter, i.e. in free space radio channel. As long as this assumption is valid, transmitted signal power from one antenna to another can be evaluated by applying Friis formula. The situation is totally different if the radio channel contains obstacles causing diffractions and reflections of radio waves, which in general gives rise to multipath propagation. Hence, both the phases of the individual waves and the array phase pattern play significant roles in the reception of the radio signal arriving from many directions. In this case the signals propagating different routes interfere and therefore the received power cannot be computed as in the free space. However, the Friis formula can be generalized if the diffraction properties of the radio channel are known. The generalized Friis formula for a multipath propagation environment could be written in the following form

$$
\begin{equation*}
\frac{P_{R}}{P_{T}}=k_{g} k_{L} \frac{1}{4 R_{a}} \frac{\eta}{r^{2}} \frac{1}{\int_{\Omega}\left|\mathbf{h}_{T}\right|^{2} d \Omega}\left|\sum_{n=1}^{M} \mathbf{h}_{R n} \cdot \overline{\bar{D}}_{n} \cdot \mathbf{h}_{T n} e^{-j k r_{n}}\right|^{2} \tag{2}
\end{equation*}
$$

where all diffractions and reflections of multipath signals arriving to the receiver are modeled by dyadics $\overline{\bar{D}}_{n}$, which includes the polarization, phase and amplitude changes of
the diffraction processes of ray $n$. The complex radiation properties of the transmitting and receiving antennas are taken into account by the effective antenna height vectors $\mathbf{h}_{T}$ and $\mathbf{h}_{R}$, respectively. The coefficient $k_{g}$ represents the losses between the generator and the transmitting antenna, whereas $k_{L}$ results from the matching of the receiver and can also include other losses between the receiving antenna and the receiver load. The term, $e^{-j k r_{n}}$, is the phase coefficient associated with the propagation path length of the ray $n . M$ represents the total number of existing propagation paths, and in general if a complex propagation environment such as a microcell in a urban city is considered, the sum should consist of infinite number of terms. The specific case $M=1$ corresponds to free space propagation, where there is only one propagation path. Hence, in that case $M=1$ and $\overline{\bar{D}}_{1}=\overline{\bar{I}}$ and equation (2) simplifies to

$$
\begin{equation*}
\frac{P_{R}}{P_{T}}=k_{g} k_{L} \frac{1}{4 R_{a}} \frac{\eta}{r^{2}} \frac{\left|\mathbf{h}_{R} \cdot \mathbf{h}_{T}\right|^{2}}{\int_{\Omega}^{\left|\mathbf{h}_{T}\right|^{2} d \Omega}}, \tag{3}
\end{equation*}
$$

which is the standard Friis formula written in terms of the effective height vectors of the transmitting and receiving antennas.

A binomial array is basically constructed by $N-1$ complex plane nulls, which all are in the same z-plane location $z=-1$. This implies that the array factor is created by binomial terms, $z+1$, in the following fashion [6]

$$
\begin{equation*}
A F=(z+1)^{N-1}, \tag{4}
\end{equation*}
$$

where $N$ is either the order of the binomial array or the number of antenna elements in the corresponding antenna array. All the array element excitations are binomial coefficients obtained from the Pascal's triangle.

|  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | 1 |  |  |  |  |  | $B_{1}$ |
|  |  |  | 1 |  | 1 |  |  |  |  | $B_{2}$ |  |
|  |  |  | 1 |  | 2 |  | 1 |  |  |  | $B_{3}$ |
|  |  | 1 |  | 3 |  | 3 |  | 1 |  |  | $B_{4}$ |
|  | 1 |  | 4 |  | 6 |  | 4 |  | 1 |  | $B_{5}$ |
| 1 |  | 5 |  | 10 |  | 10 |  | 5 |  | 1 | $B_{6}$ |

In this paper $n$th order binomial array will be denoted by $B_{n}$.

Binomial arrays have some special characteristics, which are worth mentioning here. First of all, taken that the antenna element spacing is less than a half of the wavelength the amplitude pattern of a binomial array contains only one null, which implies that the directivity of the antenna increases very slowly as a function of the element number. Hence, the amplitude pattern is weakly affected by the element number. On the contrary, the phase pattern is strongly dependent on the element number. Secondly, excitations for a binomial array are always symmetrical and real if the possibility for scanning of the beam using a progressive phase shift between elements is not taken into account. Hence, an endfire radiation pattern due to a binomial array can be implemented without any tunable phaseshifters. Also, the antisymmetry of the real excitation
permits the use of a perfectly conducting reflector, which reduces the needed element number to one half of the original one. For instance $N$-element array with a reflector corresponds to the array $B_{2 N}$ without the reflector.

## III. Antenna array

Network planning is one of the primary tasks when a new cellular network is built or a new base station needs to be added to an existing network due to increasing capacity requirements. Fundamental network planning is done by determining the locations and appropriate coverage sectors for each base station. If one is considering an urban street geometry, one possible choice would be to design coverage along the streets with highly directive beams. However, this leads to difficulties when controlling the effective cell sizes, since the transmitted power can propagate long distances along street canyons causing interference between microcells, which are using the same carrier frequency. In dense urban cities, high requirements for network capacity forces one to use microcells where the base stations are located relatively near each other. In this environment, base station antennas are situated below rooftops and their beams should be directed towards building walls in order to avoid interference. Due to the resulting reflections and diffractions, the radio signal will be spread more evenly to the surroundings of the base station. While as an undesireable effect the mean value of the signal is lowered, this method reduces interference from the neighboring cells.

In this paper, we are considering the possibility of using binomial antenna arrays to provide diversity gain in uplink situation, i.e. a mobile station is transmitting. The design goal is a diversity antenna for a base station end whose 3 dB -beamwidth should be about $60^{\circ}$ and remain unaltered in diversity reception to avoid interference from neighboring cells and to provide a constant coverage area. These conditions can be approximately realized by using binomial arrays. Figure 1 shows the directivity patterns of binomial arrays $B_{2}, B_{4}, B_{6}$ and $B_{8}$. If the pattern corresponding to $B_{2}$ is neglected, one can see that the higher order patterns $B_{4}-B_{8}$ do not differ considerably. However, their phase properties, which are illustrated below in the same figure, are quite different. The phases of the array patterns change linearly as a function of $\cos \alpha$ and as the order of the array increases, the fluctuations will become more rapid. This implies that a linear four element antenna array could be used to realize a base station receiver, whose radiation power pattern is stationary whereas its spatial phase properties can be altered by changing the element coefficients according to the binomial law. Hence, four binomial receivers can be implemented by either using DSP (adaptive diversity) or more straightforward hardware structures. In this case, the latter solution is more favorable because of the simple excitation of the elements. In Figure 2, the needed element coefficients of different order binomial array are shown. In order to be able to compare the performance of different order binomial arrays, the element excitations should be normalized to provide power pattern of equal average.

Since the variations of the phase patterns of different order binomial arrays are quite different, there is a possibility to form a complex sum of the incoming waves in several ways. When diversity antenna configurations are chosen, it becomes advantageous to have phase patterns of as high order as possible, since the optimization resolution in angular space will be increased then. If two multipath signal components are approaching at different angles with completely opposite phases, a $180^{\circ}$ phase-shift between the matching points in the phase pattern is needed in order to have a full scale of tuneability in the diversity optimization. If the incident angles are very close to each other, a very high spatial resolution of phase is needed in order to distinguish the waves from each other. Usually in microcells, the base station receiver is combining many waves at the same time, hence it is not possible to optimize the phase pattern of the receiver for all directions. However, it is quite a simple task to find an optimum among some prechosen element excitations.

If various techniques of synthesizing diverse phase patterns for a single solid power pattern are compared, binomial arrays used in the present case could grant the best angular resolution of phase for a fixed number of array elements. This result could be mathematically justified by studying the phase effects of nulls in the complex z-plane.

The symmetry of the resulting array excitations allows one to use a perfectly conducting reflector, which can be put into the final configuration as shown in Figure 2. This reduces the number of antenna elements to half from an even order asymmetric binomial array. The requirement for the progressive phase shift between the array elements is therefore $180^{\circ}$, which means that one has an endfire array. Its beam directs straight towards the opposite wall and the element spacing is $\lambda / 2$ except for the nearest element, whose distance from the reflector is $\lambda / 4$. This implies that only four antenna elements are needed to produce array excitation for even order binomial arrays up to the array order of eight. With four antenna elements one is able to obtain binomial arrays $B_{2}, B_{4}, B_{6}$ and $B_{8}$, whose directivity and phase patterns are presented in Figure 1.

## IV. Simulations

The effect of the fast-fading phenomenon and the performance of the multistate phase diversity technique, whose beams are obtained with the suggested array configuration, were simulated by using a ray-tracing radiowave propagation model of an urban city. The base station location and beam directions as well as the mobile station route and the surrounding street geometry of the simulations are shown in Figure 3.

The ray-tracing radiowave propagation model used in the simulation is a two-dimensional model, which in addition also includes the primary reflection from the ground. The polarization of the incoming waves is taken into account when calculating the wall reflections, as well as the corner diffractions, which are modeled according to the UTD [8].

The transmitting antenna of the mobile station is omnidirectional in the horizontal plane and the receiving base
station antenna is assumed to be the binomial phase diversity array. Both the mobile station and the base station antenna are vertically polarized. The transmitter power level is set to be 1 W in the simulations. The radio channel response associated with the route of the mobile station was simulated with all four base station antenna configurations. Special attention was paid to cases, where the mobile station was in a line of sight (LOS) situation with respect to the base station antenna and alternatively when one or more diffractions were needed (non line of sight, NLOS) to predict the propagated signal. The transmitter was assumed to be a very narrow-band one at 1 GHz .

Figure 4 illustrates the received field strength as a function of distance between the mobile terminal and the base station as the mobile station is moving according to the path shown in Figure 3. Vertical lines in the figure outline parts of the mobile station route that are analyzed further. These parts are displayed in more detail in Figure 5. The figure shows all the electrical field strength levels corresponding to the reception made with the binomial excitations $B_{2} \ldots B_{8}$. It clearly shows how the fast-fading phenomenon is present regardless of the choice of the receiving antenna excitation $B_{2}, B_{4}, B_{6}$ or $B_{8}$. However, the most important conclusion from this figure is that the deep fading nulls do not exist at the same positions of the mobile terminal for different order binomial receivers. With a parallel receiver system it is thus possible to temporarily select the best excitation of the antenna array and thus optimize the received signal by selection combining. Hence the discussed four-element binomial array provides four-stage diversity reception in which the difference of the received signals are due to non-similar phase properties of radiation patterns, $B_{2} \ldots B_{8}$.

The present phase diversity technique is assessed by postprocessing the fading data shown in Figure 4 and by computing the statistical probabilities of the received signal. Figures 6 and 7 show the probabilities of the received signal not exceeding a certain threshold value of the received field strength in units $\mathrm{dBV} / \mathrm{m}$. The different curves represent this probability function in cases where selection combining has been used between certain binomial receivers. Continuous line associated with $B_{1}$ corresponds to the probability of an isotropic or omnidirectional BS antenna. $B_{2}$ corresponds to the lowest order binomial excitation, i.e. one element with a reflector. Other cases represent the signal probability obtained by using the selection combining of the signals received by higher order binomial excitations. For all excitations $B_{1} \ldots B_{8}$ the radiation patterns used in the simulation have been normalized to the same average radiation power.

## V. Results

As it is shown in Figure 4, distances between the fading dips are approximately $1-2 \mathrm{~m}$, while the whole analyzed route for both LOS and NLOS paths is about 10 m long. The resulting field strength is constructed from about 700 rays in NLOS path and up to 2800 rays in LOS path and the overall step size of the mobile station in the simulation
was five centimeters. Thus, the resolution and accuracy in this analysis should be more than satisfactory.

Figure 4 shows that the positions of the fading dips associated with different binomial excitations do not coincide, which indicates that the phase diversity properties of the binomial array seem to be very good both in LOS and NLOS situations. However, the signal strength in the LOS case is naturally far higher than in the NLOS situation.

Figures 6 and 7 show the signal statistics in LOS and NLOS cases, respectively, when selection combining between binomial excitations has been used. Both illustrations show that the best signal probability is obtained by simultaneous diversity detection of excitations $B_{2}, B_{4}, B_{6}$ and $B_{8}$ in both the LOS and NLOS cases. In practice the statistics for the LOS case are insignificant due to the high field strength values compared to the NLOS case.

In microcells the good quality of service implies that the probability of the received radio signal existing below a certain threshold value should be very close to zero, e.g. 0.05. If this number is used as a network planning criteria, Figure 7 shows that to reach this probability an omnidirectional receiver should be able to detect signals as weak as $-55 \mathrm{dBV} / \mathrm{m}$. To receive the same quality of service only $-48 \mathrm{dBV} / \mathrm{m}$ signal detection level is needed if $B_{2}$ and $B_{8}$ patterns are used to provide phase diversity. If all excitations $B_{2} \ldots B_{8}$ are utilized in phase diversity reception, about 1 dB improvement in diversity gain is still obtained.

## VI. Conclusions

A binomial antenna array implementation of a multistate phase diversity technique was presented. Diversity antenna patterns were simulated in a microcellular environment by using a simulation program based on ray-tracing radio channel model. Electric field strength graphs for a moving mobile station were computed in two cases: for both LOS and NLOS radio wave propagation situations. Diversity gains achieved by the implementation were assessed by calculating signal statistics as a function of the received field strength level for both LOS and NLOS channels, respectively.

As a result, the phase diversity implemented by a simple binomial array provided approximately a 7 dB gain when only two radiation patterns were used and the probability to detect the arriving signal was set to 0.95 . If more binomial patterns are utilized, the diversity performance of the system could be still improved. It should be noted that the actual figure of the diversity gain is strongly dependent on the probability value corresponding to the signal threshold level, which needs to be exceeded in the radio network. The present study indicates that array antennas in general can be used to alleviate fading problems in mobile radio channel. The binomial array was introduced as a very simple array, whose diversity properties are practically based on the phase patterns of the array and its sub-binomial arrays. The radiation beam didn't have any significant role in the analyzed diversity reception implementation, which is one benefit of the proposed base station antenna, since changing gain would possibly increase
interference from neighboring cells. On the other hand the mean value of the received signal is approximately the same for all binomial excitations, which equalizes the use of binomial diversity branches. The other benefit of the binomial array is its ability to provide antenna diversity in one antenna location, which results in a relatively compact base station configuration.

## VII. Acknowledgements

The simulations of this paper were made by using a raytracing propagation model developed by Nokia Telecommunications. This work is supported and partly funded by IRC (Institute of Radio Communications).

## References

[1] R.G. Vaughan, J.B. Andersen, Antenna Diversity in Mobile Communications, IEEE Transactions on Vehicular Technology, vol. VT-36, no. 4, November 1987, pp. 149-172.
[2] A.M.D. Turkmani, A.A. Arowojolu, P.A. Jefford, C.J. Kellett, An Experimental Evaluation of the Performance of Two-Branch Space and Polarization Diversity Schemes at 1800Mhz, IEEE Transactions on Vehicular Technology, vol. 44, no. 2, May 1995, pp. 318-326.
[3] P. Eggers, J. Toftgrd, A. Oprea, Antenna Systems for Base Station Diversity in Urban Small and Micro Cells, IEEE Journal on Selected Areas in Communications, vol. 11, no. 7, September 1993, pp. 1046-1056.
[4] J-F. Lemieux, M.S. El-Tanany, Experimental Evaluation of Space/Frequency/Polarization Diversity in the Indoor Wireless Channel, IEEE Transactions on Vehicular Technology, vol. 40, no. 3, August 1991, pp. 569-574.
[5] W.L. Stutzman, G.A. Thiele, Antenna Theory and Design, Second Edition, John Wiley \& Sons 1998. pp. 116-121.
[6] C.A. Balanis, Antenna Theory, Analysis and Design, Second Edition, John Wiley \& Sons 1997. pp. 290-294.
[7] J.J.A. Lempiäinen, K.I. Nikoskinen, J.O. Juntunen, Multistate phase diversity microcell antenna, Electronic Letters, vol. 33, no. 6, March 1997, pp. 438-440.
[8] G.L. James, Geometrical Theory of Diffraction for Electromagnetic Waves. Third Edition Revisited., Peter Peregrinus Ltd. 1986. pp. 155-159






Fig. 1. (a) directivity patterns and (b) phase patterns of binomial arrays. Solid line: $B_{2}$. Dashed line: $B_{4}$. Dash-dot line: $B_{6}$. Dotted line: $B_{8}$.

## street direction



Fig. 2. Suggested antenna array configuration and unnormalized array excitations for diversity beams used


Fig. 3. Base station location, street geometry and simulation path. Dashed line: LOS path. Dotted line: NLOS path.


Fig. 4. Relative received field strength in dB-units for the whole simulated path. Vertical lines indicate the subareas, which are analyzed further as LOS and NLOS propagation paths.


Fig. 5. Relative received field strength in dB-units for both LOS and NLOS propagation paths. (a): LOS path. (b): NLOS path. Solid line: received with the array excitation $B_{2}$. Dashed line: $B_{4}$. Dash-dot line: $B_{6}$. Dotted line: $B_{8}$.


Fig. 6. Received signal statistics for the LOS propagation path.


Fig. 7. Received signal statistics for the NLOS propagation path.

