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**Adaptive Algorithms for Tracking Direction-of-Arrival
in Mobile Communications Systems**

This thesis has been submitted for official examination for the degree of Licentiate of Technology in Espoo, Finland

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Espoo, December 20, 1999

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Title of thesis:	Adaptive Algorithms for Tracking Direction-of-Arrival in Mobile Communications Systems
Date:	20.12.1999
Faculty:	Department of Electrical and Communications Engineering
Professorship:	S-38 Telecommunications Technology
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<p>One way to significantly increase the capacity of the third-generation mobile communications systems is to employ adaptive antenna array technologies together with sophisticated signal processing algorithms. From the added value service provider and operator point of view there are increasing demands for position location methods of mobile users in wireless systems. The position location information can be utilized, e.g., in the applications of emergency management and location sensitive billing.</p> <p>The position locating and tracking of mobile users can be realized by applying antenna arrays at multiple BSs with adaptive spatial spectrum DOA estimation methods. The classical and subspace based spatial spectrum estimation methods can be utilized for the calculation of DOA information of mobile users only in stationary signal scenarios. Because of the time-varying nature of communications channels the main focus of this licentiate thesis was directed towards the development of adaptive DOA tracking methods of mobile users in non-stationary signal scenarios.</p> <p>First we develop an adaptive method for the convergence parameter for the efficient tracking of a time-varying manifold of the antenna array. This method can combine fast convergence speed and low estimation error with low computational complexity both in the stationary and non-stationary signal scenarios. In the sequel for the similar framework, a tracker based on a step-by-step updating approach of a reference signal based CG method was developed for the tracking system. For the class of high-resolution methods a simplified gradient based approach was developed for the adaptive tracking of a parameter vector in the noise subspace from a time-varying eigenstructure. For the same tracking scheme we also developed a control strategy for noise subspace roots that can alleviate DOA association problem of mobile users caused by spurious roots. As a final point, the implementation aspects were considered for the CG-based tracking system. A suitable systolic architecture was proposed by means of which the computational complexity of the tracking system was furthermore reduced by an order of a magnitude.</p>	
Keywords:	Direction-of-Arrival estimation, Tracking systems, Subspace tracking, Signal subspace tracking, Noise subspace tracking, Beamforming, Adaptive antennas

Tekijä:	DI Petri Karttunen
Otsikko:	Adaptiiviset algoritmit tulosuunnan estimoinnissa matkapuhelinjärjestelmissä
Päivämäärä:	20.12.1999
Osasto:	Sähkö- ja tietoliikennetekniikan osasto
Professori:	S-38 Teletekniikka
Valvoja:	Prof. Timo Laakso
Tarkastaja:	Prof. Visa Koivunen
<p>Adaptiiviset antennit yhdessä kehittyneiden signaalinkäsittelyalgoritmien kanssa mahdollistavat kapasiteetin kasvattamisen kolmannen sukupolven matkapuhelinjärjestelmissä. Langattomien järjestelmien mobiilikäyttäjien paikkatiedolle on yhä suurempaa tarvetta lisäarvopalveluja tarjoavien ja operaattorin kannalta. Paikkatietoa voidaan käyttää hyväksi mm. hätäpuheluiden paikallistamisessa ja käyttäjien sijaintiin perustuvassa laskutuksessa.</p> <p>Käyttäjien paikallistaminen voidaan toteuttaa tukiasemien antennirivistöillä, joissa adaptiivisilla menetelmillä käyttäjien tulosuuntaa estimoidaan. Muuttumattomassa tukiaseman signaaliympäristössä tulosuuntaa voidaan helposti laskea perinteisillä keilanmuodostusmenetelmillä tai käyttämällä tarkan erottelukyvyn omaavia aliavaruuden menetelmiä. Aikariippuvien radiokanavien takia tämän lisensiaattityön päämääräksi rajattiin adaptiivisten algoritmien kehittäminen käyttäjien tulosuunnan estimointia varten muuttuvassa signaaliympäristössä.</p> <p>Tässä työssä kehitimme aluksi käyttäjien tulosuunnan seurantajärjestelmää varten adaptiivisen menetelmän konvergenssiparametrille. Tämä menetelmä pystyy saavuttamaan sekä nopean konvergoinnin että alhaisen estimointivirheen muuttuvissa ja vakiokanavissa. Seuraavaksi käyttäjien tulosuunnan estimoinnissa kehitettiin näyte kerrallaan päivittyvä referenssisignaali pohjainen CG-menetelmä. Korkean erottelukyvyn omaavien aliavaruusmenetelmien luokassa kehitettiin yksinkertainen gradienttimenetelmä ajassa muuttuvan kohina-aliavaruuden seuraamiseen. Samaisessa järjestelmässä kehitettiin lisäksi myös kontrollistrategia kohina-aliavaruuden juurille, mikä lievittää virheellisten juurien vaikutusta tulosuunnan estimaattien yhdistämisessä oikeisiin käyttäjiin. Käyttäjien seurantajärjestelmän laitteistollista toteuttamista varten tarkasteltiin CG pohjaista järjestelmää. Sopiva systolinen arkkitehtuuri esiteltiin, minkä avulla seurantasysteemin laskennallista tehokkutta voitiin parantaa.</p>	
Avainsanat:	Tulosuunnan estimointi, Seurantajärjestelmät, Aliavaruuden seuraaminen, Signaaliavaruuden seuraaminen, Kohina-aliavaruuden seuraaminen, Keilanmuodostus, Adaptiiviset antennit

Preface

This licentiate thesis has been great pleasure to work with at the Laboratory of Telecommunications Technology, Helsinki University of Technology. First and foremost, I am sincerely grateful to my supervisor, Professor Timo Laakso who has enthusiastically guided my research work with his valuable scientific insight and encouraging guidance.

I wish to thank the personnel of the Laboratory of Telecommunications Technology for the relaxing and inspiring working atmosphere. I wish to express my gratitude to Lic. Tech. Ramin Baghaie, a co-author of a part of the publications, for his guidance when planning the research work. I am grateful to the secretaries Arja Hänninen and Mirja Lemetyinen for taking care of all my practical matters. Finally, I am thankful to Mr. Kimmo Pitkäniemi for the workstation support and M.Sc. Marko Luoma for the software related advises.

This licentiate thesis has been part of a research project of the Institute of Radio Communication (IRC) funded by the Technology Development Center (TEKES), NOKIA Research Center, Sonera and the Helsinki Telephone Company. I would like to express my gratitude to all participants of the project with whom I have had many fruitful discussions and meetings. Especially, it is a pleasure to thank D.Sc. Jorma Lilleberg who has provided a fatherly support with his scientific advice at the beginning of the research work. Many warm thanks also to Lic. Tech. Michael Hall for the checking of the English language. This thesis has also been financially supported by a grant from the Wihuri foundation.

Lopuksi haluan kiittää rakkaita vanhempiani kaikesta heidän tuesta ja kannustuksesta. Last but not least, I wish to express my deepest gratitude to my dearest Sarah for her supporting love and patience at the times of delight and despair when preparing the thesis.

Espoo, December 20, 1999

Petri Karttunen

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List of Publications

This licentiate thesis consists of the following four publications which are referred to by [P1], ..., [P4] in the text. None of these publications are used in any other person's thesis.

- [P1] **P. Karttunen**, T. Laakso and J. Lilleberg, "Tracking of mobile users in mobile communication system using adaptive convergence parameter," *Proceedings of the International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'97)*, Vol. 3, Helsinki, Finland, September 1–4, 1997, pp. 989–993.
- [P2] **P. Karttunen** and R. Baghaie, "Conjugate gradient based signal subspace mobile user tracking," *Proceedings of the Vehicular Technology Conference (VTC'99)*, Vol. 2, Houston, Texas, USA, May 16–20, 1999, pp. 1172–1176.
- [P3] **P. Karttunen**, "An algorithm for noise subspace based mobile user tracking," *Proceedings of the International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'99)*, Vol. 2, Osaka, Japan, September 12–16, 1999, pp. 651–655.
- [P4] R. Baghaie and **P. Karttunen**, "VLSI implementation of CG based mobile user tracking," to appear in the *Proceedings of the IEEE Midwest Symposium on Circuits and Systems (MWSCAS'99)*, Las Cruces, New Mexico, USA, August 8–11, 1999.

List of Abbreviations

AIC	Akaike's Information theoretic Criterion
AR	AutoRegressive
ARMA	AutoRegressive Moving Average
AWGN	Additive White Gaussian Noise
BS	Base Station
CCI	Co-Channel Interference
CDMA	Code Division Multiple Access
CG	Conjugate Gradient
DF	Direction Finding
DOA	Direction-of-Arrival
DSP	Digital Signal Processing
ED	Eigenvalue Decomposition
EM	Expectation-Maximization
ESPRIT	Estimation of Signal Parameters via Rotational Invariance Techniques
FCC	Federal Communications Commission
FIR	Finite Impulse Response
FPE	Final Prediction Error
GLONASS	GLObal NAVigation Satellite System
GPS	Global Positioning System
GS	Gram-Schmidt
ISI	InterSymbol Interference
LMS	Least Mean Squares
LOB	Line-of-Bearing
LOS	Line-of-Sight
LS	Least Squares
MA	Moving Average
MDL	Minimum Description Length
ML	Maximum Likelihood
MMSE	Minimum Mean Squared Error
MN	Minimum Norm
MRC	Maximum Ratio Combining
MSE	Mean Squared Error

MUSIC	MUltiple Signal Classification
NLOS	Non Line-of-Sight
OC	Optimum Combining
RLS	Recursive Least Squares
RQI	Rayleigh Quotient Iteration
SAGE	Space Alternating Generalized EM
SD	Steepest Descent
SNIR	Signal-to-Noise plus Interference Ratio
SNR	Signal-to-Noise Ratio
SS	Spatial Smoothing
SVD	Singular Value Decomposition
TDMA	Time Division Multiple Access
TDOA	Time Difference-of-Arrival
TLS-ESPRIT	Total Least Squares ESPRIT
TOA	Time-of-Arrival
VLSI	Very Large Scale Integration
WSF	Weighted Subspace Fitting
WSS	Wide-Sense Stationary

1. Introduction

In recent years, the cellular communication technology has grown enormously. The customers from the mobile telephone networks want similar services that exist in the wireline networks, such as multimedia capabilities and Internet services. This growth pressure from the customers' side has compelled to make advances in the communication technology. This has resulted in that the operators and manufacturers are continuously searching for methods for improving performance, capacity and coverage of the mobile communication systems. Therefore, the current research on 3rd generation mobile systems aims at increasing the system capacity for efficiently handling the growing group of subscribers. This has resulted in a great amount of work that has been done on adaptive antenna research in the area of the mobile communications industry and within the academic world. The current research work is concentrated on employing multiple antenna elements at the Base Station (BS) and utilizing adaptive algorithms for multi-user detection, spatial processing of communication channels and channel parameter tracking [Rap98]. In the user tracking context the mobility and its management have also become an important part of the overall system design [Zon97].

Traditionally, several target position locating and tracking problems arise in a number of different applications [Rap96b]. The location information is useful for navigational services where the movement of nearby ships is tracked in order to avoid collisions. The target tracking can be used for airport applications where air traffic controllers keep track of incoming and outgoing aircrafts for which the radar applications are usually targeted. The target tracking problem also arises in the design of an optimal route in transport applications. Tracking systems have been utilized for the position locating and steering of autonomous vehicles like farm tractors without human control [Rek98].

In a similar way, in cellular applications the location information can be utilized in numerous different ways. Traffic congestion in busy traffic roads can be monitored by position locating and tracking of mobile users. Car driving can be assisted furthermore with fixed terminals installed in the cars that track the position of nearby vehicles [Abb99]. Also, network operators can bill mobile subscribers based on their location. The location information of subscribers also enables position dependent advertising.

Furthermore, target tracking algorithms can be used for assisting beamforming procedures in order to enhance the quality of a communication link to a mobile user as depicted as an example in Figure 1 [Zet97]. The system's capability of finding reliable estimates for the channel parameters is of great value in beamforming based applications [Com88].

Different applications set different demands on the accuracy of the position locating estimation. The Federal Communications Commission (FCC) has risen a requirement that the position locating of mobile users should be done within an accuracy of 125 meters within 67% of the time. Therefore, the requirements for the development of efficient algorithms and methods for tracking the location and Direction-of-Arrival (DOA) information of targets are evidently justified.

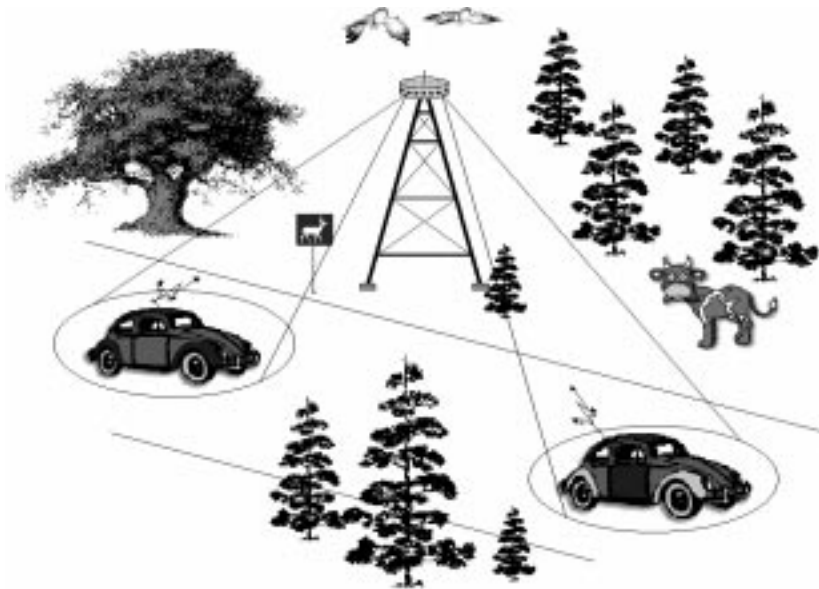


Figure 1 Illustration of a base station antenna working in the beamforming mode in a rural area

In this thesis, we concentrate on developing efficient, robust and realizable algorithms for mobile user tracking systems. We concentrate especially on developing various algorithms for user tracking systems. The interest is not directed towards network related issues. The DOA estimation of mobile users in a stationary signal scenario can be dealt with in a more straightforward way. Therefore, the main focus is on problems that arise in non-stationary channels where efficient adaptive parameter estimation schemes are required.

Therefore, in the design of the tracking system the following three aspects should be considered and are addressed in this thesis, namely:

1: The system should have fast tracking capability

2: The system should have low parameter estimation error

3: The system should be of low complexity

The thesis is organized in the following way. To start with, in Chapter 2 the research work done in the area of adaptive antennas will be discussed. In Section 2.1 the characteristics of the communication channels are briefly presented. In Section 2.2 the motivations of making use of adaptive antenna arrays in mobile communication systems are identified and some of their implications are addressed. Section 2.3 gives a brief survey of some of today's position locating systems and especially explains how the adaptive arrays at the BS site can be utilized for position locating of mobile users. In Section 2.4 the mathematical foundation is constructed for analyzing the linear antenna arrays that will act as a basis for the tracking algorithm development. As a result of the presented signal model, two categories for the correlation matrix based trackers can be identified, a spatially structured approach and an eigenstructure based approach. In Section 2.5 the stationary DOA estimation methods, with the special focus on eigenstructure based high-resolution methods, are reviewed from the literature for locating mobile users in the stationary signal scenario. In Section 2.6 we focus on adaptive DOA estimation methods for continuous tracking of mobile users. For the spatially structured methods a gradient based tracker and a Conjugate Gradients (CG) based tracker are developed for tracking a time-varying manifold of the antenna array. Furthermore, an adaptive gradient based approach is developed for the adaptive tracking of the time-varying eigenstructure. Chapter 3 summarizes the publications on mobile user tracking, together with the identification of the main results. Finally, Chapter 4 draws the conclusions with the main contributions.

2. Review of Previous Work

2.1 Communication Channels

The communication systems always comprise a *transmitter site*, a *receiver site* and a *communication channel* between them. The present understanding of communication channels is rather mature [Sil84][Pah95]. The characteristics of the radio communication channels are needed in the design of high-capacity communication systems. The radio propagation environment can be typically characterized by numerous propagation mechanisms such as reflection, refraction and scattering of radio waves. However, the complicated time-varying channel can be adequately modeled mathematically using the impulse response approach [Has93]. Modeled or measured communication channels have been typically utilized as a design criterion in the development of communication systems.

The most important signal propagation mechanism is the *multipath fading* as illustrated in Figure 2. The multipath phenomenon arises from the fact that different signal components propagate along slightly different paths. The transmitted signal is often reflected or refracted from the surrounding buildings or from other physical structures. The fading is a time-varying phenomenon which causes fluctuations in the signal power. Depending on the relative phase values, the signal components may add together either constructively or destructively causing a strong signal or deep fades to abruptly appear in the signal power [Has93]. Depending on the rapidity of the underlying fading processes these fading phenomena can be classified into *slow fading* or *fast fading*.

The *spatial* and *temporal* correlation properties of a typical multipath fading channel have been investigated through the measurements of an 8-element adaptive antenna array at a 2.154 GHz range outdoor radio channel by the authors in [Kar98]. Many authors have explored the correlation properties of the multipath channels, for example Rappaport *et al.* in [Rap91] and Dossi *et al.* in [Dos96]. However, the correlation research work here has been refined by taking into account the complex valued taps of the adaptive antenna array. The correlation properties of multipath signals are important when analyzing the performance of different diversity reception schemes.

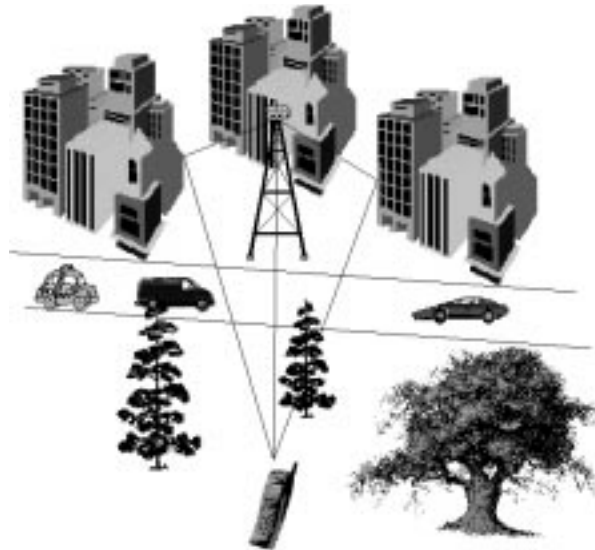


Figure 2 Illustration of a communication system comprising a transmitter (at the base station), a receiver (at the mobile unit) and the multipath fading channel with a direct path and two indirect (reflected) paths

The performance and reliability of communication systems may be degraded unless the deleterious effects of the multipath propagation are taken into account. The multipath propagation generates InterSymbol Interference (ISI) in the time domain, the effect of which can be efficiently mitigated through adaptive channel equalization [Qur85]. Diversity schemes with adaptive antenna arrays have also been successfully applied for mitigating the harmful effects of the multipath fading.

The *Co-Channel Interference* (CCI) is one of the most prohibiting factors restricting better performance gains in present communications systems. The CCI occurs as signals of the same carrier frequency is received from two different transmitters. The influence of CCI on Time Division Multiple Access (TDMA) systems and Code Division Multiple Access (CDMA) systems is quite different [Rap96a][Vit95]. In TDMA based systems CCI mainly arises from adjoining cells whereas in CDMA based systems CCI originates from the current cell and adjoining cells and, therefore, is spatially more uniformly distributed. In conclusion, the locating of co-channel interference sources will be easier in TDMA based systems with the adaptive tracking algorithms.

2.2 Adaptive Antenna Arrays

2.2.1 Single–Antenna Element Systems

Radio antennas are devices which are used to couple the electromagnetic energy received in certain frequency ranges to a suitable physical medium like coaxial cable or fiber. The most common antenna structure for wireless communication systems is the *dipole antenna*. The dipole antenna radiates uniformly for the environment having an omnidirectional radiation pattern. Therefore, a large amount of energy is lost when the transmitted signal reaches the desired user. The desired user can only be reached by increasing the signal power. Evidently, this kind of omnidirectional property wastes transmitter power and radio spectrum. In addition, interference from adjoining cells can directly couple into the receiver [Ala92]. Therefore, in order to reduce the interference problem antennas with certain fixed transmission and reception directions have been constructed. This has resulted in antenna structures having the directionality property. In sectored antenna systems directional antennas are used to cover the whole cell area. Directional antennas are commonly used to cover part of the radiation pattern of the omnidirectional antennas, usually 60 or 120 degrees which can result in a significant increase in the antenna gain.

2.2.2 Diversity Systems

One of the means to significantly increase the capacity of future telecommunication systems is to employ adaptive antenna array technologies. This concept is based on adding multiple antenna elements resulting in more sensitive receiving antenna structures. At present, the most common antenna array structure is the *phased antenna array* which can be configured in many ways and can be directly incorporated into the BS [Ala92]. By applying more antennas as such does not improve the performance and intelligence of the communication system. Therefore, diversity systems have been introduced to improve the effectiveness of communication systems [Win92b]. The diversity systems, in addition to *spatial diversity*, can be based on *frequency*, *time* or *polarization diversity* schemes [Pro85]. Spatial diversity can be implemented using spatially distributed antenna elements and temporal diversity exists in the radio channel in the form of multipath propagation [Has93]. During propagation the electromagnetic

wave includes electric field and magnetic field components which are used to define the horizontal and vertical polarization of signal. Polarization diversity systems extract the two available information bearing signals in two cross-polarized branches [Tur95].

A wide variety of diversity combining schemes exist which offer different performance gains [Pro85]. *Switched diversity* systems make switching decision among the elements so that an antenna element having the best signal will be selected. The *Maximum Ratio Combining* (MRC) method combats the Rayleigh fading of the desired signal only [Sal94]. In this method the received signal in each element is weighted according to the Signal-to-Noise Ratio (SNR) of each diversity branch. The *Optimum Combining* (OC) method combats the Rayleigh fading of the desired signal and simultaneously suppresses interfering signals [Sal94]. In another approach, instead of totally nulling of an interfering signal the interference power is reduced. The *Mean Square Error* (MSE) [Bal92] combiner does not force the interference power to zero but merely reduces the power of an interferer closer to the noise level so that the effect of the interferer can be subtracted. These diversity systems have been mostly applied in the uplink case, i.e., from the mobile unit to the BS, due to the fact that the BS can be more easily equipped with the hardware required for the computations.

2.2.3 Smart Antenna Systems

The *smart antenna* concept is based on simultaneous utilization of multiple antenna elements with the sophisticated Digital Signal Processing (DSP) algorithms [Raz99]. The present software radio architecture with smart antenna technology can provide a potential approach for wireless communication due to the presence of sufficiently powerful general purpose processors, digital signal processors and beamforming based signal processing algorithms. Smart antennas have a spatial discrimination capability because they can follow changes in the signal environment and react accordingly by appropriately adjusting the beam pattern of the antenna array as shown in Figure 3. According to the way these beam patterns are employed, smart antenna systems can be basically divided into two different groups. *Switched antenna systems* make use of preselected beam patterns with fixed directions and gains while *adaptive arrays* have full control over the formation of the beam patterns [Com88].

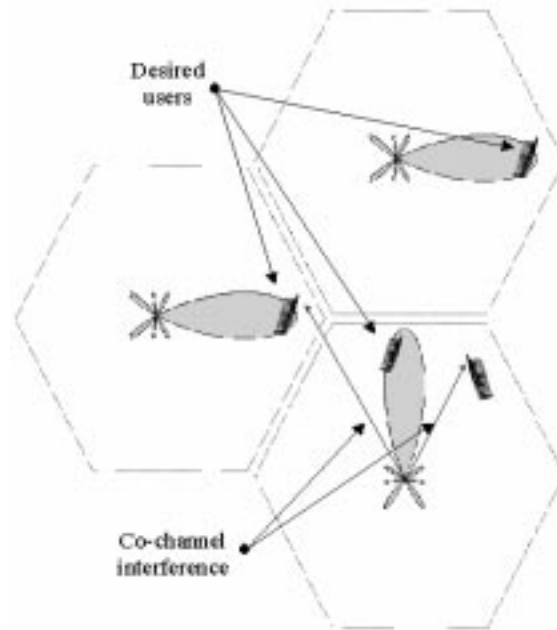


Figure 3 Illustration of beamforming procedures where the beams are established in the desired signal direction and nulls in the interference signal directions

2.2.4 Smart Antenna Systems over Single–Antenna Element Systems

Smart antennas can offer substantial increase in capacity in interference limited communication systems [Win92a]. The main advantage of smart antennas arises from the possibility to suppress the CCI of other users in the same cell or from users in the adjoining cells. The beam pattern is directed towards the desired user, while at the direction of the interfering signals signal nulls are placed. However, there must exist enough spatial separation between the desired and interfering signal sources for them to be resolved. However, the spatial separation problem can be mitigated by multipath propagation in frequency–selective channels. If the control of signal nulls can be efficiently executed in an adaptive way the high interference signal scenario becomes much more profitable for the adaptive antennas because they can gain relatively much more in relation to the antennas in switched beam systems or the sectorized antennas in a conventional communication system. This spatial directivity enables the adaptive antennas to utilize the limited radio spectrum much more efficiently.

In areas of high traffic density, communication channels are already crowded due to the rapid growth of the number of mobile users. This has traditionally resulted in the cell splitting procedure in order to maintain an acceptable service level for all the mobile

subscribers. However, this is expensive, thus placing the emphasis in the system design on the efficient reuse of frequencies.

By suitably combining the available signal power from the antenna elements, an increased gain can be achieved in the desired signal direction [Ala92]. However, this requires adaptive beam steering and interference nulling capability. The higher gain makes it possible to enlarge the coverage area of cells. The narrow beamforming property at the BS makes it possible for mobile phones to communicate at a reduced power level. This could considerably save the lifetime of the batteries in the mobile handset.

Spatial diversity reception enables mitigation of the *multipath fading* [Pro85]. Multipath combining schemes with antenna arrays reduce the delay spread of the channel allowing higher bit rate services to be deployed.

2.3 Position Locating Systems

In this section, we briefly review the techniques behind the position locating systems. There are a number of satellite based position locating systems in the world, the Global Positioning System (GPS), Qualcomm's OmniTRACS® System for the transportation industry from the USA and the GLObal NAVigation Satellite System (GLONASS) from Russia. GPS is the most well-known and popular system for locating users' terminals [Owe95]. This is due to the worldwide availability, reasonable low cost of GPS terminals and the high accuracy of position location estimates. Standard GPS receivers can achieve a horizontal position accuracy of 100 meters within 95% of time whereas the differential and military receivers can even achieve the accuracy of a few meters fulfilling the FCC's requirements. In wireless applications the GPS receivers can be integrated inside the mobile unit. However, this may cause problems because of the cost of receivers, size of the installation and additional power consumption. Furthermore, in indoor environments and certain tall building and dense foliage terrains can result in the GPS signal and the associated position location information may not be available all the time. Consequently, in order to enhance the functionality and practicality of the position location methods for mobile users, the infrastructure of the existing cellular systems may be utilized either alone or in conjunction with other position location methods.

The position location methods in wireless communications can be basically categorized into two different classes. In the first category, the methods based on the direct measurement of the DOAs at the BS site can be identified. These Direction Finding (DF) systems utilizes the adaptive antenna arrays together with sophisticated DOA estimation methods and algorithms. The second category comprises methods based on the utilization of the Time-of-Arrival (TOA) measuring techniques.

2.3.1 DOA-Based Position Locating Systems

Mobile users can be located when the minimum of two or more DOA estimates are established at multiple BSs. The position locating results in a trigonometric type of problem that can be worked out by finding the coordinates from the intersection of two or more Line-of-Bearings (LOB) where iterative techniques have been commonly applied as illustrated in Figure 4. The determination of the DOA is realized by an

adaptive antenna array at the BS in conjunction with sophisticated signal processing algorithms. The mobile phone can be located when two or more DOA estimates are established at the antenna arrays of the BS [Rap98]. As more LOBs are utilized the estimation accuracy improves, but at the cost of increased computational complexity.

DOA-based position locating systems have some inherent problems. One problem is multipath propagation which is the prevailing phenomenon especially in urban areas. The estimation of time-varying multipath channels can lead to catastrophic errors in the DOA estimates. This problem can be alleviated by sorting different multipath components, i.e., the Line-of-Sight (LOS) component from other later arriving multipath components with the aid of a channel estimation procedure. As a result, the target direction can be estimated by utilizing the first arrived multipath component. However, the DOA estimation procedure cannot be accomplished in the case of Non Line-of-Sight (NLOS) situations.

Another issue is the audibility problem [Rap96b]. When a mobile user is close to one BS, it is difficult for other BSs to measure the DOA of the distant mobile user. This is because the cells are usually designed to have a sufficiently good quality communication link only in the coverage area of one BS. This is a problem especially in rural areas where the design is dictated by coverage issues rather than capacity considerations which are important for the cell design of urban areas.

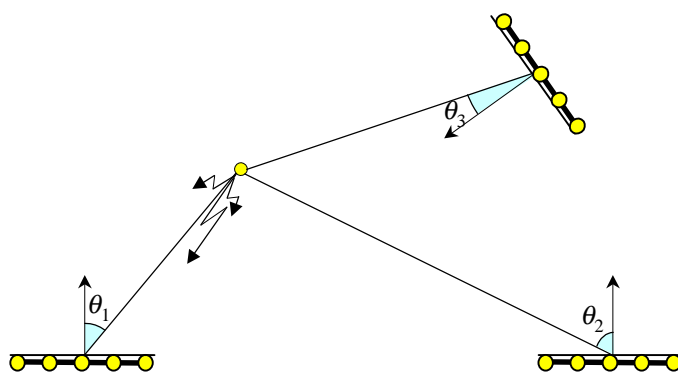


Figure 4 Locating procedure of a mobile user at the intersection of three LOBs as drawn from the BSs in the azimuth angle θ_1 , θ_2 , and θ_3

2.3.2 TOA–Based Position Locating Systems

The distances in Time–of–Arrival (TOA) systems are computed based on the time delays of the radio signal replicas propagating from the mobile unit to the BS. The design of a TOA–based system is complicated by the difficulty of implementing exact clock synchronization at the BSs and the mobile units. In addition, timestamps must be inserted into the transmitted messages to be able to calculate the absolute time of the probing signal from the mobile unit to the BS. Therefore, Time Difference–of–Arrival (TDOA) systems are used in practice. The TDOA estimation scheme eliminates the problems that exist in the TOA–based systems, such as the requirement of having clock synchronization both at the BS and the mobile unit. The TDOA method is based on the difference in the arrival times at two separate locations. The TDOA problem can be efficiently solved by using general correlation techniques [Gar92]. In the ideal case, the mobile phone can be located at the intersection of spheres by utilizing the measurements of the multiple BSs. This geometrical problem results in a set of nonlinear equations which can be solved by iterative methods or linearization techniques [Ho93].

2.4 Signal Models

In this section, the signal model is developed for antenna arrays, which acts as a mathematical foundation for the later algorithm development. Particularly, the uniform linear antenna array structure has been chosen for the following investigation. Because of its mathematical simplicity and tractability the periodic array structure results in a low computational complexity.

The *narrowband model* is an essential assumption in the formulation of signal processing algorithms for the antenna arrays [Com88]. In the narrowband model, the bandwidth of the information bearing signal compared to the carrier frequency f_0 is insignificant. Consequently, the wavelength of the propagating wave can be adequately characterized by $\lambda=c/f_0$ where c is the speed of light. The electromagnetic wave is also plane and hence this signal model is valid only for *far-field* sources. Figure 5 shows the plane wave of a source impinging on a group of M isotropic antenna elements. The time delays between signals impinging on adjacent antenna elements can be modeled using the phase delay $2\pi d\sin(\theta)/\lambda$ where d is the inter-element distance.

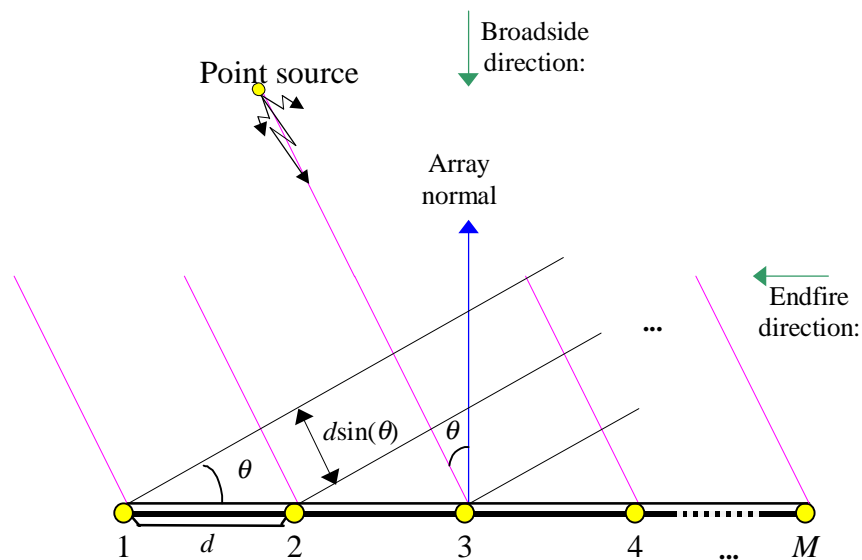


Figure 5 Plane wave impinging from a far-field source on the M antenna elements arranged in the linear array structure

Realistic channels are complex in nature but the complicated time-dependent channels can be adequately modeled by a linear Finite Impulse Response (FIR) filter model

[Has93]. The received signal waveform at the antenna array in the absence of multipath propagation can be expressed by one complex-valued channel tap as

$$\mathbf{x}(t) = \sum_{k=1}^K \sqrt{P_k} b_k(t) \mathbf{a}(\theta_k) + \mathbf{n}(t) \quad (1)$$

where K is the number of users, P_k is the signal power, $b_k(t)$ is the transmitted signal waveform, θ_k is the DOA measured with respect to the array normal and $\mathbf{n}(t)$ is Additive White Gaussian Noise (AWGN) with variance σ_n^2 , independent from the transmitted signals.

With uniform arrays, prohibitive factors in achieving better performance gains are the *grating lobe* phenomenon, i.e., an aliasing effect of sparsely located antenna elements in the spatial domain, and the *mutual coupling* effect, i.e., electric current coupling between closely located antenna elements [Com88]. Therefore, the half-wavelength spacing of elements ($d=\lambda/2$) has been chosen as a compromise between grating lobe and mutual coupling effects. Consequently, the normalized antenna response vector $\mathbf{a}(\theta_k)$ can be expressed as

$$\mathbf{a}(\theta_k) = \frac{1}{\sqrt{M}} \left[1 \exp[-j\pi \sin(\theta_k)] \cdots \exp[-j\pi(M-1)\sin(\theta_k)] \right]^T \quad (2)$$

where M is the number of antenna elements and the phase delay is parametrized with respect to θ_k . This could also be expressed as a function of elevation angle or range to the target [Sat95].

The sampling of the continuous time signal waveform of Eq. (1) provides sufficient statistics for signal detection according to Shannon's sampling theorem. As a result, the discrete-time baseband signal model is expressed as

$$\mathbf{x}(n) = \mathbf{A}(\theta) \mathbf{s}(n) + \mathbf{n}(n) \quad (3)$$

where $\mathbf{A}(\theta)$ defining the *array manifold* is a parameter matrix which arranges the array

response vectors as a *Vandermonde* matrix $\mathbf{A}(\theta)=[\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_K)]$, $\mathbf{s}(n)$ is a vector containing the received signal samples as $\mathbf{s}(n)=[s_1(n), \dots, s_K(n)]^T$ and $\mathbf{n}(n)$ is the antenna noise vector.

In our user tracking application the tracking problem is equivalent to adaptive tracking of variations in θ where $\theta=[\theta_1, \dots, \theta_K]^T$. Therefore, the remainder of the thesis is concerned with constructing mathematical models for the tracking parameter estimation.

2.4.1 Spatially Structured Correlation Matrix

The *spatially structured correlation matrix* is based on the autocorrelation sequence of the discrete–time stochastic process $\mathbf{x}(n)$ and can be defined for an AWGN channel model as follows:

$$\mathbf{R} = \mathbb{E}[\mathbf{x}(n)\mathbf{x}^H(n)] = \mathbf{A}\mathbf{S}\mathbf{A}^H + \sigma_n^2 \mathbf{I} \quad (4)$$

where $\mathbf{S}=\mathbb{E}[\mathbf{s}(n)\mathbf{s}(n)^H]$ is a full–rank covariance matrix of the signals $\mathbf{s}(n)$. However, uncorrelated sources reduce the covariance matrix \mathbf{S} into a diagonal form $\text{diag}(P_1, \dots, P_K)$. In this signal model a structured basis has been established where the $\mathbf{A}\mathbf{S}\mathbf{A}^H$ part of the correlation matrix represents the *signal subspace*. The complement of the signal subspace constitutes the *noise subspace*.

The Wide–Sense Stationary (WSS) discrete–time stochastic process $\mathbf{x}(n)$ leads to a correlation matrix which has a symmetric structure, nonnegative definite property and Toeplitz structure. The Levinson recursion, as developed in [Hay86] and [Hay89], can solve in an efficient way the Toeplitz structure of the correlation matrix in Eq. (4), inside of which the tracking parameter information is embedded.

2.4.2 Eigenstructure Based Correlation Matrix

The Eigenvalue Decomposition (ED) is an important tool for the analysis and design of tracking systems. The eigenvalues and eigenvectors can be extracted from the

correlation matrix by applying different factorization techniques based on either the diagonalization, unitary diagonalization or unitary triangularization techniques [Gol96]. The *Schur decomposition theorem* states that the eigenvalues of the correlation matrix \mathbf{R} appear on the main diagonal after applying an orthogonal similarity transformation matrix \mathbf{U} [Gol96]. Consequently, any positive definite *eigenstructure based correlation matrix* can be expressed as

$$\mathbf{R} = \sum_{m=1}^M \lambda_m^2 \mathbf{u}_m \mathbf{u}_m^H \quad (5)$$

where \mathbf{u}_m is the m :th eigenvector and λ_m the corresponding eigenvalue. In general, the eigenvectors are arranged in the order of the respectively decreasing eigenvalues. The eigenvectors \mathbf{u}_m ($m=1, \dots, K$) lie in the signal subspace while the remaining eigenvectors \mathbf{u}_m ($m=K+1, \dots, M$) lie in the noise subspace.

In the matrix notation the eigenstructure based correlation matrix can be expressed as

$$\mathbf{R} = \mathbf{U}_s \Lambda_s \mathbf{U}_s^H + \mathbf{U}_n \Lambda_n \mathbf{U}_n^H \quad (6)$$

where the matrix \mathbf{U}_s contains the desired signal and interfering signal eigenvectors orthogonal to each other and the matrix Λ_s contains the signal and interference eigenvalues λ_m ($m=1, \dots, K$) on its diagonal. In a similar way, the matrix \mathbf{U}_n contains the orthogonal noise eigenvectors and the matrix Λ_n contains the noise eigenvalues λ_m ($m=K+1, \dots, M$) on its diagonal. In the ideal case the noise eigenvalues are equal to the noise power σ_n^2 .

As a result of this subspace construction, the noise subspace will be the complement of the signal subspace. As a final point, it should be noted that the respective signal and noise subspaces created by the eigenstructured based correlation matrix and spatially structured correlation matrix can be related to each other through a linear transformation matrix. This is an important observation as the subspace decomposition also provides a way of finding the tracking parameters.

In conclusion, tracking systems can be basically divided into two different groups depending on whether the eigenstructure or spatial structure of the correlation matrix is utilized. This classification is a direct result of the interpretation of the signal model presented. Furthermore, a group of methods which avoid the computation of the correlation matrix and instead, operate directly on the received samples can be identified as a third group.

2.4.3 Sample Correlation Matrix

Practical tracking algorithms based on the spatially structured or eigenstructure based approaches utilize the available parameter information in the sample correlation matrix obtained from the antenna array output. The correlation matrix can be estimated using the Maximum Likelihood (ML) approach [The92]. The joint probability density function $p(\mathbf{X}|\theta)$ of identically independently distributed (i.i.d) Gaussian sequence $\mathbf{X} = \{\mathbf{x}(n), n=1, \dots, N\}$ with zero mean and correlation \mathbf{R} can be expressed as

$$p(\mathbf{X}|\theta) = (2\pi)^{-MN/2} \text{Det}(\mathbf{R})^{-N/2} \exp\left[-\frac{1}{2} \text{tr}\left\{\mathbf{R}^{-1} \sum_{n=1}^N \mathbf{x}(n) \mathbf{x}(n)^H\right\}\right] \quad (7)$$

where N is the number of samples [Sor80]. From this expression it can be deduced that a *sufficient statistic* of the correlation matrix estimate is given by the average of outer products of the vector $\mathbf{x}(n)$ with itself. In the practical algorithms the correlation matrix is recursively updated through the *rank-one update scheme* which can be defined as

$$\hat{\mathbf{R}}(n) = \lambda_f \hat{\mathbf{R}}(n-1) + (1 - \lambda_f) \mathbf{x}(n) \mathbf{x}(n)^H \quad (8)$$

where the time-varying nature of the data is taken into account through the forgetting factor λ_f . The rank-one update from time instant $n-1$ to time instant n results in small changes in the array manifold and eigenstructure. In the stationary signal scenario better correlation matrix estimates can be obtained with longer observation periods. However, due to the nonstationarity this is not practical and the estimation must be based on a short-term observation interval.

In the exponentially decaying data window the previous correlation matrix estimate is reduced by the factor λ_f and the new data vector is influenced by the factor $1-\lambda_f$. By selecting a suitable forgetting factor, an accurate correlation matrix estimate can be obtained. In the stationary signal scenario the rank-one update method results in a forgetting factor λ_f of $(n-1)/n$ and an influence factor $(1-\lambda_f)$ of $1/n$. However, in the nonstationary signal scenario λ_f should be chosen from the range $[0, 1]$. We can determine an asymptotic sample length for exponentially decaying window models where the practical values of λ_f are close to 1.

The methods based on the direct updating of the covariance matrix are not numerically stable owing to the squaring operation in the calculation of the outer product in Eq. (8). Therefore, the methods for directly updating the DOA parameters without the estimation of the correlation matrix are more effective, thus preferable in practice. The ED of the correlation matrix $\mathbf{R}(n)$ and the Schur decomposition theorem can be related through the *Cholesky factorization* or *QR factorization* [Gol96]. These two approaches provide a stable way of updating the sample correlation matrix. However, a better estimate of the sample covariance matrix can be obtained by imposing, for example, the Toeplitz and periodic constraints [Du92].

2.5 Stationary DOA Estimation Methods

In this section, a literature review of stationary DOA estimation methods is presented. The different methods can be categorized according to prior information they utilize and a model that underlying unknown processes are assumed to generate. Therefore, these well-known spectrum estimation methods can be basically classified into two different categories, *conventional methods* and *subspace methods*.

2.5.1 Conventional Methods

The parametric and nonparametric properties characterize algorithms in a group of the conventional methods. The *non-parametric methods* are based on the *Einstein–Wiener–Khinchine* theorem expressing a relationship between the autocorrelation sequence of the received signal and its power spectrum by the Fourier transform theory [Hay89]. These Fourier-based methods are generally applicable for a much wider range of applications than the other methods. These methods have also the spatial spectral counterpart when the output power of the antenna array is expressed as a function of the DOA that is termed as the *spatial spectrum* $P(\theta)$. The output power spectrum of the classical beamformer, i.e., *phase delay–and–sum* beamformer can be expressed as

$$P(\theta) = \mathbf{a}^H(\theta) \mathbf{R} \mathbf{a}(\theta) \quad (9)$$

where the value of θ that maximizes the output power gives the direction of the desired user. These methods do not have a very good performance because they do not make any assumptions about the underlying signal statistics. Therefore, they have poor resolution capability in the *Rayleigh criterion* sense related to the broadness of the main beam as dictated by the array parameters. However, these methods work well in the case of a single signal source scenario. In the case of several users or a multipath channel, partial power from other users' signals or from other users' multipath components is coupled through the sidelobes of the beamformer into the measured look directions.

The poor resolution was a problem in the classical beamformer. In Capon's minimum

variance method the power $\mathbf{a}(\theta)^H \mathbf{R} \mathbf{a}(\theta)$ is minimized while maintaining unity gain $\mathbf{a}(\theta)^H \mathbf{a}(\theta_0) = 1$ in the look direction θ_0 . This minimization problem can be solved by the Lagrange multiplier method, transforming the constrained minimization problem into an unconstrained one. As a result of the minimization, the output power spectrum in Capon's beamformer is given by

$$P(\theta) = \frac{1}{\mathbf{a}^H(\theta) \mathbf{R}^{-1} \mathbf{a}(\theta)} \quad (10)$$

Capon's minimum variance method has a much better resolution capability than the classical beamformer. However, correlated interfering signals may cause the cancellation of the desired signal. In addition, a computationally expensive matrix inversion is needed in the calculation of the spatial spectrum.

Parametric methods focus on the parameters of the model. The methods falling into this class comprise the AutoRegressive (AR), Moving Average (MA), AutoRegressive Moving Average (ARMA) and Prony methods [The92]. In the AR, MA and ARMA models the stochastic discrete-time processes are modeled by the rational transfer function model. On the other hand, the Prony method is based on the modeling of signals by the linear combination of exponentials that can be used to approximate ARMA models [The92]. The original input process $\mathbf{x}(n)$ can be generated by applying an innovation process, e.g., a white noise process with variance σ_n^2 [Hay89]. The output power spectrum of the ARMA model is expressed as

$$P(\theta) = \sigma_n^2 \frac{\mathbf{w}_{\text{AR}}^H \mathbf{a}(\theta)}{\mathbf{w}_{\text{MA}}^H \mathbf{a}(\theta)} \quad (11)$$

where \mathbf{w}_{AR} and \mathbf{w}_{MA} are the AR and MA model coefficients, respectively.

2.5.2 Subspace Methods

The majority of the recent research work in the field of spectrum estimation has focused on *high-resolution* subspace methods for DOA and frequency estimation [Mar89]. The

strength of these methods stem from their resolution capability. Two point sources can be separated in case of sufficient SNR which is the well-known Rayleigh criterion. The high-resolution or super-resolution capability is defined to be a limit beyond the Rayleigh criterion. Methods that are based on ED can achieve high resolution because their resolution capability is not restricted by SNR, rather by the number of available samples N .

Numerous different algorithms can be found in this category. Model based high-resolution methods realize the computationally expensive ED or Singular Value Decomposition (SVD) [Gol96]. The vast majority of the methods that can be found in this class are the statistical Maximum Likelihood (ML), Multiple Signal Classification (MUSIC), the Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT), Minimum Norm (MN) and Weighted Subspace Fitting (WSF) methods.

The optimum procedure for finding DOA estimates is through the ML approach. The ML solution can be derived using either *stochastic* or *deterministic* approaches [The92]. In general, the ML solution results in a multidimensional search problem. The problem can be solved by applying the methods of general optimization techniques, for example, Gauss-Newton or gradient methods [Sor80]. The ML method tends to be computationally expensive and the convergence of the iterative algorithms is not guaranteed. Therefore, the ML approach may not be practical for DOA-based estimation and tracking unless simplifications are done in the algorithm development [Sat95]. However, a promising approach for solving the ML problem in practical ways is by the Expectation-Maximization (EM) algorithm [Fed88] or faster converging Space Alternating Generalized EM (SAGE) algorithm [Fes94]. The EM as well as SAGE algorithm decouple the parameter estimation problem into separate simpler optimization problems.

The original work contributing to the area of subspace methods was the introduction of the *Pisarenko* method [Sto88]. This method is based on finding a minimum eigenvector \mathbf{q}_M corresponding to a minimum eigenvalue σ_M of the correlation matrix \mathbf{R} . This construction causes signal eigenvector components of the \mathbf{U}_s to be orthogonal to the minimum eigenvector. According to the theorem roots of the minimum eigenvector \mathbf{q}_M are given by a root expression $z_m = r_k \exp[-j\pi \sin(\theta_m)]$ ($m=1, \dots, M$) from which the K

roots closest to the unit circle provide the final DOA estimates.

The major improvement to the performance shortcomings of the Pisarenko method was the introduction of the MUSIC method. The MUSIC algorithm is a well-known high-resolution method for the detection and localization of signal sources and the method was originally proposed by Schmidt [Sch86]. The resolution capability of the MUSIC method has been analyzed in [Sto90]. The spatial spectrum function of the MUSIC method can be defined as

$$P(\theta) = \frac{1}{|\mathbf{a}^H(\theta)\mathbf{U}_n|^2} \quad (12)$$

The DOA parameter θ is obtained from the value of the parameter vector $\mathbf{a}(\theta)$ which maximizes the spectrum expression of Eq. (12). The MUSIC method can be applied to arrays of arbitrary geometry for estimating azimuth angles, elevation angles and polarization of the incoming signal. However, this generality causes drawbacks since the method is sensitive to deviations in the array parameters. Therefore, the method must be calibrated and a few improvements have been suggested by Tseng *et al.* in [Tse95]. From the algorithmic point of view the MUSIC method is computationally expensive which deteriorates its implementation possibilities for real-time applications. However, the MUSIC method provides better performance than the Pisarenko method. An improved version of the MUSIC algorithm is the computationally more efficient root-MUSIC method, originally developed by Barabell in [Bar83] whereas its performance has been analyzed by Rao *et al.* in [Rao89]. The algorithm involves a root-finding procedure and, therefore, can only be applied to periodic arrays [Dow96].

The ESPRIT algorithm is a very close relative to the MUSIC algorithm and it was developed by Roy *et al.* [Roy89]. This computationally efficient method utilizes the rotational invariance property of the underlying signal subspaces which can be obtained by the translational invariance of the antenna arrays. The desired DOA estimates can be directly calculated from the eigenvalues. As a result, the computationally costly search procedure and the storage requirements of the MUSIC algorithms can be relaxed. The method partially overcomes the problem of array calibration, a difficulty in the MUSIC algorithm, but the offset between antenna pairs must be known exactly. The efficient

and practical way to solve the ESPRIT algorithm is using the TLS–ESPRIT method whose performance has been investigated by Ottersten *et al.* in [Ott91].

The MUSIC and ESPRIT methods fail to work properly in the presence of multipath propagation due to the collapse in the rank of the signal correlation matrix \mathbf{S} . As a remedy, *Spatial Smoothing* (SS) techniques have been presented, first introduced by Evans *et al.* in the context of array processing [Eva82]. In this method the correlation matrix is pre–modified as $\mathbf{R} = \frac{1}{2}(\mathbf{R}_f + \mathbf{R}_b)$ where \mathbf{R}_f and \mathbf{R}_b are the translational invariant forward and backward defined correlation matrices that restore a collapse in the rank with the degrees of freedom reduced to $2M/3$ [Xu94].

The MN procedure was originally proposed by Kumaresan and Tufts in [Kum83]. The method is similar to the Pisarenko method in the sense that a single vector \mathbf{w} of the noise subspace is constructed. The spectrum function in the MN approach is given by

$$P(\theta) = \frac{1}{|\mathbf{a}^H(\theta)\mathbf{w}|^2} \quad (13)$$

where the vector \mathbf{w} is defined in the following way:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{w} \text{ subject to } \mathbf{U}_s^H \mathbf{w} = 0 \text{ and } \mathbf{w}^H \mathbf{e}_1 = 1 \quad (14)$$

where \mathbf{e}_1 has the unity value as the first element of the vector and is zero elsewhere. This represents the minimization of the squared magnitude $\|\mathbf{w}\|^2$ subject to two constraints: the vector \mathbf{w} belongs to the noise subspace and the first component of the vector \mathbf{w} has the unity value. These constraints effectively break noise subspace roots down into two different groups: a set of signal roots lying on the unit circle and a set of *spurious roots* uniformly distributed inside the unit circle.

The WSF method is a member of the same class of subspace fitting algorithms as the deterministic ML method [Vib91]. The minimization criterion function for the WSF method can be expressed as $\theta = \text{argmin} \text{trace}\{\mathbf{P}_n \mathbf{E}^H \mathbf{E}\}$, where \mathbf{P}_n is a noise subspace projection matrix and \mathbf{E} is a suitably defined matrix. In the WSF method the matrix \mathbf{E} is

chosen as $\mathbf{U}_s(\Lambda_s - \sigma_0^2 \mathbf{I})^{1/2}$ [The92]. Most of the subspace algorithms can be cast into a similar framework by defining a suitable matrix \mathbf{E} .

2.5.3 Conventional Methods over Subspace Methods

In this section, most well-known methods for DOA estimation in the stationary signal scenario were reviewed. The parametric methods have an advantage of being simple and efficient to implement but they perform drastically worse in multipath channels. The high-resolution methods have better dynamic range and superior resolution capability but they also perform poorly when the signals are correlated. The high-resolution subspace methods require an estimate for the sample correlation matrix by means of which the respective signal or noise subspaces are established. However, the ED is expensive to compute and difficult to update. Therefore, these methods can only be applied for the accurate DOA estimation of stationary signal sources. In addition, the subspace algorithms are often exploited as a periodic estimation procedure of array parameters like DOAs θ_k and powers P_k .

Most of the subspace methods and parametric methods require the knowledge of the number of incident signal sources through the statistical model order estimation methods. These methods comprise the *Final Prediction Error* (FPE), *Akaike's Information theoretic Criterion* (AIC) and *Minimum Description Length* (MDL) that are based on counting the number of occurring smallest eigenvalues [Hay89]. It was shown in [Xu94] that the SS technique applied for the multipath channels complicates the model order estimation procedure since the penalty terms associated with these criteria need to be modified accordingly.

Table 1 Comparison of conventional and subspace based approaches

	Conventional Methods	Subspace Methods
Advantages	1. Low computational complexity	1. High-resolution property
Disadvantages	1. Multipath channels and multiple users result in performance losses 2. Parametric methods require the determination of the model order	1. Subspace decomposition is expensive to establish 2. Most subspace methods require the determination of the model order

Table 1 summarizes some of the main properties discussed of the methods presented for DOA estimation. Much of the recent research related to these spectrum estimation methods has concentrated on finding fast algorithms for array beamforming and efficient implementation structures of the subspace based high-resolution algorithms. The adaptive DOA estimation algorithms, being the topic of Section 2.6, are targeted at direct tracking of a time-varying eigenstructure or array manifold of the correlation matrix.

2.6 Nonstationary DOA Estimation Methods

The requirements for well-designed tracking schemes arise especially in the case of non-stationary signal scenarios where computationally efficient adaptive algorithms are needed for the DOA-based parameter tracking. Tracking systems can be basically classified into two different groups, *spatially structured methods* utilizing an array manifold of the spatially structured correlation matrix and *eigenstructure based methods* utilizing the eigenstructure of the correlation matrix.

The tracking systems can be furthermore presented as depicted in the block diagram of Figure 6. The *tracking unit* is responsible for tracking time-varying parameter information from the received antenna response vector $\mathbf{x}(n)$ through a parameter matrix $\mathbf{W}(n)$. The *DOA unit* is responsible for estimating the DOA parameters $\boldsymbol{\theta}(n)=[\theta_1(n) \dots \theta_k(n)]^T$ from the parameter matrix $\mathbf{W}(n)=[\mathbf{w}_1(n) \dots \mathbf{w}_k(n)]$. Depending on the specific tracking or DOA extraction criterion, different realizations can be derived for these units.

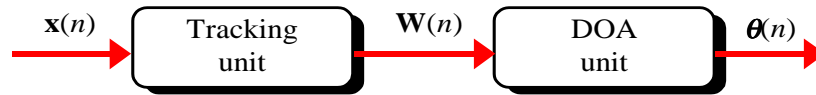


Figure 6 Block diagram showing the components of the tracking system

2.6.1 Spatially Structured Methods

Adaptive spatially structured methods track a time-varying manifold of the correlation matrix $\mathbf{R}(n)$ based on the previous array manifold information in $\mathbf{R}(n-1)$ from which the DOAs can be extracted. In this section we focus on developing two efficient spatially structured methods, the gradient based tracker and the CG based tracker.

An example of spatially structured methods is the algorithm proposed by Sword *et al.* in [Swo90] and later improved by Lo *et al.* in [Lo92]. Their approach is based on a recursive formulation for tracking DOAs where small changes in the difference of the correlation matrix estimates $\mathbf{R}(n)$ and $\mathbf{R}(n-1)$ result in small changes in the difference of the steering matrices $\mathbf{A}(\boldsymbol{\theta}(n))$ and $\mathbf{A}(\boldsymbol{\theta}(n-1))$ from which the DOAs can be extracted.

In a similar way, in a method proposed by Sastry *et al.* the DOAs are estimated by minimizing a specific cost function involving the error norm of the sample and model covariance matrices for solving of which the block CG algorithm is invoked periodically [Sas91]. However, as a drawback of both approaches the number of the users, their DOAs and especially their signal powers must be periodically estimated using a signal subspace algorithm. One of the problems with these tracking algorithms is the *data association problem*, i.e., associating the DOA estimates established at the previous time instant $n-1$ to that of time instant n [Rao94]. In the case of the gradient based tracker we rely on the recursive formulation and in the case of CG based tracker we rely on the existence of a unique reference signal.

2.6.1.1 Gradient Based Tracker

In this section a gradient based approach for tracking the parameter matrix will be derived. The formulation of this method can be approached through the general cost function given by

$$J = \arg \min_{\mathbf{W}} E[\|\mathbf{x} - \mathbf{W}(\mathbf{W}^H \mathbf{W})^{-1} \mathbf{W}^H \mathbf{x}\|^2] \quad (15)$$

where \mathbf{W} is the weight vector matrix $\mathbf{W}=[\mathbf{w}_1 \dots \mathbf{w}_K]$. This can be interpreted as the projection of the incoming signal vector \mathbf{x} into a complement subspace spanned by the columns of the matrix \mathbf{W} . Depending on how \mathbf{W} is constrained, different tracking methods can be derived. The cost function can be simplified by constraining $\mathbf{W}^H \mathbf{W} = \mathbf{I}$. However, this may not be necessary due to the self-fulfilling property of the constraining criterion. By constraining the component vectors of the weight matrix \mathbf{W} to belong to the array manifold the gradient based adaptation algorithm can be derived as originally proposed by Affes *et al.* in [Aff94] and [Aff96]. By taking the gradient with respect to \mathbf{W} in the cost function of Eq. (15), and after some algebraical manipulations the adaptive Least Mean Squares (LMS) update formula can be expressed as follows:

$$\mathbf{w}_k(n) = \mathbf{w}_k(n-1) + [\mathbf{x}(n) - \mathbf{w}_k(n-1)s_k(n)][\mu s_k(n)] \quad (16)$$

In the stationary signal scenario the method converges to the steering vectors of K users

corresponding to the signal subspace part of the correlation matrix. However, in order to guarantee the constraint the continuous structure fitting back to the array manifold is needed. In the nonstationary signal scenario the adaptation method tracks slowly variations in the steering vectors. The generation of the user signal $s_k(n)$ is needed but it will be provided explicitly from the adaptation weight vector $\mathbf{w}_k(n)$. It should be noted that the classical steering vector based methods are very sensitive to the pointing errors in the DOA estimates and even slight deviations cause significant losses at the output Signal-to-Noise plus Interference Ratio (SNIR) of the beamformer [Hon87]. However, in this method robust beamforming is achieved during the adaptation self-correcting DOA errors for the beamformer. In [P1], the focus was on this steering vector based method for which we developed an adaptive step size method for increasing the tracking performance.

2.6.1.2 CG Based Tracker

The existence of the known sequence $s_k(n)$ called the *reference signal* in a communications system is valuable because it can be utilized for the reference signal based adaptive algorithms. For these adaptive algorithms the optimum solution in the Minimum Mean Square Error (MMSE) sense is the one which minimizes the quadratic error between the reference signal $s_k(n)$ and the output of the beamformer $\mathbf{y}_k(n)=\mathbf{w}_k(n)^H\mathbf{x}(n)$, i.e., the quadratic cost function $E[(s_k(n)-\mathbf{w}_k(n)^H\mathbf{x}(n))^2]$. This minimization criterion results in the cost function that can be expressed as

$$J = \mathbf{w}_k^H \mathbf{R} \mathbf{w}_k + \mathbf{w}_k^H \mathbf{b} \quad (17)$$

where $\mathbf{b}=E[s_k(n)\mathbf{x}(n)]$ is the cross-correlation vector between the desired signal sequence $s_k(n)$ and the array response vector $\mathbf{x}(n)$. The reference signal should have low cross-correlation with all other interference signal components. As a result, this quadratic cost function can be used as a minimization criterion of interference signal components. These reference signal based algorithms relax the requirement to have more antenna elements than signals to be received. In addition, the methods are less sensitive to errors in the antenna array parameters, for example in the sensor locations. However, the major drawback of this approach is the requirement to know or generate a reference signal. Therefore, the tendency is towards blind methods.

The residual vector for the cost function of Eq. (17), i.e. the gradient with respect to \mathbf{w}_k results in a Wiener–Hopf equation $\mathbf{g}_k = \mathbf{R}\mathbf{w}_k + \mathbf{b}$. In general, the solving of this kind of time–varying Least Squares (LS) type problem can be approached by the methods of the adaptive filtering theory like LMS or Recursive Least Squares (RLS) algorithms [Din97] or SVD based adaptive subspace algorithms [Fer90],[Kav94]. The well–known CG algorithm provides an efficient way to solve the system of M linear equations. The CG method was originally developed independently by Stiefel and Hestenes [Hes52]. The method collects a block of data samples and converges to the solution after accomplishing M iteration steps. Therefore, it doesn't suite well for the estimation of time–varying parameters.

The key point for the sample–by–sample processing basing on the CG based approach is the incorporation of the time–varying estimates of the correlation matrix $\mathbf{R}(n)$ and cross–correlation vector $\mathbf{b}(n)$ into the residual vector $\mathbf{g}_k(n)$ [Cha98]. Consequently, after some algebraical manipulations the residual vector can be formulated as

$$\mathbf{g}(n) = \lambda_t \mathbf{g}(n-1) - \alpha(n) \mathbf{R}(n) \mathbf{p}(n-1) + \mathbf{x}(n) \left(d(n) - \mathbf{x}(n)^H \mathbf{w}(n-1) \right) \quad (18)$$

where $\alpha(n)$ is a convergence parameter and $\mathbf{p}(n-1)$ is a search direction vector as they are defined in a similar way in the context of the ordinary block CG algorithm [Gol96]. In [P2] the tracking unit was implemented with this expression of the residual vector for the CG algorithm.

2.6.1.3 DOA Extractor

In this section a function for the DOA extraction will be developed for the spatially structured methods. This unit serves as a two–way purpose: primarily, it can used for the extraction of θ_k :s from the array manifold \mathbf{A} . Secondly, but as essentially as in the first case, it serves as a way to restore deviations in the estimated steering vectors caused by the array manifold tracking algorithms back to the array manifold. Deviations in the steering vector $\mathbf{a}(\theta_k)$ can be modeled as

$$\mathbf{a}(\theta_k) = \mathbf{H} \theta_k + \mathbf{n} \quad (19)$$

where the column vector \mathbf{H} for the uniform linear arrays has as its elements $2\pi d(m-1)\sin(\theta_k)f/c$ with m ($m=1, \dots, M$) being an index and \mathbf{n} represents a deviation vector caused by the adaptation algorithm. The LS fitting criterion based on the time differences of the array manifold at the time instants n and $n-1$ can be formulated as

$$\hat{\theta}_k^{(LS)}(n) = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \text{Im} \left(\log \frac{\mathbf{w}(n)}{\mathbf{a}(\theta_k(n-1))} \right) \quad (20)$$

where $\mathbf{H} = [0 \ \pi \ 2\pi \ \dots \ (M-1)\pi]^T$ with $\lambda/2$ spaced antenna elements, $\mathbf{w}(n)$ is a current estimate for the k :th steering vector and the Im -operation, \log -operation and vector division are defined to be elementwise. In [P2], this unit is developed and its implementation is discussed in detail.

2.6.2 Eigenstructure Based Methods

The eigenstructure based tracking methods are based on finding eigenvalues and eigenvectors of the correlation matrix \mathbf{R} . The adaptive eigenstructure based methods track a full or partial time-varying eigenstructure of the correlation matrix $\mathbf{R}(n)$ based on prior knowledge of the eigenstructure in $\mathbf{R}(n-1)$. Because the ED is generally expensive to compute and difficult to update in an efficient way we focus on developing a simple adaptive gradient based approach. In the beginning, some of the developed subspace methods are reviewed.

2.6.2.1 Adaptive Subspace Trackers

Pisarenko was the first to prove that the harmonic frequencies contained in the white noise can be found through eigenvalue analysis [Pis73]. Later, the statistical performance of the Pisarenko harmonic decomposition method was analyzed in [Sto88]. The original concept of estimating the signal subspace \mathbf{U}_s was first developed by Owsley [Ows78]. Thompson was the first to implement a method for seeking an eigenvector \mathbf{q}_M corresponding to the minimum eigenvalue σ_M of the correlation matrix

R [Tho80]. In [Hay86] the applicability of the classical methods, i.e., the power method, the inverse power method and the Rayleigh Quotient Iteration (RQI) [Gol96] for computing Pisarenko harmonic decomposition were reviewed and an efficient algorithm based on the Levinson recursion was developed based on the Toeplitz structure. In [Com90] a class of classical algorithms like Lanczos based trackers are reviewed for tracking eigenvectors of the correlation matrix.

Thompson's method can be derived from the generalized cost function as expressed in Eq. (15). If the constraint of the weight vector matrix \mathbf{W} belonging to the array manifold is released, the minimization criterion is changed to the minimization of a cost function $J = E[|\mathbf{x} - \mathbf{U}_s \mathbf{U}_s^H \mathbf{x}|^2]$ [Yan95]. This cost function can be interpreted as the projection of the incoming signal vector \mathbf{x} onto the complement of the signal subspace \mathbf{U}_s . This method, first proposed by Yang *et al.*, extended the idea of tracking a single eigenvector for tracking a restricted set of eigenvalues and eigenvectors or a signal subspace spanned by K eigenvectors corresponding to the largest eigenvalues or a noise subspace spanned by $M-K$ eigenvectors corresponding to the smallest eigenvalues respectively [Yan88]. Yang's method can be regarded as a generalization of Thompson's approach.

The tracking of the whole signal or noise subspace structure with the previous eigenstructure methods is computationally demanding. In the way of reducing the computational complexity of the expensive updating of the full eigendecomposition in the subspace trackers Karasalo was first to introduce a concept of the noise subspace averaging [Kar86]. The noise subspace averaging is realized by averaging the smallest $M-K$ eigenvalues and replacing them with an average value. Instead, DeGroat and Dowling have introduced the subspace tracking method where the concept was extended furthermore to the signal subspace [DeG92]. In their approach both the signal and noise subspaces are averaged. An arbitrary orthonormal basis spanned by eigenvectors of the signal and noise subspace has been established where the respective signal and noise eigenvalues are replaced by their average values. This is desirable especially in the DOA estimation methods that require only some arbitrarily oriented space. Due to the spherical subspaces there is a decrease in performance but in the case of closely located sources these methods perform even better as shown in [Dow92][DeG93]. In addition, the deflation of both the signal and noise subspace leads into an eigenvalue problem of

size 2 which can be efficiently solved noniteratively, but the estimate of the subspace dimension K is needed [DeG90][DeG92].

The signal or noise subspace tracking problem can be formulated as a constrained minimization problem. The constrained type of optimization problem expresses the general quadratic cost function as

$$J = \text{tr}(\mathbf{W}^H \mathbf{R} \mathbf{W}) \text{ subject to } \mathbf{W}^H \mathbf{W} = \mathbf{I} \quad (21)$$

For solving of this minimization problem many adaptive signal processing algorithms have been utilized, e.g., gradient, Gauss–Newton and RLS methods [Din97] have been employed. Yang *et al.* in [Yan88] has applied the gradient based approaches together with the Gram–Schmidt (GS) orthonormalisation procedure for accomplishing the orthonormality constraint [Tre97]. Mathew and Reddy in [Mat95] transformed the optimization problem into the unconstrained one first by estimating a single eigenvector by the Gauss–Newton based algorithms and then extending the estimation to that of the total subspace through the inflation based approach. In the single eigenvector case the optimization problem can be transformed into an unconstrained minimization one through Lagrange’s theorem. In [P3], based on this cost function, a step–by–step adaptation method was developed for tracking a minimum eigenvector using the high–resolution noise subspace approach.

2.6.2.2 DOA Extractor

In this section, a function for the DOA extraction is developed for the noise subspace root–tracking. When the dominant subspace estimates have been established the subspace spectrum estimation methods can be applied for finding the necessary DOA estimates. However, in the case of the linear antenna arrays the problem of parameter extraction can be expressed as that of a polynomial root–finding. It has been shown by Rao *et al.* in [Rao89] that even a higher resolution capability can be achieved with the root versions of the estimators than with their spectral counterparts of Section 2.5.2. The coefficients of the null–spectrum function of the spectral estimators are expressed as

$$\mathbf{w}_m = \text{tr} \left\{ \sum_{m_1, m_2 = m} \mathbf{P}(m_1, m_2) \right\} \quad (22)$$

where \mathbf{P} is a spectrum matrix. As an example the spectrum matrix \mathbf{P} can be defined as $\mathbf{P} = \mathbf{U}_n \mathbf{U}_n^H$ or $\mathbf{P} = \mathbf{q}_M \mathbf{q}_M^H$ for the MUSIC and the Pisarenko methods, respectively [Yan89].

One of the most difficult aspects in the DOA computation is that of solving the polynomial root-finding problem in an efficient and stable way. It can be expressed as an eigenvalue problem for which the *companion matrix* based approaches can be applied [Str97]. Eigenvalues of the companion matrix are the same as roots of the null-spectrum function. The fundamental difficulty with the application of the methods is that many familiar root-finding algorithms will only converge to a particular root if a sufficiently close initial guess is provided. In addition, these methods experience difficulties in not always converging into the correct root locations, especially, if initial parameter estimates are not close enough to the true values. The Gauss-Newton, secant, the classical power method, the inverse power method and the RQI methods etc. have been modified for solving the root finding problem [Sta89][Yan89].

Therefore, in [P3] we concentrated on employing a zero-tracking method originally proposed by Orfanidis [Orf86]. In this method, after locating the first root the root polynomial can be deflated. However, the zero-tracking based approach for the polynomial root tracking has two inherent problems. One of the problems arises due to the recursive nature of the deflation procedure causing computation errors of root estimates to propagate to the following roots. However, this problem can be solved by the appropriate modification of the null-spectrum function [Ho92]. The second problem is caused by unpredictable movement of these spurious roots resulting in spiky estimation errors. In [P3] an adaptive control procedure for controlling the movement of these spurious roots was developed that was based on the grouping of the roots into two distinct categories.

2.6.3 Spatially Structured over Eigenstructure Based Approaches

One of the largest differences between the spatially structured and eigenstructure based approaches arises from the way how the DOA unit has been implemented. In the case of

the eigenstructure based approaches the implementation of the DOA unit must be carefully considered. Since these methods adaptively track the signal or noise subspaces, ordinary DOA estimation methods of Chapter 2.5 can be applied for finding the DOAs. As an example, the MUSIC method finds DOAs through a search procedure over the parameter space. However, this computationally expensive search procedure can be reduced by using the root version of the MUSIC method still requiring a solution to an ill-conditioned polynomial root-finding problem. The problem with the adaptive polynomial root-finding approach for the eigenstructure based methods is that they can only be applied to linear array structures, therefore, reducing the applicability of these methods whereas the spatially structured methods can be applied to antenna arrays of arbitrary geometry. Altogether, the eigenstructure methods require a separate carefully designed DOA unit adding some computational complexity to the overall system whereas for the spatially structured methods the DOA unit can be implemented in a more straightforward and robust way.

The convergence behavior of the tracking methods depends drastically on the proper parameter initialization of the algorithms in both approaches. The performance of the stochastic algorithms is greatly affected by the proper selection of the convergence parameter. In [P1] the performance of the enhanced tracking system with an adaptive convergence parameter was compared with the system with a fixed convergence parameter. For the method with a fixed convergence parameter, performance losses were experienced both in the stationary and the non-stationary signal scenarios. Therefore, in parameter tracking problems initial DOA information is valuable for providing faster convergence closer to the optimal solution. However, the initial DOA estimates must be close to the true values for both approaches to converge. This suggests that the adaptive methods can be initialized by using a computationally complex DOA estimation method.

Round-off errors tend to propagate in recursively defined algorithms and a periodical re-initialization procedure may be needed. For the ill-conditioned problems the commonly used technique is to apply a technique of the square-root filtering for the correlation matrix based algorithms. In this method the correlation matrix is decomposed into an upper triangular matrix and its transpose and a diagonal matrix [Gol96]. Somewhat more complex expressions have been obtained but the nonnegative

definiteness is guaranteed.

Many adaptive beamforming algorithms are sensitive to errors in the DOA estimates. Different beamforming methods based on the subspace approach have been developed for enhancing the performance in the presence of pointing errors. These methods require adaptive tracking of the signal subspace and good performance can be achieved when the steering vectors fall into the tracked signal subspace [Cho97]. In the spatially structured case the self-robustness for the beamforming is achieved through adaptation.

The tracking systems can be categorized based on their complexity. The computational complexity of the subspace based approaches originates from a need to realize the ED or SVD in every update step. These eigenstructure based subspace trackers can be categorized into two groups of computational complexity, $O(M^2K)$ and $O(MK^2)$. The complexity $O(M^2K)$ is the result of matrix-vector multiplications. Also, the tracking system presented in [P2] falls into this category. The methods of the computational complexity $O(MK^2)$ are based on a low-rank approximation of the correlation matrix. However, both the signal and noise subspace dimensions can be deflated reducing the computational complexity. The spatially structured methods can generally reach a lower implementation complexity. This results from the fact that the implementation of the tracking unit can be based directly on the adaptive algorithms and the implementation of the DOA unit is simple in nature. Therefore, in the spatially structured case the computational complexity of $O(MK)$ can be achieved.

One of the problems for both approaches is the lack of adaptive tracking of the subspace dimension. Stewart *et al.* have proposed a URV based decomposition method acting as a reliable substitute to the subspace based algorithms. The method can be applied for the adaptive estimation of both the rank and noise subspace [Ste92]. The rank is automatically detected when the sequence of plane rotations is applied to the correlation matrix.

To sum up, Table 2 characterizes the main properties discussed between the spatially structured and eigenstructured based approaches.

Table 2 Comparison of spatially structured and eigenstructured based approaches

	Spatially Structured Methods	Eigenstructure Based Methods
Advantages	Arrays of arbitrary geometry Simplified DOA procedure	High-resolution property Array geometry not necessarily uniform
Disadvantages	Model order estimation No high-resolution property	Complicated DOA procedure Model order estimation

3. Summary of Publications

3.1 Publication 1

In the first paper, the configuration of an adaptive antenna system with multi-target tracking is presented. The DOA estimation algorithms can be classified into two groups, a group of stationary DOA estimation methods such as the MUSIC method and a group of non-stationary DOA estimation methods such as Sword's algorithm or the Kalman approach. Few disadvantages of them were identified from the user tracking point of view. The problem with the conventional adaptive tracking algorithms is that the adaptation performance is greatly influenced by the proper choice of the convergence parameter. A fixed choice of convergence parameter may be problematic since it causes extra noise called misadjustment during the adaptation.

Therefore, in the paper we employed the multi-target tracking concept originally proposed by Affes *et al.* that is based on the spatial structured correlation matrix. The tracking system was modified by the inclusion of an adaptive control mechanism for the convergence parameter. The method is based on DOA error function for each user comprising a block of signs of DOA error samples from the model fitting unit. The convergence parameters of all the users are accordingly adjusted depending on the frequency of zero crossings in the error function, a similar approach for the adaptive calculation of the convergence parameter as developed by Harris *et al.* in [Har86] in the context of the adaptive filtering.

For the performance comparison the same simulation configuration has been utilized as also in the later publications [P2] and [P3]. The performance comparison has been accomplished in terms of convergence speed and misadjustment. The mobile users have the initial 5° pointing errors in their DOA estimates. The simulation results showed that the proposed adaptive control of the convergence parameter provided fast convergence and small misadjustment errors that cannot be achieved by utilizing a fixed convergence parameter. In the case of stationary sources, the adaptive convergence parameter method can achieve slightly faster convergence and smaller DOA errors than methods with fixed settings. By carefully selecting a fixed convergence parameter, a similar

performance can be achieved. However, in the nonstationary case the method with adaptive convergence parameter can easily outperform any fixed setting for the convergence parameter, especially when mobile users move along curved trajectories.

3.2 Publication 2

In the previous publication [P1], a gradient method with an adaptive step size control was developed for the user tracking system. The adaptive step size method was based on the block processing form. For achieving better performance a block of samples was gathered and one update iteration was performed. This might be disadvantageous due to two reasons. Practical systems may speak in favor of a step-by-step update mode rather than block update mode. More importantly, the performance of the block update model decreases in the nonstationary signal scenario.

In this paper, an adaptive step-by-step update scheme of the reference signal based CG method is implemented for the user tracking system. In addition, a function was developed for the DOA calculation in the array manifold adaptation methods. In the proposed system the convergence parameter will be automatically adjusted for better performance during the adaptation process. As a result of the updating process, the method converges to the dominant signal subspace spanned by the signal subspace part of the full rank correlation matrix. The simulation results confirm that the proposed CG based system achieves better tracking performance than gradient methods both in the stationary and non-stationary signal scenario. The better tracking performance is achieved in terms of faster and smoother convergence and smaller misadjustment. In addition of the analysis of the tracking performance, the computational complexity of the system is also investigated.

3.3 Publication 3

In this paper we have derived a high-resolution noise subspace algorithm based on a step-by-step adaptation scheme for tracking a minimum eigenvector of the correlation matrix. The tracking algorithm is based on the adaptive version of the Pisarenko harmonic decomposition method. The minimization of the *Rayleigh quotient* is solved

through the unconstrained cost function as defined by Lagrange's theorem. The residual vector is defined and the respective convergence parameter is calculated through the line search procedure. As a result of the minimization, the update equations similar to the method of Steepest Descent (SD) are obtained. The convergence parameter is similar to that existing in the block CG algorithm but simpler than the expression in [Fu95]. In the case of small eigenvalue spread the performance of the SD method will be comparable to that of the CG method in the tracking unit.

In the DOA extraction unit for the zero-tracking based method [Orf86] we have developed a control strategy for noise subspace roots eliminating the DOA association problems with spurious roots. In this method, roots are classified into two different groups, a set of roots corresponding to users lying close to the unit circle and a set of spurious roots which are uniformly distributed inside the unit circle. The adaptive control procedure for governing the movement of all the roots has been developed. This decomposition of noise subspace roots into two different groups is similar to the decomposition done for the MN method. However, in contrast to the MN spatial spectrum estimator of Section 2.5 the movement of different roots is adaptively controlled.

In the noise subspace the closely located sources can be generally separated with the lower mean DOA error in the stationary signal scenario. The performance comparisons confirm that the proposed noise subspace approach can achieve a similar tracking performance in terms of convergence speed and final misadjustment level as compared to the tracking algorithms of the spatially structured case in the publication [P3].

3.4 Publication 4

In this paper we focus on Very Large Scale Integration (VLSI) implementation of the tracking system by developing a novel systolic architecture suitable for real-time applications. For our user tracking system we have estimated the computational complexities of different units. The computationally most complex operation is the calculation of the convergence parameter in the algorithm of the tracking unit. Based on these estimations the software and hardware implementation parts have been identified. For the hardware implementation the systolic array is developed for the computationally

intensive matrix–vector multiplications. The operations in the algorithm are distributed into an array of distributed memory elements providing the fine–grain parallelism.

The proposed systolic architecture reduces the required computational complexity of the tracking unit by an order of magnitude to $O(M)$. Furthermore, the Strength Reduction (SR) transformation technique is utilized for the implementation of the complex multipliers that allows remarkable savings in silicon area and consumed power [Par99].

3.5 Author’s Contributions

In Publication [P1], the first author’s contribution was essential in designing the tracking system, preparing the text for the article and planning the simulation software. During the research work Prof. Timo Laakso and D.Sc. Jorma Lilleberg acted as supervisors and they were also involved in setting the goals of this research project.

In the research work of the Publication [P2], the first author was responsible for designing the CG based tracking system and performing the simulations while the second author, Lic. Tech. Ramin Baghaie was responsible for a section in the article discussing the computational complexity issues of the proposed tracking system.

The ideas presented in Publication [P3] initially originate from the author. The author himself wrote the text in the article and was responsible for carrying out the simulations. Prof. Timo Laakso provided comments on the manuscript.

In Publication [P4], the first author Lic. Tech. Ramin Baghaie was responsible for carrying out the estimation of the computational complexities and for designing the practical systolic array while the second author was responsible for the software development of the user tracking application and assisted in the preparation of the article.

4. Conclusions and Discussions

The main focus of this licentiate thesis was directed towards the development of adaptive tracking algorithms for DOA estimation of mobile users in non-stationary signal scenarios. Tracking systems should support fast tracking capability, have small estimation errors for the tracking parameters and should have low implementation complexity. In order to meet these criteria we focused on the construction of three different tracking methods, adaptive convergence parameter method and step-by-step CG method for the array manifold tracker of the spatially structured approach and a simplified gradient based approach for the adaptive tracking of a minimum eigenvector in the noise subspace.

An adaptive step size method was developed for more efficient tracking of a time-varying manifold of the antenna array. The method with the fixed convergence parameter experiences the performance losses in the MSE sense both in the stationary and the non-stationary signal scenarios because it can not combine simultaneously the requirements of the fast convergence speed and low estimation error. Consequently, the adaptive step size method performs better. Furthermore, in another approach a reference signal CG based tracker based on the step-by-step adaptation mode was also developed that can unambiguously associate different DOA estimates, relax a requirement to have the number of signals less than the number of antenna elements and is not sensitive to nonidealities of the antenna array parameters. With these methods fast convergence speed and small DOA estimation errors can be obtained. It can be deduced based on the geometrical considerations of the BS environment that the tracking speed in the developed adaptive algorithms is more than adequate also for closely located sources near the BS.

We reviewed a class of adaptive eigenstructure based methods from the literature that track a full or partial time-varying eigenstructure of the correlation matrix based on the prior knowledge of the eigenstructure. For the utilization of the high-resolution approach we developed a simplified SD based method for the adaptive tracking of a minimum eigenvector from the time-varying eigenstructure of the correlation matrix. Furthermore, the developed control strategy for the noise subspace roots alleviates the DOA association problems due to the spurious roots.

Differences in the spatially structured and eigenstructure based approaches lead us toward different implementation structures for the tracking and DOA extraction units. When considering the implementation aspects of our tracking system a suitable systolic architecture for the CG based tracker was developed. The existing fine-grain parallelism in the algorithms helped to reduce the computational complexity of the tracking system by an order of the magnitude $O(M)$.

As a future research work we propose DOA tracking of mobile users in more realistic multipath channels associated with difficult challenges of the multipath propagation. The research activity is especially directed towards the development of tracking algorithms for CDMA based applications.

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Appendix A: List of Publications

Publication 1

P. Karttunen, T. Laakso and J. Lilleberg, "Tracking of mobile users in mobile communication system using adaptive convergence parameter," *Proceedings of the International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'97)*, Vol. 3, Helsinki, Finland, September 1–4, 1997, pp. 989–993.

Publication 2

P. Karttunen and R. Baghaie, "Conjugate gradient based signal subspace mobile user tracking," *Proceedings of the Vehicular Technology Conference (VTC'99)*, Vol. 2, Houston, Texas, USA, May 16–20, 1999, pp. 1172–1176.

Publication 3

P. Karttunen, "An algorithm for noise subspace based mobile user tracking," *Proceedings of the International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'99)*, Vol. 2, Osaka, Japan, September 12–16, 1999, pp. 651–655.

Publication 4

R. Baghaie and P. Karttunen, "VLSI implementation of CG based mobile user tracking," *Proceedings of the IEEE Midwest Symposium on Circuits and Systems (MWSCAS'99)*, Las Cruces, New Mexico, USA, August 8–11, 1999