AN ALGORITHM FOR NOISE SUBSPACE BASED MOBILE USER TRACKING

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Abstract − In this paper for the user tracking application we develop a step−by−step adaptation method for tracking the parameter vector in the noise subspace. Also, we develop the control strategy for the noise subspace roots eliminating the problems with the spurious roots. The performance comparison of the developed user tracking system will be investigated in terms of the tracking capability. As the performance criterion, the mean Direction−of−Arrival (DOA) error will be invoked in the stationary and non−stationary signal scenarios. The simulation results confirm that the proposed noise subspace approach can achieve similar tracking performance as the signal subspace approach in terms of the convergence speed and the final misadjustment level.

I. INTRODUCTION

Adaptive array processing techniques can provide more system capacity by reducing co−channel interference. This can be realised by the beamforming based communication [1]. For this aim the parameter estimation must be appropriately carried out for all the users of interest both in the stationary and non−stationary signal scenarios. These parameters could be estimated by any of the well−known spectrum estimation methods like MUltiple SIgnal Classification (MUSIC) algorithm but their inherent computational complexity makes their continuos invocation infeasible [1]. Therefore, the adaptive target tracking methods continuously update tracking parameters of all the users. There are numerous methods that can be applied for the multi−target tracking problems. In [2], for the user tracking system, a step−by−step update mode of Conjugate Gradient (CG) based algorithm was developed for directly updating the signal subspace related array response vector. Instead, in this paper, a different viewpoint has been taken. For our user tracking system, we will develop an adaptive high−resolution noise subspace approach.

The paper is organised as follows: in Section 2, the signal model is introduced on which the paper will be based. Section 3 introduces the overall system design by partitioning the user tracking problem into three different separate system blocks, namely tracking, Direction−of−Arrival (DOA), and the beamforming units. The design and the analysis of these different blocks will be carried out. Section 4 presents some numerical results comparing both the signal and the noise subspace approaches. Section 5 draws conclusions about the proposed system design.

II. SIGNAL MODEL

The observation vector $\mathbf{x}(t)$ of a one−tap channel model for the antenna receiver can be expressed as

$$\mathbf{x}(t)=\sum_{k=1}^{K}\sqrt{P_k}\,b_k(t)\,\mathbf{a}(\theta_k)+\mathbf{n}(t) \qquad (1)$$

where the column vectors $\mathbf{a}(\theta_k)$ and $\mathbf{n}(t)$ are the array response vector for kth user (k=1, ..., K) and the Additive White Gaussian Noise (AWGN) vector, respectively [1]. The total number of co−channel users has been denoted as K. The array response vector has been parameterised by θ_k representing DOA on the azimuth plane. The transmitted signal waveform $b_k(t)$ is modelled as a zero−mean Gaussian distributed process with the variance P_k. Similarly, the additive noise vector $\mathbf{n}(t)$ independent from user signals is also drawn from a Gaussian distribution with the variance $\sigma_n^2\mathbf{I}$. Signal−to−Noise Ratio (SNR) for the presented signal model is defined to be $10\log(P_k/\sigma_n^2)$ [3].

For the Uniform Linear Antenna (ULA) arrays the normalised array response vector $\mathbf{a}(\theta_k)$ with the $\lambda/2$ spaced elements can be defined in the following way:

$$\mathbf{a}(\theta_k)=\frac{1}{\sqrt{M}}\left[1 \;\; e^{-j\pi\sin(\theta_k)} \;\cdots\; e^{-j\pi(M-1)\sin(\theta_k)}\right]^{\mathrm{T}} \qquad (2)$$

In the matrix notation, Eq (1) can now be expressed more compactly as $\mathbf{x}(t)=\mathbf{A}(\theta)\mathbf{s}(t)+\mathbf{n}(t)$, where the antenna response vectors of each users are collected into a single matrix $\mathbf{A}(\theta)=[\mathbf{a}(\theta_1)\;\ldots\;\mathbf{a}(\theta_K)]$. The antenna response matrix $\mathbf{A}(\theta)$ is assumed to be slowly varying with respect to the parameter θ. Finally, the model correlation matrix for the antenna array, which is symmetric, non−negative definite and Toeplitz can be defined as $\mathbf{R}=E[\mathbf{x}(t)\mathbf{x}(t)^H]=\mathbf{ASA}^H+\sigma_n^2\mathbf{I}$, where $\mathbf{S}$ is a diagonal signal correlation matrix [1].

The user tracking problem is that of continuously tracking the parameter θ_k of the linear model in Eq (1). In the following sections, the adaptive algorithms are developed for tracking variations in the parameter value. There exists Cramer−Rao Lower Bound (CRLB) on the minimum variance of the parameter θ that any unbiased estimator can only asymptotically approach [3]. Based on the presented signal model the CRLB can be calculated by some algebraical manipulation as

$$var(\theta_k)=\left[N\left(\frac{\sigma_s^2}{\sigma_n^2}\right)\left\|\frac{\partial a(\theta_k)}{\partial\theta_k}\right\|^2\right]^{-1} \quad (3)$$

where N is the total number of samples. This bound is too loose in the sense that it gives optimistic performance values compared to the bounds of the adaptive estimators. However, this bound reflects many interesting properties about the effect of the presented signal model on the minimum variance of the parameter.

III. SYSTEM MODEL

In this section, the user tracking system will be formulated. In [2], we investigated the step−by−step adaptation scheme for updating the signal subspace vector. We utilise similar construction for the tracking system but we concentrate on the design and the implementation of the step−by−step update scheme for the noise subspace. Figure 1 describes the system components of the noise subspace based user tracking. The *tracking unit* deals with the adaptive algorithms for tracking the noise subspace. In our approach, it essentially consists of *Rayleigh solver* for seeking the vector $\mathbf{w}(n)$ corresponding to the minimum eigenvalue of the model correlation matrix. The *DOA unit* deals with the extraction procedures for finding the necessary parameter values. Here, we develop a recursive rooting method for the parameter tracking. The *beamforming unit* deals with the beamforming procedures for enhancing the desired signal reception and nulling the interference. However, the beamforming unit is not

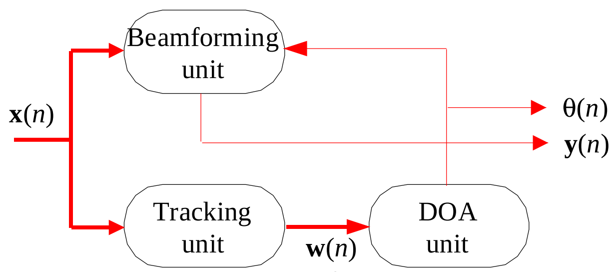

Figure 1 *Overall block diagram for the noise subspace based mobile user tracking system (n: iteration index)*

necessary for the functionality of the tracking system and its consideration will be discarded in this paper.

Tracking unit

In this section, we derive a step−by−step update algorithm for tracking the minimum eigenvector corresponding to the minimum eigenvalue of the model correlation matrix $\mathbf{R}$. As the performance criterion we minimize the total output power at the output of the array. In order to prevent the power decreasing to the zero the constraint for the weight vector will be introduced. The quadratic cost function can be expressed as in Eq (4) and will be used as the minimization criterion of interference and noise components.

$$J=\mathbf{w}^H\mathbf{R}\mathbf{w} \quad (4)$$

The minimisation of this performance criterion subject to the constraint $\mathbf{w}^H\mathbf{w}=1$ leads us into the minimum eigenvector. This can be deduced by diagonalising the model correlation matrix by invoking the Eigenvalue Decomposition (ED) as

$$J=\mathbf{w}^H\left[\sum_{m=1}^{M}\sigma_m^2\mathbf{u}_m\mathbf{u}_m^H\right]\mathbf{w} \quad (5)$$

where $\mathbf{u}_m$ is an eigenvector. As a result, the eigenvectors $\mathbf{u}_m$ (m=1, ..., K) will form the orthonormal basis for the signal subspace. In the similar way, the eigenvectors $\mathbf{u}_m$ (m=K+1, ..., M) will form the orthonormal basis for the noise subspace. Clearly, the cost function will be minimised as $\mathbf{w}$ converges into $\mathbf{u}_M$. Typically, the ordinary adaptive eigensubspace algorithms track the whole subspace. Because of the orthonormal subspace requirement, many of these subspace algorithms will eventually need to invoke for example the *Gram−Schmidt* orthonormalisation procedure. The computational complexity may be prohibitive when utilising the high−dimensional subspace. Due to the fact of not utilising the whole subspace we experience some performance losses. However, as the signal subspace dimension K approaches M the available information in the noise subspace will anyway diminish. Furthermore, the model order estimation methods, i.e., the estimation of number of users will be required by these methods. If the model order has been underestimated greater performance losses can be expected compared to the overestimation, because part of the signal subspace has been included on the estimation. Therefore, in this case by using only one noise eigenvector will be more robust and also enough in terms of performance.

The model correlation matrix can be estimated through *exponentially decaying data window*. The rank one update scheme has been chosen for the exponentially decaying correlation matrix estimate $\mathbf{R}(n)$ with the forgetting factor λ_f. The forgetting factor λ_f reflects the amount of signal non−stationarity in the correlation matrix estimate. The model correlation matrix $\mathbf{R}(n)$ can be expressed as

$$\mathbf{R}(n) = \lambda_f \mathbf{R}(n-1) + \mathbf{x}(n)\mathbf{x}(n)^{\mathrm{H}} \qquad (6)$$

We use *Lagrange's theorem* for solving the minimisation problem. The Lagrange's unconstrained cost function is defined as

$$J_L = \mathbf{w}(n)^{\mathrm{H}}\mathbf{R}(n)\mathbf{w}(n) - \lambda(n)[\mathbf{w}(n)^{\mathrm{H}}\mathbf{w}(n) - 1] \qquad (7)$$

The negative gradient of the modified cost function of the Lagrange theorem can be calculated as

$$\mathbf{g}(n) = \frac{-\partial J_L}{\partial \mathbf{w}(n)} = 2[-\mathbf{R}(n)\mathbf{w}(n) + \lambda(n)\mathbf{w}(n)] \qquad (8)$$

This residual vector $\mathbf{g}(n)$ will be used for the gradient estimate in our adaptive minimisation method. Lagranges's multiplier can be respectively evaluated from the gradient of the cost function J_L as

$$\lambda(n) = \frac{\mathbf{w}(n)^{\mathrm{H}}\mathbf{R}(n)\mathbf{w}(n)}{\mathbf{w}(n)^{\mathrm{H}}\mathbf{w}(n)} \qquad (9)$$

The expression in Eq (9) is called Rayleigh quotient of vector $\mathbf{w}(n)$. The weight vector update step will be taken in the search direction $\mathbf{g}(n)$ as

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \alpha(n)\mathbf{g}(n) \qquad (10)$$

where $\alpha(n)$ is the optimal step size and $\Delta\mathbf{w}(n)=\alpha(n)\mathbf{g}(n)$ is the deviation imposed on the minimum eigenvector. The quantity $\alpha(n)$ can be determined in the following way: In the ideal case, when the eigenvalue spread σ_1/σ_M is close to 1, the cost function is smooth and the optimum solution could be achieved in one step. Therefore, the gradient of the Lagrange's cost function will be orthogonal to the previous search direction, i.e., to the residual direction $\mathbf{g}(n-1)$. By some algebraical manipulation and using the assumption of slowly changing signal scenario the optimal step size $\alpha(n)$ can be expressed as

$$\alpha(n) \approx \eta \frac{\mathbf{g}(n)^{\mathrm{H}}\mathbf{g}(n)}{\mathbf{g}(n)^{\mathrm{H}}[\mathbf{R}(n) + \lambda(n)\mathbf{I}]\mathbf{g}(n)} \qquad (11)$$

where the parameter η is an auxiliary step size introduced for increasing the performance. More search direction vectors could be evaluated for gaining the better performance and finally, we would end up in the CG like algorithm. However, quite often one update step and sometimes one residual vector computation for the parameter vector $\mathbf{w}(n)$ is enough because of not obtaining very much additional performance gain.

The normalisation of the weight vector is required in order to guarantee the constraint. Therefore, the normalisation factor $\mathbf{w}(n-1)^{\mathrm{H}}\mathbf{w}(n-1)$ in the denominator of Eq (9) will be of unity value and can be discarded.

$$\mathbf{w}(n) = \frac{\mathbf{w}(n)}{\sqrt{\mathbf{w}(n)^{\mathrm{H}}\mathbf{w}(n)}} \qquad (12)$$

The developed tracking method is the suboptimal one. Therefore, we have chosen to compare the performance of the developed method to the performance of high−resolution root−MUSIC spectrum estimation method. The weight vector estimate $\mathbf{w}(n)$ for a block of N collected samples with the application of ED is

$$w(n) = \mathrm{trace}_{\forall d}\{\sum_{m=K+1}^{M} \mathbf{u}_m \mathbf{u}_m^{\mathrm{H}}\} \qquad (13)$$

where $\mathrm{trace}_{\forall d}$ operation is targeted on all the dth subdiagonals (d=0, ..., $M-1$) and scalars from the trace operator are stacked on the column vector $\mathbf{w}(n)$ [4].

It should be noted that the estimation of only $M-1$ tracking parameters can be realised in the noise subspace. In addition, suitable recursive formulation is needed for discarding the *data association* problem which does not exist with the reference signal based methods. Therefore, the noise subspace approach makes it more difficult in the efficient and robust way to extract the tracking parameters. In the next section, we concentrate on developing the parameter tracker.

DOA unit

In this section, the DOA extraction method will be developed. As the weight vector $\mathbf{w}(n)$ of the tracking unit has converged it can be used for extracting the location parameters. The location parameters can be basically estimated by using two different approaches, the *companion matrix* based methods [5] or *zero−tracking* based methods [6]. Here, we concentrate especially on the zero−tracking method and develop a control strategy for controlling motion of the roots. This effectively discards the problems with the *spurious roots* and alleviates some numerical problems.

Companion matrix methods are based on the special structure of the non−symmetric companion matrix. Eigenvalues of the companion matrix are the same as the roots of the minimum eigenvector and therefore, it can be used for the parameter estimation [5]. Finding eigenvalues of the unsymmetric matrix is ill−conditioned in nature. Ordinary adaptive algorithms cannot be utilised in the computation unless introducing symmetric matrix. However, for solving the roots the *classical power method*, the *inverse power method* or the *Rayleigh Quotient Iteration* (RQI) method or their modifications have been generally utilised for the root extraction [7]. These methods experience difficulties in not always converging into the right root locations, especially, if initial parameter estimates are not close enough to true estimates.

By using the well−known *Pisarenko theorem* we can find the harmonic frequencies contained in the correlation matrix by solving the roots of the minimum eigenvector [7]. The respective Autoregressive (AR) power spectrum equation corresponding to the minimum eigenvector is

$$W(z) = w_1 + \ldots + w_M z^{-M+1} \tag{14}$$

This expression corresponds to the *minimum phase* filter although the root estimator is not necessarily constrained to have all the zeros inside the unit circle. The spatial user locations are to be extracted from the roots of this minimum eigenvector. For this aim the spectrum equation is first expressed in terms of zeros as

$$W(z) = w_1 \sum_{m=1}^{M} \left(1 - z_m z^{-1}\right) \tag{15}$$

where $z_k(n)=r_k\exp(-j\pi\sin(\theta_k(n)))$ are the root locations. The final location extraction function corresponding to the presented model can be expressed as

$$\Theta(n) = -\arcsin(\mathrm{imag}(\log(\mathrm{roots}(\mathbf{w}(n))))/\pi) \tag{16}$$

where the roots−operator extracts M roots from which K roots closest to the unit circle are chosen. The extracted parameters are stacked in the column vector $\theta(n)$.

In the zero−tracking method, small change of $\Delta\mathbf{w}(n)$ in the minimum eigenvector results in small changes in the root values $\Delta\mathbf{z}(n)$. The method is based on the derivative of $W(z)$ and can be expressed as

$$\frac{\partial W(z)}{\partial \mathbf{z}} = \frac{1}{w_1}\left[\frac{z_1^{-(M-1)}}{\prod_{m=1,m\neq 1}^{M}(z_m - z_1)} \cdots \frac{z_M^{0}}{\prod_{m=1,m\neq M}^{M}(z_m - z_M)}\right]^{\mathrm{T}} \tag{17}$$

The recursive zero−update formula for the kth user can expressed as

$$z_k(n) = z_k(n-1) + \Delta\mathbf{w}(n)\frac{\partial W(z)}{\partial \mathbf{z}_k(n)} \tag{18}$$

In Eq (16), we process the total M roots of which we have $M-K$ additional spurious roots. Sometimes, these spurious roots approach the unit circle and confuse us to choose some of them instead of the true roots. Therefore, we focus on developing a control strategy for roots. The method is based on the division of the unit semicircle on two different regions as determined by the limit parameter r ($r\leq 1$). This has been illustrated in Figure 2. In addition, the M users have been divided into two groups, the group of the K real users and the group of $M-K$ virtual users. These real users have been constrained to be in the annulus close to the unit circle. In the initialisation process, the real users will be placed on the unit semicircle at their initial angular positions whereas the $M-K$ virtual users are evenly distributed on the unit semicircle at the magnitude distance of r. The control procedure will be executed in the following manner: If kth virtual user cross the boundary the normalisation operation will be carried out on $z_k(n)$. Real users are usually wandering in the vicinity of the unit semicircle and if the boundary crossing happens the normalisation of the zeros will be executed.

It is well−known that the root−tracking problems are ill−conditioned. As a fact, the zero tracking methods have numerical problems because of a long product chain of subtractions in the denominator. This will evidently introduce some computational errors for the estimated parameters. The problem can be alleviated by developing the recursive estimation formula for the zero−tracking method [4]. For our root−tracking strategy the condition number of kth user and mth coefficient can be evaluated as

$$K_{(k,m)} = \frac{|w_m z_k^{m-1}|}{|W'(z_k)|} \leqslant \left[\prod_{m=1,m\neq k}^{M}(z_k - z_m)\right]^{-1} \tag{19}$$

which can be further simplified as $K\leq(1-r)^{-M+1}$. This is the worst−case situation and reflects an occasion when

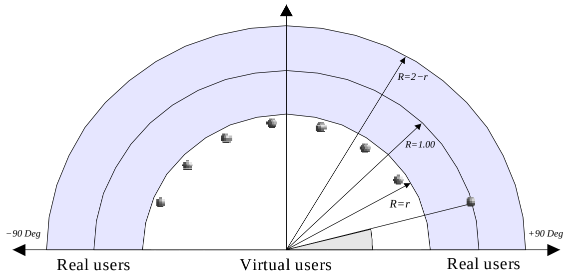

Figure 2 *Schematic indicating the permitted regions for the virtual and real users. Annulus of the width 2(1−r) around the unit semicircle is reserved for the real users whereas the virtual users are inside the semicircle of radius r*

all the virtual users are on the shortest distance from the desired real user. As an advantage, this kind of control strategy for zeros eliminates the spurious root problem. However, as a drawback we need a criterion for the model order estimation.

IV. NUMERICAL RESULTS

The ULA array with M=8 elements have been utilised for the simulation system setting. The mobile users have the moderate SNR of 20 dB. The numerical results have been computed for two closely located sources at the azimuthal locations of 10° and 0°. As the performance measure, the mean DOA error criterion has been utilised and the results have been averaged over 2000 independent realisations. For the performance comparison, the mobile users possess the 5° pointing errors in their initial azimuthal location estimates. The performance comparison has been accomplished in terms of convergence speed and misadjustment. In the case of the stationary signal scenario, the users are at their fixed locations whereas in the non−stationary signal scenario the users are moving with the constant angular velocity of 0.025 samples/deg.

Figure 3 shows the numerical results for the CG based signal subspace method, our noise subspace method and root−MUSIC method. Figure 3a) and b) compares the tracking performance in the stationary and non−stationary signal scenario, respectively. The performance of our noise subspace method gives higher mean DOA errors when in the low SNR. This is due to the fact that in the signal subspace approach the rooting process can be eliminated, thus, making the implementation of the DOA tracking unit more simple and robust. However, in the noise subspace the closely located sources can generally be separated with the lower mean DOA error than in the signal subspace. In general, the increase in the mean DOA error will take place in both estimators when the parameter value θ_k approach the endfire of array. This behaviour, as also suggested by the CRLB limit of Eq (2), will reduce the applicability of these kind of parameter estimators for the signal directions close to the broadside. The computational complexity for the tracking and DOA units in the proposed system is O(M^2) and O(KM) whereas for the CG based signal subspace system it is O(KM^2) and O(KM), respectively.

V. CONCLUSIONS

The system component design has been done for the user tracking application by dividing the tracking problem into three system blocks. The tracking problem has been cast into the form of the constrained minimisation problem and the adaptive step−by−step update scheme has been derived for the tracking unit. For the DOA unit the control scheme for the root tracking was developed. Two different system approaches were compared by the simulations showing similar tracking performance.

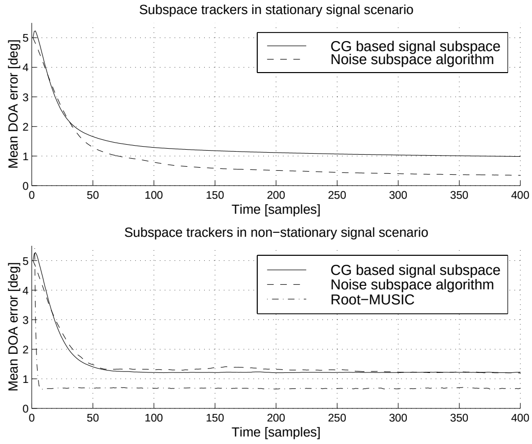

Figure 3 *Comparison of CG based signal subspace approach, our noise subspace approach and root−MUSIC method. a) Stationary signal scenario (λ_f=1), b) Non−stationary signal scenario (λ_f=0.8)*

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