# Tracking of Mobile Users in a Mobile Communications System Using Adaptive Convergence Parameter

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## ABSTRACT

In future telecommunication systems, one of the most important ways to increase system capacity is to use adaptive antennas. They enable reception and transmission using narrow beams which drastically reduce interference for other users. The problem with the conventional adaptive tracking algorithms is that the adaptation performance is dictated by the choice of the convergence parameter or step size. In this paper we employ a multi-target tracking method and modify its tracking performance by adaptive step size. The simulation results show that the proposed adaptive control of convergence parameter gives fast convergence and small misadjustment errors, which cannot be combined by using fixed step size.

## **1. INTRODUCTION**

Adaptive antennas provide more system capacity by reducing co-channel interference. The target tracking problem arises, e.g., in mobile applications where for each moving user appropriate radio beams must be formed. Target tracking enables continuous locating of mobile terminals as they move around. There are numerous methods to locate and track multiple moving targets.

The mobile phone can be located, e.g., by using the highresolution MUltiple SIgnal Classification (MUSIC) algorithm. However, the computational complexity is prohibitive if the location estimate is to be updated at every iteration round. Furthermore, the problem with these conventional target tracking methods is the data association problem, i.e., finding out which location estimates belong to which mobile terminals. As a result, the beamformer may start to follow a wrong mobile phone when they cross each other. The Eigenvalue Decomposition (ED) or the Singular Value Decomposition (SVD) of the sample correlation matrix calculated from the antenna array output can be associated to the location information of moving targets. These methods are not generally suitable for real-time applications due to large computational demands unless efficient adaptive algorithms are exploited for extracting and tracking of eigenvalues [1].

A different viewpoint has been taken by Sword *et al.* in their tracking system. First, it makes use of the MUSIC algorithm to estimate the number of sources, target angles, and associated user and noise powers from the sample

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correlation matrix [2]. Then, at each iteration round it makes use of the direction and signal correlation matrices for determining new tracking estimates. Sword's algorithm has also drawbacks because it assumes a constant signal correlation matrix for the whole tracking period which is not an appropriate assumption for rapidly and widely varying signal dynamics.

The target tracking can also be implemented in other ways. With the aid of time delays of impinged signals, the Maximum Likelihood (ML) method can estimate the angle of arrivals of moving targets. Known reference signals received from mobiles can also be used to track the position of the users requiring however an explicit reference signal generation method [3]. The state-space approach exploits a dynamical model for traveling targets and tracking of moving sources is done by updating state information via e.g. Kalman filtering in each tracking interval [4]. The basic problem with all these methods is the computational complexity.

The multi-target tracking method proposed in [5] is attractive because it does not require prior information for example about the users' associated power levels. Furthermore, it can track multiple moving sources efficiently by using a conventional beamforming method without any greater losses in performance in the case of pointing errors. In this paper we utilize this tracking method by introducing a control strategy for the adaptive step size.

The paper is organized as follows: in Section 2, the simple signal model is developed for communication signals. Section 3 presents the tracking model that is used in simulations. Section 4 introduces a new model to improve tracking performance. Section 5 shows the simulation results for Uniform Linear Array (ULA). Finally, in Section 6, conclusions about the performance of the tracking system are drawn.

## 2. SIGNAL MODEL

The signal model for the antenna array used in the simulations will be developed in this section. The antenna configuration is illustrated in Figure 1, where the antenna array receiver with M elements at the base station and N surrounding mobile users are shown.



Figure 1 M-element antenna array surrounded by N moving mobile terminals.

The signal impinged on the antenna array can be expressed as

$$\mathbf{x}(t) = \sum_{i=1}^{N} \mathbf{a}(\theta_i) s_i(t) + \mathbf{n}(t)$$
(1)

where the quantities  $\mathbf{n}(t)$  and  $\mathbf{x}(t)$  are the noise and antenna observation column vectors of size M respectively. The uncorrelated communication signals  $s_i(t)$  are modeled as a zero-mean Gaussian distributed process with variance  $\gamma_i^2$ . Obviously, this is a crude model for a typical modulated signal but sufficient for our purposes. The noise process  $\mathbf{n}(t)$ with variance  $\sigma^2$  is also drawn from a Gaussian distribution. Futhermore, the noise samples are assumed to be independent of the signal samples. The Signal-to-Noise Ratio (SNR) for *i*:th user in this model is defined as  $20\log(\gamma_i/\sigma)$ .

The quantity  $\mathbf{a}(\theta_i)$  is the array response vector for each azimuth direction  $\theta_i$ . In the case of ULA it can be expressed as

$$\mathbf{a}(\theta_i) = \frac{1}{\sqrt{M}} \Big[ 1 \exp(j\psi_i) \dots \exp(j(M-1)\psi_i) \Big]^{\mathrm{T}}$$
(2)

The quantity  $\psi_i = (2\pi d \sin(\theta_i) f/c)$  determined by the array factor includes the communication frequency f and the speed of light c. All the steering vectors are collected into matrix A which can be expressed as

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}(\theta_1) \dots \mathbf{a}(\theta_N) \end{bmatrix}$$
(3)

Eq. (1) can now be expressed in a more compact form as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{4}$$

The true correlation matrix for the antenna array can be expressed as

$$\mathbf{R} = \mathbf{E} \Big[ \mathbf{x}(t) \mathbf{x}(t)^{\mathrm{H}} \Big] = \mathbf{A} \mathbf{S} \mathbf{A}^{\mathrm{H}} + \sigma^{2} \mathbf{I}$$
<sup>(5)</sup>

where  $S = E[s(t)s(t)^{H}]$  is the signal correlation matrix. The sample correlation matrix estimated by averaging from a set of L sampled vectors can be expressed as

$$\hat{\mathbf{R}} = \frac{1}{L} \sum_{i=1}^{L} \mathbf{x}(t_i) \mathbf{x}(t_i)^{\mathrm{H}}$$
(6)

#### **3. TRACKING MODEL**

In this section, a target tracking model is developed which solves the data association problem for location estimates. The signals impinged on the antenna array are digitized and sent to the tracking unit (see Figure 2).



Figure 2 Illustration of the tracking system.

It is assumed that the process variance does not change very much during the period of collecting L data samples so that the new tracking angles can be determined without any greater performance loss. Communication signals are estimated in the beamforming unit at time instant t=kTaccording to

$$\mathbf{Y}_k = \mathbf{W}_k^{\mathrm{H}} \mathbf{X}_k \tag{7}$$

where  $\mathbf{W}_k$  is a weight matrix of size  $M \times N$  and  $\mathbf{X}_k$  is an observation matrix of size  $M \times L$ . The beamforming weights can be determined according to different criteria. The block algorithms are computationally expensive but can usually give good nulling properties [7]. The reference signal based methods are neither computationally demanding nor do they need explicit location information [3]. The conventional block beamforming method which is expressed in Eq. (8) was chosen for simplicity and for convenience. It should be noted that the matrix inversion is computationally a heavy operation and thus adaptive methods are generally preferable. In the first iteration round the  $A_0$  can be initialized by using MUSIC algorithm.

$$\mathbf{W}_{k} = \left(\mathbf{A}_{k}^{\mathrm{H}} \mathbf{A}_{k}\right)^{-1} \mathbf{A}_{k}^{\mathrm{H}}$$
(8)

From the beamforming unit, the estimated signals along with the input data samples are sent to the tracking unit for the computation of the new steering vectors. The interference and noise can be reduced from the array output by minimizing all the components orthogonal to the desired signal component. The resulting cost function can be expressed as

$$\mathbf{J} = \mathbf{E} \left[ \left\| (\mathbf{I} - \mathbf{A}_k \mathbf{A}_k^{\mathrm{H}}) \mathbf{X}_k \right\|^2 \right]$$
(9)

Gradient methods like steepest descent can be applied for Eq. (9) [8]. Thus, the steering matrix can be updated by means of a resulting LMS-like adaptation procedure which can be expressed as

$$\mathbf{A}_{k+1} = \mathbf{A}_k + \mu (\mathbf{X}_k - \mathbf{A}_k \mathbf{Y}_k) \mathbf{Y}_k^{\mathrm{H}}$$
(10)

where  $\mu$  is a suitably chosen positive convergence parameter.

The estimated steering matrix  $\mathbf{A}_{k+1}$  is sent to the model fitting unit where new tracking angles are determined according to mobile movements. In the model fitting unit, the estimated steering matrix is projected back to the array manifold. As a measure of distance, log-metric can be used, which results in the linear regression over the positions of the antenna elements. This is needed because nonidealities results deviations in the elements of steering vector from the real array manifold.

From the model fitting unit, the new tracking angle estimates  $\boldsymbol{\theta}_{k+1}$  are sent to the dynamical model unit which takes care of the quality of the location estimates. As mobile users in the real environment may cross each other, the rank of the steering matrix  $\mathbf{A}_{k+1}$  may collapse due to linear dependence. Therefore, the tracking estimates become worse unless some precautions are taken. This problem can be avoided, e.g., by using an appropriate dynamical model which includes velocity and possibly also acceleration estimates [4].

#### 4. PROPOSED MODEL

The choice of the convergence parameter may be problematic by causing noise due to adaptation called misadjustment [6]. A small fixed parameter value may lead to a slow convergence rate. On the other hand, large values affect the accuracy of tracking estimates.

By observing that the adaptation parameter  $\mu$  in Eq. (10) is basically the same for each antenna element. The proposed method relies on  $M \times N$  matrix  $\mathbf{M}_{\mu}$  to control individually the convergence parameter of each source and is expressed as

$$\mathbf{M}_{\mu} = \begin{bmatrix} \mu_1 \mathbf{u} \dots \mu_N \mathbf{u} \end{bmatrix}$$
(11)

where  $\mathbf{u} = [1 \ 1 \ ... \ 1]^{\mathrm{T}}$  is *M*-length vector with unity elements. However, in the case of a fading channel, the different antenna elements of matrix  $\mathbf{M}_{\mu}$  would have tendency to follow the envelope of the faded signals impinged on the antenna elements. By incorporating  $\mathbf{M}_{\mu}$  into the steering matrix update Eq. (10) the resulting update rule can be expressed as

$$\mathbf{A}_{k+1} = \mathbf{A}_k + \mathbf{M}_{\mu} \otimes \left[ (\mathbf{X}_k - \mathbf{A}_k \mathbf{Y}_k) \mathbf{Y}_k^{\mathrm{H}} \right]$$
(12)

where  $\otimes$  is a Hadamard (elementwise) product. Basically, the method checks variations in the sign of the error function and accordingly adjusts step sizes individually depending on the occurrence of zero crossings in the error function [9]. This tracking error quantity in Figure 2 is denoted as a column vector  $\mathbf{e}_k$  of size *N*. Intuitively, the operation can be written as follows: if the same sign occurs frequently in the error function  $\mathbf{e}_k$ , the step size is too small and the convergence is too slow. On the other hand, if the sign alternates at every iteration, the step size is too large and should be reduced. The adaptive step size control unit co-operating together with the tracking unit performs the following operations for each blocks of received samples:

- 1. If *m* last error samples for the *n*:th source  $e_k(n)$  have the same sign, multiply  $\mu_n$  by a factor  $\alpha$ .
- 2. If *m* last error samples for the *n*:th source  $e_k(n)$  have alternating signs, divide  $\mu_n$  by a factor  $\alpha$ .
- 3. Process the next block (k=k+1).
- 4. Compute new error estimate  $e_{k+1}$ .
- 5. Return to 1.

The control window parameter *m* is introduced to determine the allowed rate of change in the convergence parameter. The factor  $\alpha$  determines the proportional permitted increase or decrease in the step size. Better tracking properties can be expected with the adaptive step size method in a nonstationary signal scenario.

#### 5. SIMULATION RESULTS

The antenna array consists of M=8 linearly arranged elements with  $\lambda/2$  spacing. The simulation settings makes the following assumptions: two Gaussian distributed sources with equal variance with SNR=10 dB are at the starting azimuth locations of  $10^{\circ}$  and  $40^{\circ}$ . The initial pointing errors of  $5^{\circ}$  are introduced to the location estimates of both sources so that the convergence behavior can be inspected. For each simulation setting, 2000 independent simulations have been averaged.



**Figure 3** Illustration of tracking of stationary sources with adaptive convergence parameter (m=3). One realization.



**Figure 4** DOA error as a function of time in the case of stationary sources. a) Fixed convergence parameter. b) Adaptive convergence parameter.

The first simulation setting is for the case of stationary sources. Figure 3 illustrates the tracking curves in the stationary signal scenario. Figure 4a) shows Direction Of Arrival (DOA) error as a function of time with the fixed convergence parameter. It can be seen that a smaller step size yields smaller estimation errors whereas a big step size results in faster convergence. Figure 4b) shows DOA error as a function of time with adaptive step size method with three different kinds of window parameters (m=2, 3 and 4). It starts its adaptation from the initial convergence parameter  $\mu=1$ . This kind of parameter selection ensured fast convergence for a wide range of signal dynamics. The simulation results revealed that the smaller window parameter selection gave faster convergence with small DOA estimation errors. A bigger window parameter in turn does not respond so fast to changes in the error function. The difference between the non-adaptive and adaptive control step parameter was that faster convergence rate and much smaller DOA estimation errors could be achieved. Figure 5 shows the evolution of the step parameter. It converges to zero as it should do in the case of stationary sources. It also ensures that the adaptive algorithm stabilizes.



Figure 5 Behavior of step size for stationary sources.



**Figure 6** Illustration of tracking of moving sources with adaptive convergence parameter (m=3). One realization.

The second simulation setting includes the inspection of the tracking performance in the case of moving targets. The target tracking starts from the initial azimuth locations of  $10^{\circ}$  and  $40^{\circ}$  and follows the solid lines as shown in Figure 6. Otherwise, the simulation settings were the same as for the previous one. The simulation results for DOA estimation errors have been plotted in Figure 7a) for fixed step parameter and for adaptive step parameter in Figure 7b). Too small or too large fixed value settings give larger DOA estimation errors. The fixed parameter value setting cannot give good tracking properties due to the movement of the sources. The adaptive convergence parameter gives the best results in terms of DOA estimation errors especially for sources moving in curvaceous trajectories. Because the users are moving at a constant speed, the adaptation parameter decays and finally converge to some constant level after slight oscillation. Figure 8 shows this behavior for the convergence parameter with three different kinds of control window parameters. The smaller control window parameter stabilizes the step size faster than the bigger ones as was also the situation in the stationary signal scenario.

## 6. CONCLUSIONS

This paper reports a study of the performance of the adaptive antenna system with the tracking system. The model was modified to include the control window parameter for adaptive step size. In the case of stationary sources, the adaptive step size method could achieve slightly faster convergence and smaller DOA errors. However, with the careful selection of fixed convergence parameter similar performance could also be achieved. In the case of moving sources the adaptive step size method can easily outperform any fixed parameter value setting, especially when mobile users are traveling curvaceous trajectories.

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**Figure 7** DOA error as a function of time in the case of moving sources. a) Fixed convergence parameter. b) Adaptive convergence parameter.



Figure 8 Behavior of step size for moving sources.