Iterative Multiuser Receiver Utilizing Soft Decoding Information

Kimmo Kettunen and Timo Laakso
Helsinki University of Technology
Laboratory of Telecommunications Technology
email: Kimmo.Kettunen@hut.fi, Timo.Laakso@hut.fi

Abstract—**In this paper we propose a novel receiver structure that utilizes the soft information provided by the channel decoder in the multiuser detection. This receiver structure is based on a single-user version of the uncoded *maximum a priori* criterion. We derive the optimal multiuser likelihood calculation algorithm for this structure. In addition, suboptimal algorithms are proposed that are based on interference cancellation. Numerical simulations are reported which indicate that these suboptimal algorithms have a close-to-optimal coded BER performance even in the presence of high-power interferers.**

I. Introduction

The past research on code division multiple access (CDMA) multiuser detection has mainly concentrated on the uncoded case, that is, the channel coding is assumed to be totally independent from the multiuser detection and is thus ignored in the analysis and algorithm design [1]. Recently, there has been a growing interest for an integrated approach, where the channel coding is taken into account in the design and analysis of the multiuser receivers. An optimal detector/decoder for a convolutionally coded CDMA system was derived in [2] by Giallorenzi and Wilson, who also proposed several suboptimum multiuser receiver structures in [3]. Different suboptimum approaches have been studied in [4-6,10,12,13].

On the other hand, in the area of coding theory iterative decoding has become a popular research topic, mainly because of the Turbo codes [7]. A number of approaches combining iterative decoding and multiuser detection have also been studied. In [9], an iterative structure consisting of a combining algorithm followed by parallel MAP-decoders (one for each user) was investigated. In [8], an iterative multiuser receiver with channel decoding was derived by using the uncoded *maximum a posteriori* (MAP) criterion jointly for all users. In these receivers, the channel decoder output can be used to further improve the receiver performance. This is done in an iterative fashion similar to the iterative decoding algorithms used for concatenated codes [11].

In this paper we propose a novel receiver structure that utilizes the soft information provided by the channel decoder in multiuser detection. In this structure, the uncoded MAP criterion is used separately for each user for multiuser likelihood calculations instead of the joint uncoded MAP criterion that was used in [8]. We derive the optimal *multiuser likelihood calculation* (MULC) algorithm for this structure. This algorithm has a rather high complexity and thus several suboptimal algorithms are proposed that are based on interference cancellation. Numerical simulations show that even these suboptimal algorithms have a close-to-optimal coded BER performance, especially when the variance estimation of the data is performed individually for each iteration round.

II. System model

The CDMA system modeled in this paper is the uplink chip and symbol-synchronous direct sequence DS-CDMA communication system with K users. We assume BPSK modulation. The model uses convolutional channel coding to improve the BER performance of the system. The channel is modeled as a time-invariant single-path channel where Gaussian noise with zero mean and variance σ^2 is added. The block diagram of the system is shown in Figure 1.

In a synchronous CDMA system, the matched filter output at time i can be expressed as

$$\mathbf{y}_i = \mathbf{R}\mathbf{A}\mathbf{x}_i + \mathbf{n}_i \tag{1}$$

where $\mathbf{x}_i=(x_i^{(1)},\dots,x_i^{(k)})^T$ is the coded data vector containing the transmitted data symbols of every user and $\mathbf{n}_i$ is the Gaussian noise. Furthermore, $\mathbf{R}$ is the correlation matrix and $\mathbf{A}$ is the channel matrix, that is

$$\mathbf{R} = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1K} \\ \rho_{21} & 1 & \cdots & \rho_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{K1} & \rho_{K2} & \cdots & 1 \end{pmatrix} \tag{2}$$

$$\mathbf{A} = \mathrm{diag}\left(a_1,\dots,a_K\right), \tag{3}$$

where ρ_{ij} is the cross-correlation between users i and j. The channel matrix is diagonal since we assume single-path propagation.

This work is part of a research project of the Institute of Radio Communication (IRC) funded by Technology Development Center (TEKES), NOKIA Research Center, Telecom Finland and the Helsinki Telephone Company.

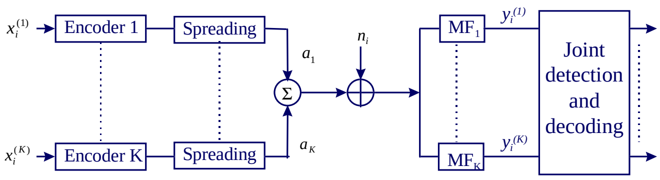

Figure 1: Block diagram of the communication system

III. ITERATIVE IC-TYPE RECEIVER STRUCTURE

The decoder structure used in the iterative decoding algorithms of (serially) concatenated codes provides the principal model for our receiver structure. Naturally, there is no need for the inner constituent decoder in a system using non-concatenated channel coding. However, substituting the inner decoder with a *multiuser likelihood calculation* (MULC) unit allows the utilization of the feedback from the channel decoder in the likelihood calculations. This approach is similar to the receiver structure considered in [8].

In [8], the multiuser likelihood calculations are based on the joint uncoded MAP criterion

$$\tilde{\mathbf{x}}_i = \arg\max_{\mathbf{x}_i} p(\mathbf{x}_i | \mathbf{y}_i) \tag{4}$$

Thus the soft information input provided for each channel decoder contains the information about the whole matched filter sample vector $\mathbf{y}_i$. In traditional interference cancellation algorithms for uncoded data, the symbol decision is based only on the code matched filter output for user k and the tentative symbol decisions for the interfering users. Since our receiver structure is designed for MULC algorithms that are based on these interference cancellation algorithms, we adopt the corresponding approach in multiuser likelihood calculation. It means that a single-user version of the uncoded MAP criteria

$$\tilde{x}_i^{(k)} = \arg\max_{x_i^{(k)}} p(x_i^{(k)} | y_i^{(k)}) \tag{5}$$

is used for each user separately. For each user k, the solution of this optimization problem requires, in addition to the knowledge of the noise variance σ^2, only the knowledge of the user k matched filter output $y_i^{(k)}$ and the estimates of the symbol probability distributions for every interfering user. These estimates of the symbol probability distributions are presented as *log-likelihood ratios* (LLR), where the LLR for symbol s is defined as

$$L(s) = \log \frac{\mathrm{P}(s=+1)}{\mathrm{P}(s=-1)} \tag{6}$$

where $\mathrm{P}(s=\pm1)$ is the probability for the symbol s to be ±1. Where necessary, we extend this notation also to the case with conditional probabilities.

The proposed receiver has an iterative structure with the mth stage of the receiver for user k is shown in Figure 2. We use the term *iterative IC-type receiver* to emphasize the fact that the receiver has a disjoint user-by-user structure and the channel symbol information is shared between the single-user receivers and used for interference cancellation.

During the mth iteration, the MULC unit calculates the symbol likelihoods based on the matched filter output $y_i^{(k)}$ of the respective user and the estimates $I_i^{(k)}(m-1)$ of the LLRs for the channel symbols transmitted by the interfering users. These estimates are calculated during the previous iteration round by adding the output of the outer unit with the earlier likelihood estimate produced by the MULC unit. The outer unit is a soft-input-soft-output (SISO) decoder, a unit that is commonly used as a constituent decoder when decoding concatenated codes, and it produces its output by adjusting the symbol likelihoods based on its knowledge of the channel code trellis. The SISO unit also produces the final data symbol decisions after the last iteration round.

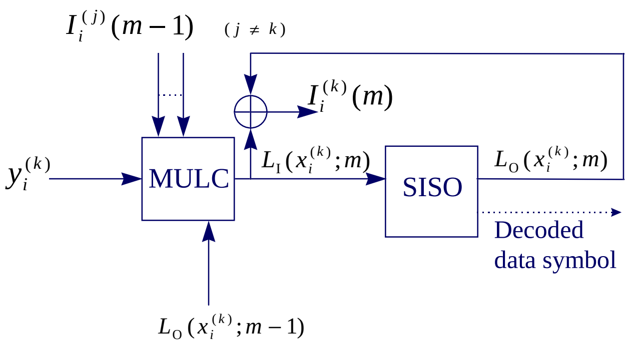

Figure 2: The *m*th stage of the iterative IC-type receiver structure utilizing channel decoding

There are several possibilities for the actual algorithm to be used in the SISO. We have selected the well-known MAP algorithm [11] that uses two-way recursions to calculate both the information symbol and channel symbol probability distributions based on *a priori* channel and information symbol probability distributions and the knowledge of the code trellis. The calculation is based on minimizing the information symbol error probability. We use a variant where all the input and output distributions are expressed in the form of log-likelihood ratios. In our case the information symbols are *a priori* equiprobable and can thus be ignored in the SISO unit. Furthermore, we use the extrinsic channel bit probabilities instead of the channel symbol probabilities as the output of the SISO unit. These are calculated by using the formula derived in [11]. Thus if $\mathrm{P_I}(x_i^{(k)})$ (resp. $\mathrm{P_O}(x_i^{(k)})$) is the *a priori* (resp. extrinsic *a posteriori*) probability distribution for channel bits then we can form the corresponding log-likelihood ratios as:

$$L_\mathrm{I}(x_i^{(k)}) = \log \frac{\mathrm{P_I}(x_i^{(k)} = +1)}{\mathrm{P_I}(x_i^{(k)} = -1)} \tag{7}$$

$$L_O(x_i^{(k)}) = \log\frac{P_O(x_i^{(k)}=+1)}{P_O(x_i^{(k)}=-1)} \tag{8}$$

As was mentioned above, the MULC unit uses for each user k the code matched filter output y_k and the channel bit LLRs I_k of the interfering users for likelihood calculation. In the next section we present alternative algorithms for the MULC unit.

IV. Multiuser likelihood calculations

In this section we derive alternative likelihood calculation algorithms for the receiver structure given in the previous section. Although our primary focus is on algorithms that use some form of interference cancellation procedure for these calculations, we also derive an algorithm that is optimal solution with respect to the uncoded separated MAP criteria. This provides a useful reference point for evaluating suboptimal algorithms and gives an upper limit for the performance of this kind of a receiver structure.

A. Optimal Likelihood Calculation

The optimal likelihood calculation algorithm is derived from the separated uncoded MAP criteria (5). Thus the algorithm requires the knowledge of the noise variance σ^2, the matched filter output sample $y_i^{(k)}$ and the probability distributions of the channel symbols $x_i^{(j)}$ for $j\neq k$. By using the well-known Bayes formula, the (*a posteriori*) LLR can be presented as

$$L(x_i^{(k)}|y_i^{(k)}) = L(y_i^{(k)}|x_i^{(k)}) + L(x_i^{(k)}) \tag{9}$$

$$= \log\left(\frac{P(y_i^{(k)}|x_i^{(k)}=+1)}{P(y_i^{(k)}|x_i^{(k)}=-1)}\right) + \log\left(\frac{P(x_i^{(k)}=+1)}{P(x_i^{(k)}=-1)}\right) \tag{10}$$

where the second term contains the *a priori* information about the transmitted information symbol and the first term is the LLR for the reception of $y_i^{(k)}$ on the condition that $x_i^{(k)}$ was transmitted. In an AWGN channel the first term is just a multiplication of the matched filter output sample $y_i^{(k)}$ with the *channel reliability coefficient* $2a_k/\sigma^2$. A simple calculation shows that the presence of multiple access interference (MAI) adds an extra term to the AWGN case:

$$L(y_i^{(k)}|x_i^{(k)}) = \frac{2a_k y_i^{(k)}}{\sigma^2} + L_{MAI,i}^{(k)} \tag{11}$$

where

$$L_{MAI,i}^{(k)} = \log\left(\frac{\sum_{\mathbf{e}} P_i(\mathbf{e})\, e^{1/(2\sigma^2)\left(2(y_k - a_k)\mu_k(\mathbf{e}) - \mu_k(\mathbf{e})^2\right)}}{\sum_{\mathbf{e}} P_i(\mathbf{e})\, e^{1/(2\sigma^2)\left(2(y_k + a_k)\mu_k(\mathbf{e}) - \mu_k(\mathbf{e})^2\right)}}\right) \tag{12}$$

The summations are over all such $\mathbf{e}$ that $e_k = 0$ and $e_j = \pm 1$ for $j \neq k$ and $P_i(\mathbf{e})$ is the probability of the transmission of symbol e_j by user j for all users $j\neq k$ at time i. The function $\mu_k(\mathbf{e})$ is defined as

$$\mu_k(\mathbf{e}) = \mathbf{P}_k \mathbf{RAe} \tag{13}$$

where the $\mathbf{P}_k$ is the projection operator that returns the kth element of a vector. Thus $\mu_k(\mathbf{e})$ is the total interference signal in the case the symbol transmitted by user i was e_i.

For any iteration round m, the $L_{MAI,i}^{(k)}$ term can be obtained by using the estimates $I_i^{(k)}(m\text{-}1)$. Denoting the estimate by $L_{MAI,i}^{(k)}(m)$, the following likelihood calculation step at stage m can be derived:

$$L_I(x_i^{(k)};m) = \frac{2a_k y_i^{(k)}}{\sigma^2} + L_{MAI,i}^{(k)}(m) + L_O(x^{(k)}_i;m-1) \tag{14}$$

B. Suboptimal Approaches

In this section we consider the likelihood calculation algorithms that use an interference cancellation step during the calculation. The simplest such algorithm (the hard decision cancellation) can in fact be obtained as an approximation of the optimal calculation. Consider the optimal algorithm which includes the calculation of the logarithm of an exponential sum (12). Since the sum is taken over all possible bit combinations sent by all the users, the number of terms in the sum is 2^K, where K is the number of users. This kind of log-exponential sums are often encountered in MAP algorithms and they can be approximated by the maximum of the exponents [11]. If one assumes that the bit estimates are highly reliable for all users then for each i there exists $\hat{\mathbf{e}}_\mathbf{i}$ such that $P(\hat{\mathbf{e}}_\mathbf{i}) \approx 1$ and $P(\mathbf{e}) = 0$ for all other $\mathbf{e}$ and we can approximate the $L_{MAI,i}^{(k)}$ term as

$$L_{MAI,i}^{(k)} \approx -\frac{2a_k}{\sigma^2}\mu_k(\hat{\mathbf{e}}_\mathbf{i}) \tag{15}$$

This gives

$$L(y_i^{(k)}|x_i^{(k)}) \approx \frac{2a_k y_i^{(k)}}{\sigma^2} - \frac{2a_k \mu_k(\hat{\mathbf{e}}_\mathbf{i})}{\sigma^2} = \frac{2a_k}{\sigma^2}\left(y_i^{(k)} - \mu_k(\hat{\mathbf{e}}_\mathbf{i})\right) \tag{16}$$

This approximation can thus be obtained by a hard decision interference cancellation followed by multiplication with the channel coefficient.

During the iteration round m, the vector $\hat{\mathbf{e}}_\mathbf{i}$ can be estimated by sign($\mathbf{I}_i$), defined as a vector with the jth element sign($I_i^{(j)}(m\text{-}1)$) for $j\neq k$ and 0 for the kth element . With this notation and when using the approximation (16), the likelihood calculation step becomes:

$$L_I(x^{(k)}_i;m) = \frac{2a_k(y_i^{(k)} - \mu_k(\text{sign}(\mathbf{I}_i)))}{\sigma^2} + L_O(x^{(k)}_i;m-1) \tag{17}$$

The simulations show (see Section VI) that the performance of this hard decision cancellation algorithm is rather poor when the base station received different users with different powers. One possible reason for this can be the fact that the algorithm actually consists of two steps: the interference cancellation step and the multiplication with the channel reliability coefficient. We can view the interference cancellation step as a method to transform a channel with MAI into a pure AWGN channel, after which the likelihood calculation for the transformed channel is done in the second step. The transformed channel is naturally no longer an AWGN channel and there will be a certain approximation error introduced in the likelihood calculations.

However, an even more serious performance degrading is introduced because the variance of the transformed channel is not the same as that of the original added noise. Using the original variance σ^2 in the channel reliability coefficient introduces an error to (17) that has a severe degrading effect on the performance of the algorithm especially when the amount of interference to be canceled is large, for instance when a high power interferer is present. This will be illustrated in Section VI. We can remove this source of error with a simple modification to the algorithm. All that needs to be done is to estimate the variance separately after each cancellation procedure and use this estimate in the channel reliability coefficient. As a result, we get the following two-step variant of the likelihood calculation at stage *m*. First, we perform the interference calculation step to obtain the corrected matched filter output samples

$$\hat{y}_i^{(k)} = y_i^{(k)} - \mu_k(\mathrm{sign}(\mathbf{I}_i)) \quad (18)$$

and calculate a new variance estimate $\hat{\sigma}^2$ based on the modified sample set $\hat{y}_i^{(k)}$. Then calculate the likelihoods using formula

$$L_{\mathrm{I}}(x_i^{(k)};m) = \frac{2a_k(\hat{y}_i^{(k)})}{\hat{\sigma}^2} + L_{\mathrm{O}}(x_i^{(k)};m-1) \quad (19)$$

As will be seen in Section VI, this improves the performance dramatically.

The hard decision interference calculation step can be substituted with some other interference calculation method. In this paper we only consider one variant of the basic interference cancellation procedure, called here the "soft" interference cancellation. This variant is obtained by using the mean $E(\mu_k(\mathbf{e}))$ instead of $\mu_k(\hat{\mathbf{e}})$ that was used above. This results in an algorithm where the interference calculation step (18) is replaced by:

$$\hat{y}_i^{(k)} = y_i^{(k)} - \mu_k(\tanh(\mathbf{I}_i/2)) \quad (20)$$

VI. Numerical results

In this section we will report some numerical results obtained through simulations. The channel coding used here is a rate 1/2 convolutional code with the generator (7,5). The SNR is 2 dB and the number of simultaneous users is K=4. The correlation coefficient ρ=0.3 is used for each user pair.

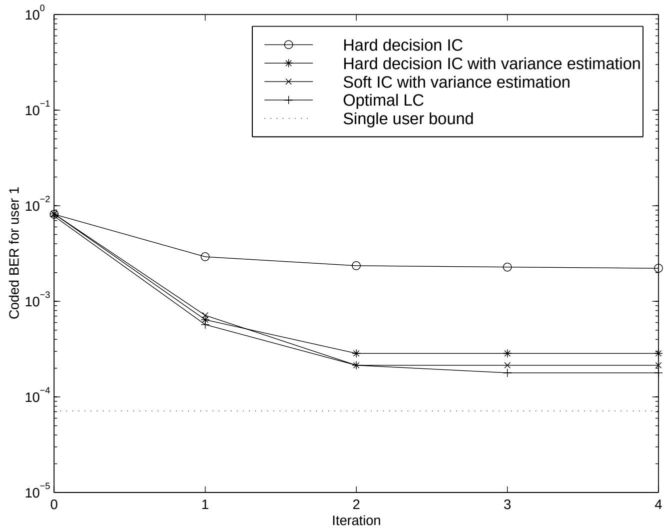

Figure 3: Receiver performance with equal received powers

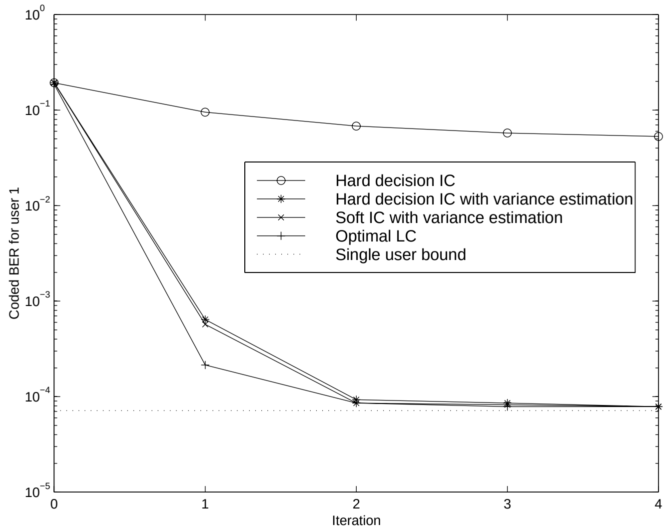

Figure 4: BER performance for a user with a decreased (-3dB) power level

Figure 3 shows the case, where the received power levels of all users are the same. All tested MULC algorithms, except the one using (17), give approximately the same BER performance. The performance decrease of the "hard decision IC" algorithm is at least partially due to heavy interference cancellation caused by relatively large correlation coefficients.

Figure 4 gives the coded BER for user 1 after each iteration step in a situation, where user 1 is received with a power level 3 dB lower than that of the other users that are received with equal powers. One can see that the optimal likelihood calculation obtains a near single user performance after few iteration rounds. The performance is in fact better than in the equal power case, mainly because the increased power levels of the interfering users also increase their respective SNRs and thus provide more reliable symbol estimates to be used in the multiuser likelihood calculations. It is also worth noting that while the hard decision IC without variance re-estimation after each iteration round performs rather badly, the performance improves dramatically when variance is estimated from the sample data after each iteration round. The performance improvement achieved by using the "soft" IC instead of hard decision IC is almost none since both soft and hard variants of the algorithm have near-optimal performance.

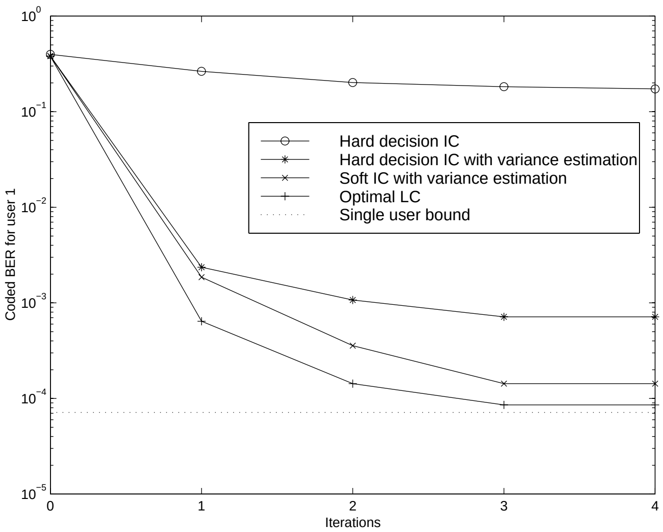

Figure 5: BER performance with three interfering users (with power levels 0dB, 3dB and 6dB)

Figure 5 shows the performance of the simulated algorithms in a situation, where there are three interfering users: one received with equal power level, one receiver with a power level 3dB higher and one with a power level 6dB higher. This situation is more demanding for the efficient interference cancellation than the previous case, since now even the tentative symbol decisions of two other users are distorted by one high power level user. This is reflected as decreased performance of the IC algorithms. The soft IC algorithm has somewhat better performance than the one using hard tentative decisions, which is natural in the presence of a high-power interfering user.

VII. Conclusions

We have studied an iterative receiver structure that utilizes the soft information provided by the channel decoder in the multiuser detection. The simulations presented in this paper support the conclusion made in [8] that such an iterative receiver structure combined with channel decoding gives a great increase of performance. The results show that the performance gain is substantial even with a simple convolutional channel coding. Furthermore, traditional interference cancellation techniques can be applied to produce algorithms that have a close to optimal coded BER performance, even with unequal received powers, when the variance estimation of the data is performed individually for each iteration round.

Acknowledgment

The authors want to thank Dr. J. Lilleberg and Mr. A. Hottinen for valuable discussions.

References

[1] A. Duel-Hallen, J. Holtzman, and Z. Zvonar, "Multiuser detection for CDMA systems", *IEEE Pers. Comm.*, vol. 2, pp. 46-58, April 1995.

[2] T. R. Giallorenzi, and S. G. Wilson, "Multiuser ML sequence estimator for convolutionally coded asynchronous DS-CDMA systems", *IEEE Trans. Comm.*, vol. 44, pp. 997-1008, August 1996.

[3] T. R. Giallorenzi, and S. G. Wilson, "Suboptimum multiuser receivers for convolutionally coded asynchronous DS-CDMA systems", *IEEE Trans. Comm.*, vol. 44, pp. 1183-1196, September 1996.

[4] A. Hafeez, and W. E. Stark, "Soft-output multiuser estimation for asynchronous CDMA channels", *Proc. IEEE Vehicular Technology Conference*, pp. 465-469, May, 1997.

[5] M.-R. Koohrangpour, and A. Svensson, "Joint interference cancellation and Viterbi decoding in DS-CDMA, *Proc. of PIMRC'97*, pp. 1161-1165, September 1997.

[6] R. Herzog, "Interference cancellation for a high data rate user in coded CDMA systems", *Proc. of ICC'98*, June 1998.

[7] R. Lucas, M. Bossert, and M. Breitbach, "On iterative soft-decision decoding of linear binary block codes and product codes", *IEEE JSAC*, vol.16, pp. 276-296, February 1998.

[8] M. C. Reed, P. D. Alexander, J. A. Asenstorfer, and C. B. Schlegel, "Near single user performance using iterative multi-user detection for CDMA with turbo-code decoders," *Proc. of PIMRC'97*, pp. 740-744, September1997.

[9] M. L. Moher, "Turbo-based multiuser detection", *Proc. of ISIT 1997*, p. 195, June. 1997.

[10] M. Brandt-Pearce, and M.-H. Yang, "Soft-decision multiuser detector for coded CDMA systems", *Proc. of ICC'98*, June 1998.

[11] S. Benedetto, G. Montorsi, D. Divsalar, and F. Pollara, "A soft-input soft-output maximum a posteriori (MAP) module to decode parallel and serial concatenated codes", *The Telecommunications and Data Acquisition Progress Report 42-127, July-September 1996*, Jet Propulsion Laboratory, Pasadena, California, pp.1-20, November 15, 1996.
http://tda.jpl.nasa.gov/tda/progress_report/42-127/127H.pdf

[12] W. Haifeng, J. Lilleberg and K. Rikkinen, "Joint multiuser detector with decoding and feedback in asynchronous CDMA systems", *Proc. of ISSSTA'98*, September 1998.

[13] J. Lilleberg , and W. Haifeng, "Novel iterative multi-user receiver for asynchronous CDMA systems in Rayleigh fading channels", *Proc. of PIMRC'98*, September 1998.