ITERATIVE MULTIUSER RECEIVERS IN CODED CDMA SYSTEMS

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ABSTRACT

In this paper, we consider the possibility of utilizing the channel decoder output in the cancellation of multiuser interference. The paper contains a summary of our earlier numerical results followed by a discussion about some observations based on these simulations.

I. INTRODUCTION

The past research on code division multiple access (CDMA) multiuser detection has mainly concentrated on the uncoded case, that is, the channel coding is assumed to be totally independent from the multiuser detection and is thus ignored in the analysis and algorithm design [1]. Recently, there has been a growing interest for an integrated approach, where the channel coding is taken into account in the design and analysis of the multiuser receivers. This approach may be combined with the iterative decoding techniques, which are a new popular research topic in the area of coding theory, mainly because of the Turbo codes [2]. A number of approaches combining iterative decoding and multiuser detection have been studied including the optimal receiver derived by Giallorenzi and Wilson in [3]. Various sub-optimal approaches have also been proposed ([4,5]), including the receiver structure proposed in [6,7] by the author.

In this paper we present a summary of our earlier numerical results ([6,7]) followed by a discussion and analysis of some main observations that were made based on these simulations.

II. SYSTEM MODEL

The CDMA system modelled in this paper is the uplink chip and symbol-synchronous direct sequence DS-CDMA communication system with *K* users. We assume BPSK modulation. The model uses convolutional channel coding to improve the BER performance of the system. The channel is modelled as a time-invariant single-path channel where Gaussian noise with zero mean and variance σ^2 is added. The block diagram of the system is shown in Figure 1.

In a synchronous CDMA system, the matched filter output at time *i* can be expressed as

$$\mathbf{y}_i = \mathbf{R} \mathbf{A} \mathbf{x}_i + \mathbf{n}_i \tag{1}$$

where $\mathbf{x}_i = (\mathbf{x}_i^{(1)}, \dots, \mathbf{x}_i^{(k)})^{\mathsf{T}}$ is the coded data vector containing the transmitted data symbols of every user and \mathbf{n}_i is a Gaussian random vector with covariance matrix equal to

$$E[\mathbf{n}_i \mathbf{n}_i^{\mathrm{T}}] = \sigma^2 \mathbf{R}. \tag{2}$$

Furthermore, \mathbf{R} is the correlation matrix and \mathbf{A} is the channel matrix, that is

$$\mathbf{R} = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1K} \\ \rho_{21} & 1 & \cdots & \rho_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{K1} & \rho_{K2} & \cdots & 1 \end{pmatrix}$$
(3)
$$\mathbf{A} = \operatorname{diag}(a_1, \dots, a_K),$$
(4)

where ρ_{ij} is the cross-correlation between users *i* and *j*. The channel matrix is diagonal since we assume single-path propagation.



Figure 1: Block diagram of the communication system

III. RECEIVER STRUCTURE

The decoder structure used in the iterative decoding algorithms of (serially) concatenated codes provides the principal model for our receiver structure. Naturally, there is no need for the inner constituent decoder in a system using non-concatenated channel coding. However, substituting the inner decoder with a multiuser likelihood calculation (MULC) unit allows the utilization of the feedback from the channel decoder in the multiuser likelihood calculations. This approach is shown in Figure 2. The receiver has a disjoint user-by-user structure and the channel symbol information is shared between the single-user receivers and used for the multiuser likelihood calculations. At each stage, these calculations are based on the channel symbol probabilities estimated during the previous stage.



Figure 2: Iterative receiver structure utilizing channel coding

Our approach for multiuser likelihood calculation is similar to the one considered in There, [8]. the multiuser likelihood calculations are based on the joint uncoded a posteriori probabilities, meaning that the soft information input provided for each channel decoder contains the information about the whole matched filter sample vector \mathbf{y}_i . In traditional interference cancellation algorithms for uncoded data, the symbol decision is based only on the code matched filter output for user k and the tentative

symbol decisions for the interfering users. Since our receiver structure is designed for MULC algorithms that are using these interference cancellation algorithms, we adopt the corresponding approach in multiuser likelihood calculation. It means that uncoded *a posteriori* probabilities are used for each user separately. For each user *k*, the multiuser likelihood calculation thus requires, in addition to the knowledge of the noise variance σ^2 , only the knowledge of the user *k* matched filter output $y_i^{(k)}$ and the estimates of the symbol probability distributions for every interfering user.

The detailed iterative structure of the mth stage of the receiver for user k is shown in Figure 3. During the *m*th iteration, this unit calculates the symbol likelihoods based on the matched filter output $y_i^{(k)}$ of the respective user and the previous likelihood estimates $I_{i}^{(k)}(m-1)$ for the symbols transmitted by the interfering users. These estimates are calculated during the previous iteration round by adding the output of the outer unit with the earlier likelihood estimate produced by the MULC unit. The outer unit is a soft-input-soft-output (SISO) decoder, a unit that is commonly used as a constituent decoder when decoding concatenated codes, and it produces its output by adjusting the symbol likelihoods based on its knowledge of the channel code trellis. The SISO unit also produces the final data bit decisions after the last iteration round.

There are several possibilities for the actual algorithm to be used in the SISO. We use a variant of the well-known MAP algorithm [9], where all the input and output distributions are expressed in the form of log-likelihood ratios (LLR). Furthermore, we use the extrinsic channel bit probabilities instead of the channel symbol probabilities as the output of the SISO unit. These are calculated by using the formula derived in [9]. Thus if $P_1(x_i^{(k)})$ (resp. $P_0(x_i^{(k)})$) is the *a priori* (resp. extrinsic *a posteriori*) probability distribution for channel bits then we can form the corresponding log-likelihood ratios as:

$$L_{\rm I}(x_i^{(k)}) = \log \frac{{\rm P}_{\rm I}(x_i^{(k)} = +1)}{{\rm P}_{\rm I}(x_i^{(k)} = -1)}$$
(5)

$$L_{\rm O}(x_i^{(k)}) = \log \frac{{\rm P_O}(x_i^{(k)} = +1)}{{\rm P_O}(x_i^{(k)} = -1)} \tag{6}$$

As was mentioned above, the MULC unit uses for each user k the code matched filter output y_k and the channel bit LLRs I_k of the interfering users for likelihood calculation.



Figure 3: The detailed structure of the mth stage for user k

During the iteration round *m*, we have a twostep likelihood calculation algorithm. Given the previous estimates in the form of a vector $\operatorname{sign}(\mathbf{I}_i)$, where the *j*th element of the vector is $\operatorname{sign}(I_i^{(i)}(m-1))$ for $j \neq k$ and the *k*th element is 0, we first perform the interference calculation step to obtain the corrected matched filter output samples

$$\hat{y}_i^{(k)} = y_i^{(k)} - \mu_k(\operatorname{sign}(\mathbf{I}_i))$$
(7)

The function μ_k estimates the multiple access interference (MAI) and is thus defined as $\mu_k(\mathbf{v}) = \mathbf{P}_k \mathbf{RAv}$, where the \mathbf{P}_k is the projection operator that returns the *k*th element of a vector. Given an estimate $\hat{\sigma}^2$ for the noise variance we can then calculate the new likelihoods by using the formula

$$L_{I}(x_{i}^{(k)};m) = \frac{2a_{k}(\hat{y}_{i}^{(k)})}{\hat{\sigma}^{2}} + L_{O}(x_{i}^{(k)};m-1)$$
(8)

The hard decision interference calculation step (7) can be substituted with some other interference calculation method. In this paper we only consider one variant of the basic interference cancellation procedure, called here the "soft" interference cancellation. There, the interference calculation step (7) is replaced by:

$$\hat{y}_{i}^{(k)} = y_{i}^{(k)} - \mu_{k} (\tanh(\mathbf{I}_{i} / 2))$$
 (9)

In the numerical simulations, we will also use the optimal likelihood calculation algorithm derived in [6]. This is useful in the evaluation of the different IC algorithms. For any iteration round *m*, the optimum likelihood calculation algorithm calculates

$$L_{I}(x_{i}^{(k)};m) = \frac{2a_{k}y_{i}^{(k)}}{\sigma^{2}} + L_{MAI,i}^{(k)}(m) + L_{O}(x_{i}^{(k)};m-1)$$
(10)

where

$$L_{MAI,i}^{(k)}(m) = \left(\frac{\sum_{\mathbf{e}} P_{i}(\mathbf{e};m) e^{1/(2\sigma^{2})(2(y_{k}-a_{k})\mu_{k}(\mathbf{e})-\mu_{k}(\mathbf{e})^{2})}}{\sum_{\mathbf{e}} P_{i}(\mathbf{e};m) e^{1/(2\sigma^{2})(2(y_{k}+a_{k})\mu_{k}(\mathbf{e})-\mu_{k}(\mathbf{e})^{2})}}\right)$$
(11)

And the summations are over all such **e** that $e_k = 0$ and $e_j = \pm 1$ for $j \neq k$ and $P_i(\mathbf{e};m)$ is the *m*th stage estimation for the probability of the transmission of symbol e_j by user *j* for all users $j \neq k$ at time *i*.

IV. NUMERICAL RESULTS

In this section we will report some numerical results obtained through simulations. The channel coding used here is a rate 1/2 convolutional code with the generator (7,5). The SNR is 2 dB and the number of simultaneous users is K=4. The correlation coefficient $\rho=0.3$ is used for each user pair.

Figure 4 shows the case, where the received power levels of all users are the same. The performance is given for the hard IC decision algorithm (7) both with and without the variance estimation after each IC stage, as well as for soft IC algorithm (9) with variance estimation after each IC stage. The optimal likelihood calculation and the single user bound are shown for reference.

The level of interference is the same for all users, but in general rather high due to the high cross-correlation. Without the variance estimation after each IC stage, the performance of the hard IC algorithm is unacceptable.



Figure 4: Receiver performance with equal received powers

On the other hand, all the different IC algorithms have near-optimal performance if the variance re-estimation is used.

Figure 5 gives the coded BER for user 1 after each iteration step in a situation, where all other users are received with a power level 3 dB higher than that of the user 1. This means that whereas the user 1 experiences an increase of interference, the channel quality of all other users is actually improved when compared to the equal-power case. This is due both to decreased interference from user 1 and the inreased signal-to-noise ratio, since the noise variance is kept constant with respect to user 1 power level. The hard decision IC algorithm without the variance re-estimation suffers from the increased interference and cannot use the improved symbol estimates of the other users. The IC algorithms using variance re-estimation again have nearoptimal performance.

In addition, the optimal algorithm has now practically the single user performance. This improvement is caused by the improved symbol estimates of the high-power users.

Figure 6 shows the performance of the simulated algorithms in a situation, where there are three interfering users: one received with equal power level, one receiver with a power level 3dB higher and one with a power level 6dB higher.



Figure 5: BER performance for a user with a decreased (-3dB) power level



Figure 6: BER performance with three interfering users (with power levels 0dB, 3dB and 6dB)

This situation is more demanding for the efficient interference cancellation than the previous case, since now also some interfering users have decreased channel quality due to the high-power interfering user. This means that the tentative symbol decisions of those other users are distorted. This is reflected as decreased performance of the IC algorithms. The soft IC algorithm has somewhat better performance than the one using hard tentative decisions. This is natural, since the tentative symbol decisions of the two low-power interferes are initially unreliable in the presence of a high-power interfering user. The use of soft decision mitigates the impact of the incorrect tentative decisions resulting a more likely convergence towards correct symbol decision than when the hard decisions are used.

V. CONCLUSIONS

In this paper, we have considered the possibility of utilizing the channel decoder output in the cancellation of multiuser interference. A summary of our earlier numerical results was presented and some observations based on those results were made. The main observation was that the investigated algorithms perform near-optimal when the variance is re-estimated after each iteration round. Also worth noting is that the equal-power case didn't produce the best performance. This was due to the improved tentative decisions of the interfering users. The algorithm using soft tentative decisions was found to provide a performance gain when there is a high power interferer present in the system.

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