Outage Probability in GSM-GPRS Cellular Systems with and without Frequency Hopping

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Abstract—General Packet radio service (GPRS) is designed for transmitting packet data and supposed to take its radio resource from the pool of unused channels by GSM voice services. Obviously, the introduction of GPRS has impact on the voice services. In this paper, we present a method to calculate the outage probability of the GSM-GPRS network for both the non-frequency hopping and the frequency hopping systems. This method takes into account the Rayleigh fading, power control (with error), discontinuous transmission, and frequency hopping (if applied). The outage probability of voice services affected by the introduction of GPRS is discussed. The number of unused voice channels allocated to GPRS depends on the difference between the outage level of the existing GSM network and the maximum acceptable level. The frequency hopping system can accommodate more GPRS traffic than the non-frequency hopping system. The power control error has more impact on the system performance when more channels are allocated to GPRS. Beyond our expectations, for the non-frequency hopping system, the channels provided to GPRS are not much different between the high channel occupancy and low channel occupancy of voice services. However, for the frequency hopping system, the system can provide more channels to GPRS at low channel occupancy. The cell service area decreases by about 10%~20% for each additional channel allocated to GPRS.

Key word: *GPRS, outage probability, quality of service.*

1. Introduction

Since cellular systems were introduced in 1970s, the main application of most mobile communications systems has been the voice-oriented service. Recently, the use of data applications in mobile terminals, such as facsimile transmission, Internet surfing, short message exchange, multimedia services, etc., is becoming more and more popular. As the human mobility becoming even more important, the mobile communications systems are demanding all-embracing applications, such as wireless personal computers, mobile offices, mobile electronic funds transfer, road transport telematics, fleet management, and remote telematics, in addition to the existing services.

The current method of data transmission in the pan-European Global System for Mobile Communications (GSM) and the American Advanced Mobile Phone Standard (AMPS) cellular networks is circuit switching. This technique reserves the traffic channel for the entire communication time, which wastes the radio resource when data traffic occurs in bursts with long silent intervals. In the development of GSM phase 2+, the European

Telecommunications Standard Institute (ETSI) has specified [1] a general packet radio service (GPRS) over the GSM to increase the utilization efficiency of the radio resource and provide a set of additional services. Similarly, in North America, Cellular Digital Packet Date is specified [2] to provide packet data services utilizing the AMPS infrastructure.

A cell supporting GPRS may allocate resources on one or several physical channels. Those physical channels, shared by the GPRS MSs, are taken from the common pool of physical channels available in the cell. The physical channels unused by circuit switched services are allocated dynamically to the GPRS according to the needs for actual packet transfers which is referred to as the "capacity on demand" principle. Up to eight packet data traffic channels (PDTCHs), with different timeslots but with the same frequency carrier, may be allocated to one MS at the same time.

Whenever the signaling is not carried by the existing GSM common control channel (CCCH), at least one packet data channel (PDCH, e.g., physical channel) is needed to act as a master. This master channel accommodates packet common control channel (PCCCH), which carries all necessary control signaling for initiating packet transfer, as well as user data and dedicated signaling. Other PDCHs act as slaves for user data transfer and for dedicated signaling. A new protocol, the medium access control (MAC) protocol, is introduced in order to multiplex up to eight different MSs onto one slave channel. Different packet data logical channels and the PDTCH can be multiplexed on the same physical channel.

When a circuit switched network is operating at a blocking probability of 0.02, the average channel load is in the range of 60%-80% depending on the total number of channels used in the cell. Thus, there are 20%-40% idle channels on average which may be used for data services. Both GPRS and CDPD are designed to utilize those unused channels to transmit short bursts of packet data. However, the introduction of the packet data services into existing circuit switched networks without allocating new spectrum may cause the degradation of either voice quality or voice capacity. More specifically, it may increase the blocking probability, or degrade the service of voice quality and reduce the cell service area, defined as the area over which a specified outage probability limit is achieved.

The introduction of packet data services must have no effects or just very small effects on the existing voice services. In order to guarantee the quality of service (QoS), it is necessary to allocate dedicated channels for the packet data service, especially for the GPRS with multiple applications and multiple class services. However, such a scheme with dedicated channels will reduce the number of channels provided for voice services and increase the blocking probability to an undesirable level. The dynamic sharing of the channels between voice services and packet data services seems not to have much impact on the capacity of voice services and creates additional capacity for packet data services [3]. However, the system performance will be degraded, e.g., outage probability increase and cell service area decrease, due to the additional interference contributed by packet data transmission [4], [5], [6]. When the operator plans a system, the outage probability distributed in a cell is normally designed within the some limit for a certain traffic load. There are two possible cases in existing circuit switched cellular networks, i.e., 1) the outage level has reached the maximum acceptable value, 2) the outage level is below the maximum acceptable value.

In the first case, since the voice services have already achieved the network capacity limit, overlaying the packet data services will degrade either voice quality or voice load due to the

additional cochannel interference. Even though in the low channel occupancy, some channels unused by voice services might be used for GPRS, the obtained capacity is quite limited and unpredictable. In the second case we may obtain additional capacity by overlaying the packet data services onto voice services, but the tolerable level of the outage limits the capacity extension so that it does not exceed the maximum acceptable value.

When a channel is allocated to GPRS, it is shared by a few data users simultaneously. The cochannel interference to the voice users might vary rapidly and dramatically in the time interval from 20 ms to a few seconds depending on the transmitted packet data size, since the locations of those packet data users could be very different. This effect could drive the system into an unpredictable and unstable situation except for causing a degradation of the QoS of voice services. Therefore, the admission control of the packet data traffic is necessary to guarantee the quality of service for voice users. In this paper, we focus on discussions of the outage probability affected by GPRS traffic in order to provide some guidelines for system evaluation and planning, as well as the admission control. Numerous papers [7]-[10] have presented methods of the outage probability calculation for different considerations separately, such as, Rayleigh fading and voice activity factor. However, very few papers can give a method integrating the considerations of Rayleigh fading, voice and data activity, and power control, with and without frequency hopping, especially for systems, where voice and packet data coexist. In section 2, we present a detailed analytical approach for the outage probability calculation of GSM-GPRS networks. Power control, Rayleigh fading, and voice activity detection, with and without slow frequency hopping are taken into consideration. In section 3, numerical results are given, and the QoS of voice services affected by GPRS is discussed by outage probability calculations. Finally, some conclusions are drawn in section 4.

2. Outage Probability of the GSM - GPRS Network

A. Assumptions

All signals are assumed to have experienced Rayleigh fading with respect to a local mean signal strength, while the local mean signal strength experiences log-normal slow fading based on a mean value, which is determined by the propagation loss law of an inverse *n-th* power of distance. The desired signal and interfering signals, and different individual interfering signals are all statistically independent. The slow fading is assumed to be uncorrelated and the variance is the same for all cells. Voice activity detection is assumed to be used in the system. Using the discontinuous transmission (DTX) method, the transmitted power of a user is reduced to a low level during speech pauses, and its interference to other users is assumed as zero. For a voice user, the active time of speech is about 40% of his call duration in average, and the rest of the call duration is in listening state. Therefore, when a channel is used for transmitting voice, the activity factor is about 40%.

The selective automatic repeat request (ARQ) protocol is used for GPRS data transfer. The MAC protocol [13] is developed to multiplex up to eight different MSs of GPRS onto one physical channel. Different packet data logical channels and the PDTCH can also be multiplexed on the same physical channel. The MAC function provides collision avoidance, detection and recovery procedures. On the downlink, multiplexing is controlled by a scheduling. On the uplink, multiplexing is controlled by medium allocation to individual users. When a channel is reserved for an individual MS, it allows to transmit a predetermined

(limited or unlimited) amount of time without interruption. When the MS (or BS) has sent the last Radio Link Control (RLC) Data Block (perhaps not the final data packet), the channel is released and it is waiting for the acknowledgment. A new Packet Channel Request is needed to be issued in order to continue to transfer the remaining data block and erroneous data block. When the service-initiating request of a GPRS user is accepted, it can be in two states: active and idle. In the active state, it may either be in transmitting or backlogged (retransmitting) state. In the idle state, it is waiting for acknowledgment or waiting for transfer. However, a channel accommodating a few GPRS users, can be assumed as to be transmitting some kind of information, if the number of users requesting service is large enough. The reason is that the time scheduling a user into the transmitting state is so small (one burst period) that the idle time of this channel can be neglected compared with its active time. The traffic load of GPRS may vary in a day, however, for the system planner's point of view, the network situation with a high GPRS load should be considered in order to guarantee the QoS of voice services. Therefore, the activity factor of a channel carrying the packet data traffic is taken as 100% here.

B. Probability Density Functions of the Signal and Interference

A signal from the transmitter to the receiver has experienced the path attenuation, slow fading and fast fading. Thus, the received power S of a desired signal and the interference power I can be described as:

$$S = A r_{00}^{-\eta} \xi_{00} Z_{00} P_{T_{00}} \tag{1}$$

$$I = \sum_{k=1}^{M} \sum_{i=1}^{N_k} I_{ik} = \sum_{k=1}^{M} \sum_{i=1}^{N_k} A r_{ik}^{-\eta} \xi_{ik} Z_{ik} P_{T_{ik}} X_{ik} \tag{2}$$

where P_T is the transmitted power; ξ is the slow fading variable; Z is the fast fading variable; r is the distance from the transmitter to the receiver; η is a factor of transmitted power attenuated with distance. X is the channel activity factor; A is a proportionality constant. The indexes i and k refer to the i-th mobile in the k-th interfering cell and $i = k = 0$ corresponds to the desired mobile and the desired cell. M and N_k are the number of interfering cells and the number of mobiles in the k-th cell respectively. One should note that, when a channel is allocated to GPRS, it is shared by a few data users. The cochannel interference to the voice users might fluctuate frequently in the time interval from 20 ms to a few seconds depending on the transmitted packet data size, since the locations of those packet data users could be very different.

r_{00} is the distance from the desired mobile to its host base station; r_{ik} is the distance from the desired mobile to the k-th interfering base station of the i-th mobile in downlink, or the distance from the host base station of the desired mobile to the i-th mobile in the k-th interfering cell in uplink.

ξ_{00} *and* ξ_{ik} are log-normal random variables which refer to the slow fading of the desired signal and the interference signal from the i-th mobile in the k-th interfering cell respectively.

Z_{00} and Z_{ik} are fast fading random variables of desired signal and interference signal from the *i-th* mobile in the k-th interfering cell respectively. The distribution of those variables for

Rayleigh fading normally is the exponential distribution, i.e., the probability of density function (PDF) is:

$$f_{Z_{ik}}(Z_{ik}) = \exp(-Z_{ik}), \qquad Z_{ik} > 0, \qquad i = 0, 1, \cdots, N_m \text{ and } k = 0, 1, \cdots, M \qquad (3)$$

X_{ik} is a binary random variable, which takes values in {0, 1} and representing whether channel used by the i-th mobile in the k-th cell is active ($X_{ik} = 1$) or inactive ($X_{ik} = 0$). The PDF of X_{ik} is

$$f_{X_{ik}}(X_{ik}) = p_k^{X_{ik}}(1 - p_k)^{1 - X_{ik}}, \qquad X_{ik} = 0,1 \qquad (4)$$

where p_k is the probability that the i-th mobile in the k-th interfering cell is active at the same frequency and time slot as that of the desired mobile.

Power control compensates the long term attenuation (i.e., attenuation due to the path-loss and shadowing) by different power level transmissions. The power control algorithm, which is based on the SIR measurement, has a risk of system instability [11]. Some radio links of the system may end up transmitting at, or altering between, their maximum or minimum power. The power control algorithm for GSM and GPRS systems is recommended [12], [13] on the base of the received strength power:

$$P(t) = \max\{P_{\min}, \min(\Gamma_0 - \alpha \cdot C, P_{\max})\} \qquad \text{in } dB$$

or

$$P(t) = \max\{P_{\min}, \min(\frac{\Gamma_0}{C^\alpha}, P_{\max})\} \qquad (5)$$

where $P_{\max}$, $P_{\min}$ are the maximum and minimum transmitted power allowed by the system respectively; Γ_0 is a pre-designed constant; C is the received signal level; α is a system parameter varying from 0 to 1. If the power control fully compensates the path loss and slow fading, the α is equal to 1. However, if the path loss and slow fading is only partly compensated ($0<\alpha<1$), some gain in capacity is obtained [14]. More specifically, the transmitted power of the desired signal, $P_{T_{00}}$ is inversely proportional to $r_{00}^{-\alpha\eta}\xi_{00}^{\alpha}$. For user i in cell k, its transmitted power $P_{T_{ik}}$ is inversely proportional to $r_{0ik}^{-\alpha\eta}\xi_{ik}^{\alpha}$, where r_{0ik} and ξ_{0ik} refer to distance and the slow fading on the path from mobile i in cell k to its own base station respectively. However, the power control is imperfect in the practical system due to the measurement error, and a power control error P^e may exist. The power control error P^e is normally considered as a lognormal random variable with a zero mean and a standard deviation σ_e [20], [22]. Therefore, we can rewrite (1) and (2) as:

$$S = A'r_{00}^{-\eta(1-\alpha)}\xi_{00}^{1-\alpha}P_0^e Z_{00} = A'r_{00}^{-\eta(1-\alpha)}\beta_{00}Z_{00} \qquad (6)$$

and

$$I = \sum_{k=1}^{M}\sum_{i=1}^{N_k} I_{ik} = \sum_{k=1}^{M}\sum_{i=1}^{N_k} A'(\frac{r_{ik}}{r_{0ik}^{\alpha}})^{-\eta}\xi_{ik}\xi_{0ik}^{-\alpha}P_{ik}^e Z_{ik}X_{ik} = \sum_{k=1}^{M}\sum_{i=1}^{N_k} A'(\frac{r_{ik}}{r_{0ik}^{\alpha}})^{-\eta}\beta_{ik}Z_{ik}X_{ik} \qquad (7)$$

where $A' = A\Gamma_0, \beta_{00} = \xi_{00}^{1-\alpha} P_0^e$, and $\beta_{ik} = \xi_{ik} \xi_{0ik}^{\alpha} P_{ik}^e$. Because ξ_{00}, P_0^e, ξ_{ik}, ξ_{0ik} and P_{ik}^e are independent lognormal random variables, β_{00} and β_{ik} are independently lognormal random variables with the zero mean and the standard deviation $\sigma_{00} = \sqrt{(1-\alpha)^2 \sigma_s^2 + \sigma_e^2}$ and $\sigma_{ik} = \sqrt{(1+\alpha^2)\sigma_s^2 + \sigma_e^2}$ (both in dB) respectively [15]. Here we assume a slow fading factor and power control error with standard deviation σ_s and σ_e respectively for all mobile stations.

For (6), we consider the lognormal random variable β_{00} to be superimposed on $A' r_{00}^{-\eta(1-\alpha)}$. This superimposition produces the local mean power $\bar{S}$ which is lognormally distributed with a standard deviation σ_{00} and a mean value $10\log(A' r_{00}^{-\eta(1-\alpha)})$, i.e.,

$$f_{\bar{S}}(\bar{S}|r_{00}) = \frac{1}{\sigma_{00}\sqrt{2\pi}(\bar{S}\ln 10 / 10)} \exp\left\{-\frac{\left[10\log(\bar{S}) - 10\log(A' r_{00}^{-\eta(1-\alpha)})\right]^2}{2\sigma_{00}^2}\right\} \qquad (8)$$

Applying the same principle into (7), the local mean individual interference power $\bar{I}_{ik}$ is obtained by superimposing the β_{ik} into the term of $A'[r_{ik}/(r_{0ik})^{\alpha}]^{-\eta}$. The PDF of the local mean individual interference power $\bar{I}_{ik}$ is

$$f_{\bar{I}_{ik}}(\bar{I}_{ik}|r_{ik}, r_{0ik}) = \frac{1}{\sigma_{ik}\sqrt{2\pi}(\bar{I}_{ik}\ln 10 / 10)} \exp\left\{-\frac{\left[10\log(\bar{I}_{ik}) - 10\log(A' r_{ik}^{-\eta} / r_{0ik}^{-\eta\alpha})\right]^2}{2\sigma_{ik}^2}\right\} \qquad (9)$$

Eq. (6) and (7) can be rewritten as

$$S = \bar{S}Z_{00} \qquad (10)$$

$$I = \sum_{k=1}^{M}\sum_{i=1}^{N_k} \bar{I}_{ik} Z_{ik} X_{ik} \qquad (11)$$

C. Outage Probability

The outage probability of a system is the probability that the instantaneous signal power to interference power ratio (S/I) falls below a specified threshold γ and denoted as

$$P_o(outage) = \Pr\left\{S / I < \gamma\right\} \qquad (12a)$$

i.e.,

$$P_o(outage) = \Pr\left\{\frac{\bar{S}Z_{00}}{\sum_{k=1}^{M}\sum_{i=1}^{N_k}\bar{I}_{ik}Z_{ik}X_{ik}} < \gamma\right\} \qquad (12b)$$

From (12), we have

$$P_o(outage|\bar{S}, \bar{I}, Z, X) = \Pr\{Z_{00} < \frac{\gamma}{\bar{S}}\sum_{k=1}^{M}\sum_{i=1}^{N_k}\bar{I}_{ik}Z_{ik}X_{ik}\} \qquad (13)$$

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where $\bar{I} = \left[\bar{I}_{11}, \bar{I}_{12}, \cdots \bar{I}_{N_k M}\right]$, $Z = \left[Z_{11}, Z_{12}, \cdots Z_{N_k M}\right]$, $X = \left[X_{11}, X_{12}, \cdots X_{N_k M}\right]$. Because Eq. (12) refers to the probability of the signal to interference ratio (SIR), when applying (8) and (9) to (12), we can let $A' = 1$ without loss of generality. Using the PDF of Z_{00} in (3) into (13) we have

$$P_o(outage|\bar{S}, \bar{I}, Z, X) = \int_0^{\frac{\gamma}{\bar{S}} \sum_{k=1}^{M} \sum_{i=1}^{N_k} \bar{I}_{ik} X_{ik} Z_{ik}} \exp(-Z_{00}) dZ_{00} = 1 - \prod_{k=1}^{M} \exp\left\{-\sum_{i=1}^{N_k} \frac{\gamma}{S} \bar{I}_{ik} Z_{ik} X_{ik}\right\} \quad (14)$$

- Non-Frequency Hopping Systems

For a non-frequency hopping system, there is only one co-channel user allowed in a cell. Thus, $N_k = 1$, and

$$P_o(outage|\bar{S}, \bar{I}, Z, X) = 1 - \prod_{k=1}^{M} \exp\left\{-\frac{\gamma}{S} \bar{I}_k Z_k X_k\right\} \quad (15)$$

Averaging (15) over the Rayleigh fading and shadowing using (3), (8) and (9) as shown in the Appendix, we have average outage probability:

$$\overline{P}_o(outage) = 1 - \pi^{-\frac{1+N_d}{2}} \sum_{l=1}^{n} w_l \left\{ \prod_{k=1}^{N_v} \left[1 - p_k + \pi^{-\frac{1}{2}} p_k \sum_{j=1}^{n} w_j \frac{1}{1 + d_k \exp(cx_j - ax_l)} \right] \right.$$

$$\left. \bullet \prod_{q=1}^{N_d} \sum_{i=1}^{n} w_i \frac{1}{1 + d_q \exp(cx_i - ax_l)} \right\} \quad (16)$$

where w_i is the weight of the n-point Gauss-Hermite quadrature formula and x_i is the abscissa of the i-th zero of Gauss-Hermite polynomial; N_v and N_d are the number of cochannels used by voice users and data users in M interfering cells respectively. $a = \sqrt{2}\sigma_{00} \ln 10 / 10$, $c = \sqrt{2}\sigma_k \ln 10 / 10$, $d_k = \gamma(r_{00}^{1-\alpha} r_{0k}^{\alpha} / r_k)^{\eta}$, $d_q = \gamma(r_{00}^{1-\alpha} r_{0q}^{\alpha} / r_q)^{\eta}$, r_{00} is the distance from the desired mobile to its host base station; r_k (or r_q) is the distance from the desired mobile to the k-th (or the q-th) interfering base station of the voice (or data) user in downlink, or the distance from the host base station of the desired mobile to location of the cochannel voice (or data) user in the k-th (or the q-th) interfering cell in uplink; r_{0k} (or r_{0q}) refers to distance from location of the cochannel voice (or data) user in the k-th (or the q-th) interfering cell to its own base station. The outage probability is mainly dependent on the locations of mobiles, the frequency reused factor and the channel load by GSM voice services and GPRS.

- Frequency Hopping Systems

For frequency hopping systems, each channel in a cell is occupied by voice or data users with the same probability A_k,

$$A_k = \frac{N_v(k) \cdot V_f + N_d(k)}{N_t \cdot N_{hop}(k)} \quad (17)$$

where N_t is the number of time slots per TDMA frame and $N_{hop}(k)$ is the number of distinct frequency carriers in cell k; $N_v(k)$ and $N_d(k)$ are the number of channels used in cell k by voice users and data users respectively.

In GSM frequency hopping system, each channel within a cell is orthogonal. The intercell interference is considered only. Because of the non-uniform call traffic in practical networks, the number of frequency carriers allocated to different cells may not be the same. Therefore, the frequency set used in the desired cell may not be exactly the same as the frequency sets in interfering cells. Let $N_{co}(k)$ be the number of frequency carriers used in both the desired cell and interfering cell k; N_{de} is the total number of frequency carriers used in the desired cell. We define a fractional factor f_k as:

$$f_k = \frac{N_{co}(k)}{N_{de}} \times \frac{N_{co}(k)}{N_{hop}(k)} \qquad (18)$$

Therefore, the interference probability caused by all users in cell k is $p_k = A_k \cdot f_k$. However, each of $N_k = N_v(k) + N_d(k)$ co-channel users in cell k is equally likely to cause interference and they are mutually exclusive, the probability of interference from the i-th mobile in the k-th cell should use p_k/N_k. Combining (4) and (14), we have

$$P_o(outage|\overline{S}, \overline{I}, Z) = 1 - \prod_{k=1}^{M} \{(1 - p_k) + \frac{p_k}{N_k} \sum_{i=1}^{N_k} \exp(-\frac{\gamma}{S} \overline{I}_{ik} Z_{ik})\} \qquad (19)$$

Averaging (19) over the Rayleigh fading and shadowing using (3), (8) and (9) as shown in the Appendix, we have average outage probability:

$$\overline{P}_o(outage) = 1 - \pi^{-\frac{1}{2}} \sum_{l=1}^{n} w_l \left\{ \prod_{k=1}^{M} \left[1 - p_k + \pi^{-\frac{1}{2}} \frac{p_k}{N_k} \sum_{i=1}^{N_k} \sum_{j=1}^{n} w_j \frac{1}{1 + d_{ik} \exp(cx_j - ax_l)} \right] \right\} \qquad (20)$$

where $d_{ik} = \gamma(r_{00}^{1-\alpha} r_{0ik}^{\alpha} / r_{ik})^{\eta}$, r_{00} is the distance from the desired mobile to its host base station; r_{ik} is the distance from the desired mobile to the k-th interfering base station of the i-th mobile in downlink, or the distance from the host base station of the desired mobile to the i-th mobile in the k-th interfering cell in uplink; r_{0ik} refers to distance from mobile i in interference cell k to its own base station. Obviously, the outage probability is mainly dependent on the number of hopping frequencies, the locations of mobiles, the frequency reuse factor and the channel load by GSM voice services and GPRS.

3. Numerical Results

Introducing GPRS into GSM networks may bring some effects on the quality of service and capacity of the existing circuit switched services. In this section, we will give some numerical results of the outage probability of the uplink affected by transmitting the packet data traffic in the GSM radio network resource.

The hexagonal cell cellular system with 7 cell reused pattern is considered in this calculation. A central cell which is taken as the cell with desired mobiles has six interfering cells.

Omnidirectional antennas are assumed to be used and the propagation loss exponent and slow fading factor are same for all cells. The propagation loss exponent η and standard deviation σ_s of the slow fading are assumed to be 4 and 8 dB respectively. The discontinuous transmission (DTX) method is assumed to be used in the system. The voice channel activity factor is assumed to be 0.4, and the packet data channel activity factor is used 100% as we discussed in section 2. For power control, the algorithm partly compensating the path loss and shadowing is chosen $\alpha = 0.5$ in (5).

A typical GSM system normally has 3 or 4 carriers per cell. Here the number of carriers is assumed to be 4 in each cell. Then $4 \times 8 = 32$ physical channels are available in a cell. Suppose 3 channels are reserved for the network signaling, thus, only 29 traffic channels are available for carrying users' information in a cell. The mobile stations are uniformly distributed with an identical number in each cell.

Suppose the GSM network operating at a blocking probability of 0.02 for voice services. For a cell with 29 traffic channels, an average traffic load of 21.04 Erlangs is supported. The average number of calls in the system is $E(n) = \rho(1-P_b) = 21.04 \times 98\% = 20.62 \approx 21$. Our goal is to investigate how the performance of circuit switched services are affected by the introduction of the GPRS traffic into existing GSM networks. Our interest is the interference statistics of voice services affected by GPRS. Therefore, we assume that the call blocking probability of voice services is not affected by GPRS due to the dynamic resource allocation between voice services and GPRS.

The GSM specification [18] recommends a SIR threshold of 9 dB and 2 dB implementation margin has been included in the simulated residual BER threshold. A threshold lower than 9 dB, such as 7 dB [19], [20], corresponding to a bit error rate of 10^{-3}, should be considered in the theoretical calculations. However, in practical network planning, operators normally use a SIR threshold higher than the recommended value of 9 dB, which is up to 12-14 dB. In addition, only one ring of the interference cells is considered in our calculation. Therefore, a threshold value of 10 dB is used in the following calculations.

The outage probability is calculated through a series of Monte Carlo simulations based on generating a large number of snapshots. In each snapshot locations of users are randomly generated, and a pure random channel allocation algorithm is used to assign channels to users in the central (desired) cell as well as in each interfering cell, according to the considered number of simultaneous users. Due to the random channel allocation, for the non-frequency hopping system there always exist some "good" channels with less cochannel users and "bad" channels with more cochannel users, but for the frequency hopping system the quality of every channel is the same because of the interference diversity. From the system planner's point of view, the system should guarantee the quality of the "worst" channel with the largest number of cochannel users. Therefore, for the non-frequency hopping system the outage probability of each snapshot is calculated from the "worst" channel. In a simulation, 10000 snapshots are generated and the outage probability of each snapshot is calculated by Eq. of (16) or (20). The distribution of the outage probability in a given cell depends on locations of the mobile stations in this cell and its interfering cells. Instead of giving a cumulative distribution function (CDF) of the outage probability, we only show the 0.9 percentile value [21] of the outage probability, called 90% worst case outage probability. The 90% worst case value is obtained by sorting those 10000 snapshots' values of the outage probability in increasing order and choosing the 9000th value.

For an average situation of 21 traffic channels used by voice services in each cell simultaneously, Figure 1 shows the 90% worst case outage probability of the non-frequency hopping system with the SIR value for different GPRS channel occupancy. Figure 1a and 1b correspond to perfect power control and 2 *dB* standard deviation of power control error in the system respectively. As seen from Figure 2, with the 10 *dB* SIR threshold value, the outage probability increases by about 5%-10% whenever the number of channels used for GPRS increases by one. In addition, comparing Figure 1a and 1b, about 0.5 *dB* gain for the SIR threshold value is found for perfect power control system.

Similarly, Figure 2a and 2b show the 90% worst case outage probability of the frequency hopping system with the SIR value for different GPRS channel occupancy. Figure 2a and 2b correspond to perfect power control, and 2 *dB* standard deviation of power control error in the system respectively. The performance of the frequency hopping system in the outage probability is much better than that of the non-frequency hopping system. However, its characteristic of the outage probability affected by GPRS is similar to that of the non-frequency hopping system.

From Figure 1 and Figure 2, we find that the power control error has more impact on the system performance when more channels are allocated to GPRS.

Figure 3 shows the 90% worst case outage probability of the non-frequency hopping system with different number of channels occupied by voice services and GPRS. A SIR threshold value of 10 *dB* and perfect power control are assumed in the simulation. For the same number of channels used by GPRS, the outage probability does not vary very much from high channel occupancy to low channel occupancy of voice services. The main reason is that the outage probability is calculated with the worst channel in each snapshot here. This result implies that for the non-frequency hopping system the channels provided to GPRS are not much different between high channel occupancy and low channel occupancy of voice services. The result is out of our expectations because we expect more resource available for GPRS at the low channel occupancy by voice traffic. Therefore, multislot GPRS services might cause large degradation of the quality of existing voice services even at low channel occupancy of voice services. In order to guarantee the QoS of existing voice users, some observations, such as GPRS admission control, need to be taken, before allocating unused channels to GPRS.

Figure 4 shows the 90% worst case outage probability of the frequency hopping system with a different number of channels occupied by voice services and GPRS. For the same number of channels used by GPRS, the outage probability increases as the channel occupancy of voice services gets higher. From Figure 4, we find that the quality of voice services may not be affected by the introduction of GPRS into GSM with proper admission control of GPRS.

Cell coverage is normally determined by the received signal strength, however, the received value of the signal to interference ratio (SIR) is more relevant to describe the service area of a cell. Therefore, a parameter, the cell service area, is defined as the area over which a specified outage probability limit is achieved. In order to investigate the cell service area of existing voice services affected by GPRS, we simulate the outage probability distributed with the normalized radius (r/R, R is the cell radius) in the desired cell. In each simulation, the location of the mobile station in the desired cell is restricted to a circle with a radius r, and locations of mobiles in interfering cells are randomly generated in those cells. 21 simultaneous voice users

are assumed in each cell and a SIR threshold value of 10 *dB* is used. Perfect power control is assumed in the system. Figure 5a and 5b show the 90% worst case outage probability and average outage probability of the non-frequency hopping system distributed with normalized radius (*r/R*) respectively. Comparing the 90% worst case outage probability with the average outage probability, we find that the 90% worst case outage probability is much higher. With the same outage probability limit, both Figure 5a and 5b show that the cell service area decreases by about 10%~20% whenever the number of channels used for GPRS increases by one. As more channels are allocated to GPRS, the cell service area decreases dramatically.

Figure 6a and 6b show the 90% worst case outage probability and average outage probability of the frequency hopping system distributed with normalized radius *r/R* respectively. Though the frequency hopping system has better performance in the cell service area than the non-frequency hopping system, the characteristics of the cell service area shrinking as the introduction of GPRS is similar to that of the non-frequency system. The cell service area decreases by about 15% whenever the number of channels used for GPRS increases by one.

4. Conclusions

GPRS is designed for transmitting the packet data and supposed to take its radio resources from the pool of unused channels by GSM voice services, in order to increase the effective capacity of the digital cellular system. However, introducing GPRS into the existing GSM network is at the expense of the degradation in either voice quality or voice capacity. In this paper, we present a method to calculate the outage probability of the GSM-GPRS network for both the non-frequency hopping and frequency hopping systems. This method takes into account the fast (Rayleigh) fading, power control (with error), discontinuous transmission, and frequency hopping (if applied), and can be utilized to the evaluation of network performance and network planning for GSM-GPRS cellular systems. The analytic expressions of the outage probability are obtained by consideration of the high load of GPRS (the data channel activity factor = 1), but with a small change they can be applied in the situation when the load of GPRS is low (the data channel activity factor < 1). Obviously, in the latter, the GPRS is less aggressive, but it still increases the interference statistics of voice services.

The effects on the quality of voice services due to the introduction of GPRS into GSM network are evaluated by calculating the outage probability. Obviously, GPRS increases the outage probability of existing GSM voice services. Therefore, all those unused voice channels might not be used for carrying GPRS traffic. The number of unused voice channels allocated to GPRS depends on the difference between the outage level of the existing GSM network and the maximum acceptable level.

The frequency hopping system can accommodate more GPRS traffic than the non-frequency hopping system. The power control error has more impact on the system performance when more channels are allocated to GPRS. Beyond our expectations, for the non-frequency hopping system the channels provided to GPRS are not much different between the high channel occupancy and the low channel occupancy of voice services. However, for the frequency hopping system the system may provide more channels to GPRS at the low channel occupancy. The cell service area is decreased by about 10%~20% whenever the number of channels used for GPRS increases by one. As more channels are provided to GPRS the cell service area decreases dramatically.

Appendix: Derivations of the Outage Probabilities

1) Non-Frequency Hopping Systems

Averaging (15) over the Rayleigh fading by the PDF of Z_k in (3) we obtain

$$P_o(outage|\overline{S},\overline{I},X) = 1 - \prod_{k=1}^{M}\int_0^\infty \exp\{-(\frac{\gamma}{\overline{S}}\overline{I}_k X_k + 1)Z_k\}dZ_k = 1 - \prod_{k=1}^{M}\frac{\overline{S}}{\gamma\overline{I}_k X_k + \overline{S}} \tag{A1}$$

Let N_v and N_d are the total number of cochannel users used by voice and data in all interfering cells respectively. For voice users, the activity factor V_f is about 40%, but for the data users, the activity factor $V_d = 100\%$. Therefore, for the data users, $X_k \equiv 1$, but for the voice user, the probability p_k in (4) is equal to the voice activity V_f. Combining (A1) and (4), we obtain

$$P_o(outage|\overline{S},\overline{I}) = 1 - \prod_{k=1}^{N_v}\{(1-p_k) + p_k\frac{\overline{S}}{\gamma\overline{I}_k + \overline{S}}\}\prod_{q=1}^{N_d}\frac{\overline{S}}{\gamma\overline{I}_q + \overline{S}} \tag{A2}$$

Averaging (A2) over the shadowing by (8), (9), we have

$$\overline{P}_o(outage) = 1 - \int_{-\infty}^{\infty}\frac{1}{\sigma_{00}\sqrt{2\pi}(\overline{S}\ln 10/10)}\exp\left\{-\frac{\left[10\log(\overline{S}) + 10\eta(1-\alpha)\log(r_{00})\right]^2}{2\sigma_{00}^2}\right\}$$

$$\bullet \prod_{k=1}^{N_v}\int_{-\infty}^{\infty}\frac{(1-p_k + p_k\frac{\overline{S}}{\gamma\overline{I}_k + \overline{S}})}{\sigma_k\sqrt{2\pi}(\overline{I}_k\ln 10/10)}\exp\left\{-\frac{\left[10\log(\overline{I}_k) + 10\eta\log(r_k/r_{0k}^\alpha)\right]^2}{2\sigma_k^2}\right\}d\overline{I}_k d\overline{S}$$

$$\bullet \prod_{q=1}^{N_d}\int_{-\infty}^{\infty}\frac{\frac{\overline{S}}{\gamma\overline{I}_q + \overline{S}}}{\sigma_q\sqrt{2\pi}(\overline{I}_q\ln 10/10)}\exp\left\{-\frac{\left[10\log(\overline{I}_q) + 10\eta\log(r_q/r_{0q}^\alpha)\right]^2}{2\sigma_q^2}\right\}d\overline{I}_q d\overline{S} \tag{A3}$$

Here, $\sigma_k = \sigma_q = \sqrt{(1+\alpha^2)\sigma_s^2 + \sigma_e^2}$. Let

$$a = \sqrt{2}\sigma_{00}\ln 10/10, b = \eta(1-\alpha)\ln r_{00}, x = (10\log\overline{S} + 10\eta(1-\alpha)\log r_{00})/\sqrt{2}\sigma_{00},$$
$$c = \sqrt{2}\sigma_k\ln 10/10, d_k = \gamma(r_{00}^{1-\alpha}r_{0k}^\alpha/r_k)^\eta, y = \{10\log\overline{I}_k + 10\eta\log(r_k/r_{0k}^\alpha)\}/\sqrt{2}\sigma_k,$$
$$d_q = \gamma(r_{00}^{1-\alpha}r_{0q}^\alpha/r_q)^\eta, z = \{10\log\overline{I}_q + 10\eta\log(r_q/r_{0q}^\alpha)\}/\sqrt{2}\sigma_q.$$

We have

$$\overline{P}_o = 1 - \pi^{-\frac{1+N_d}{2}} \int_{-\infty}^{\infty} dx \cdot \exp(-x^2) \prod_{k=1}^{N_v} \{1 - p_k + \pi^{-\frac{1}{2}} p_k \int_{-\infty}^{\infty} \frac{\exp(-y^2)dy}{1 + d_k \exp(cy - ax)}\}$$

$$\bullet \prod_{q=1}^{N_d} \int_{-\infty}^{\infty} \frac{\exp(-z^2)dz}{1 + d_q \exp(cz - ax)} \qquad (A4)$$

Those integrals can be calculated approximately by the Gauss-Hermite polynomial method [16, 17], i.e.,

$$\int_{-\infty}^{\infty} e^{-x^2} f(x)dx = \sum_{i=1}^{n} w_i f(x_i) + R_n, \quad R_n = \frac{n!\sqrt{\pi}}{2^n (2n)!} f^{(2n)}(\xi) \qquad (-\infty < \xi < \infty) \qquad (A5)$$

where w_i is the weight of the n-point Gauss-Hermite quadrature formula and x_i is the abscissa of the i-th zero of Gauss-Hermite polynomial, which are tabulated in [16] or calculated according to [17]. It is not difficult to find that the remainder R_n is sufficiently small for $n > 10$ (in this paper, we use $n = 20$), if the formula (A5) is applied into (A4). Applying (A5) into (A4), we obtain

$$\overline{P}_o(outage) = 1 - \pi^{-\frac{1+N_d}{2}} \sum_{l=1}^{n} w_l \{ \prod_{k=1}^{N_v} [1 - p_k + \pi^{-\frac{1}{2}} p_k \sum_{j=1}^{n} w_j \frac{1}{1 + d_k \exp(cx_j - ax_l)}]$$

$$\bullet \prod_{q=1}^{N_d} \sum_{i=1}^{n} w_i \frac{1}{1 + d_q \exp(cx_i - ax_l)} \} \qquad (A6)$$

- Frequency Hopping System

Averaging (20) over the Rayleigh fading by (3), we obtain

$$P_o(outage | \overline{S}, \overline{I}) = 1 - \prod_{k=1}^{M} \{ (1 - p_k) + \frac{p_k}{N_k} \sum_{i=1}^{N_k} \frac{1}{1 + \frac{\gamma}{\overline{S}} \overline{I}_{ik}} \} \qquad (A7)$$

Comparing (A7) and (17), we can easily obtain the outage probability according to last session result of (A4),

$$\overline{P}_o = 1 - \pi^{-\frac{1}{2}} \int_{-\infty}^{\infty} dv \cdot \exp(-v^2) \prod_{k=1}^{M} \left\{ p_k + \pi^{-\frac{1}{2}} \frac{p_k}{N_k} \sum_{i=1}^{N_k} \int_{-\infty}^{\infty} \frac{\exp(-u^2)du}{1 + d_{ik} \exp(cu - av)} \right\} \qquad (A8)$$

where $d_{ik} = \gamma (r_{00}^{1-\alpha} r_{0ik}^{\alpha} / r_{ik})^{\eta}$. Using the integral result of (A5), we have

$$\overline{P}_o(outage) = 1 - \pi^{-\frac{1}{2}} \sum_{l=1}^{n} w_l \left\{ \prod_{k=1}^{M} \left[1 - p_k + \pi^{-\frac{1}{2}} \frac{p_k}{N_k} \sum_{i=1}^{N_k} \sum_{j=1}^{n} w_j \frac{1}{1 + d_{ik} \exp(cx_j - ax_l)} \right] \right\} \qquad (A9)$$

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Acknowledgments

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* All these figures can be put inside the section 3.
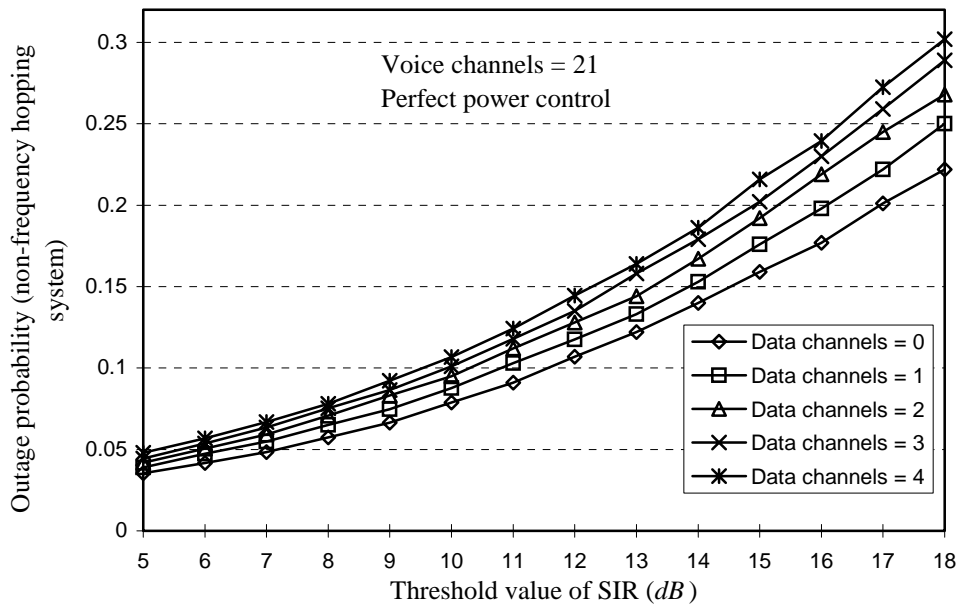
Figure 1a. Outage probability (90% worse case) of the non-frequency hopping system vs. SIR and different GPRS channel occupancy (21 channels been occupying by voice services, perfect power control).
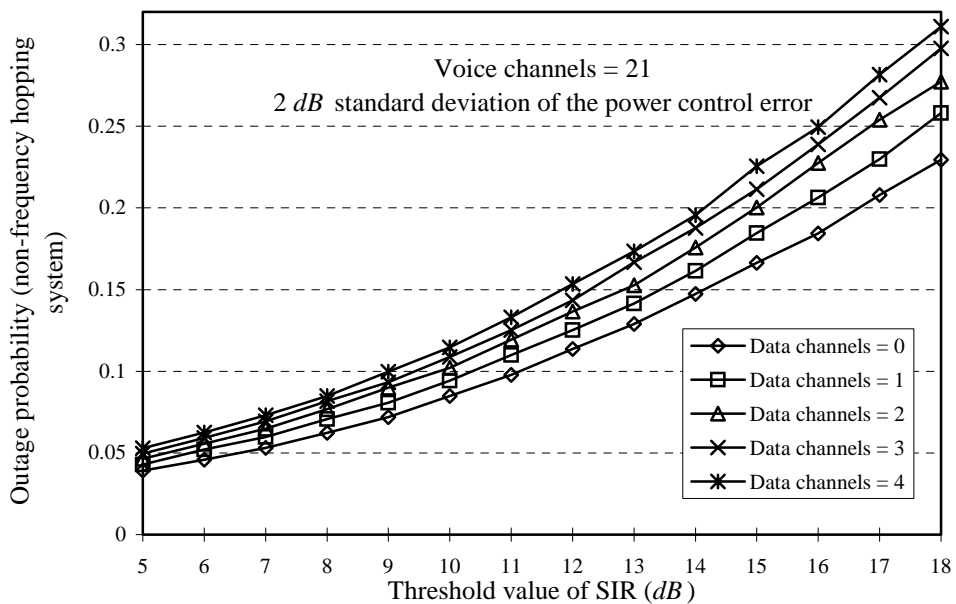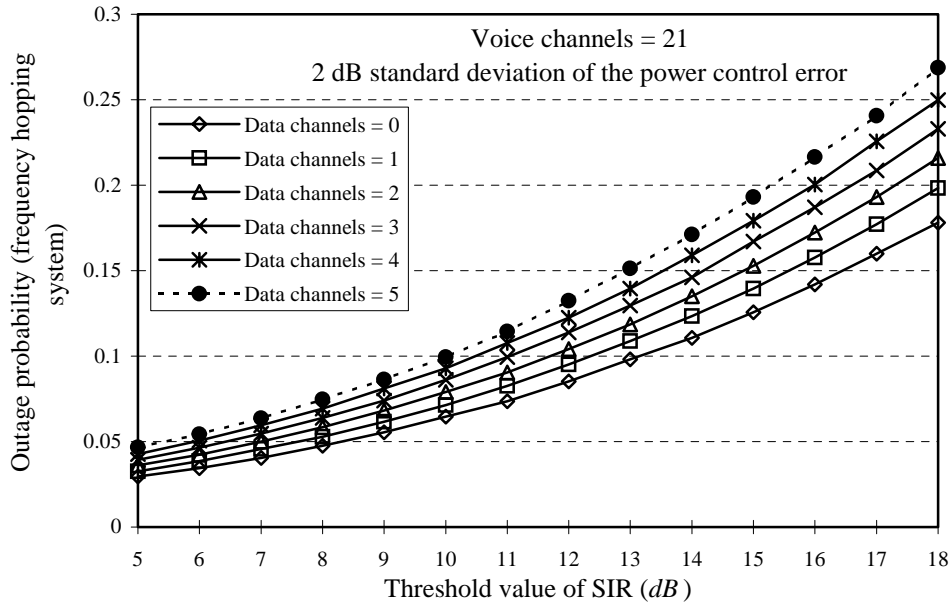

Figure 1b. Outage probability (90% worse case) of the non-frequency hopping system vs. SIR and different GPRS channel occupancy (21 channels occupied by voice services, standard deviation of the power control error = 2 *dB*).
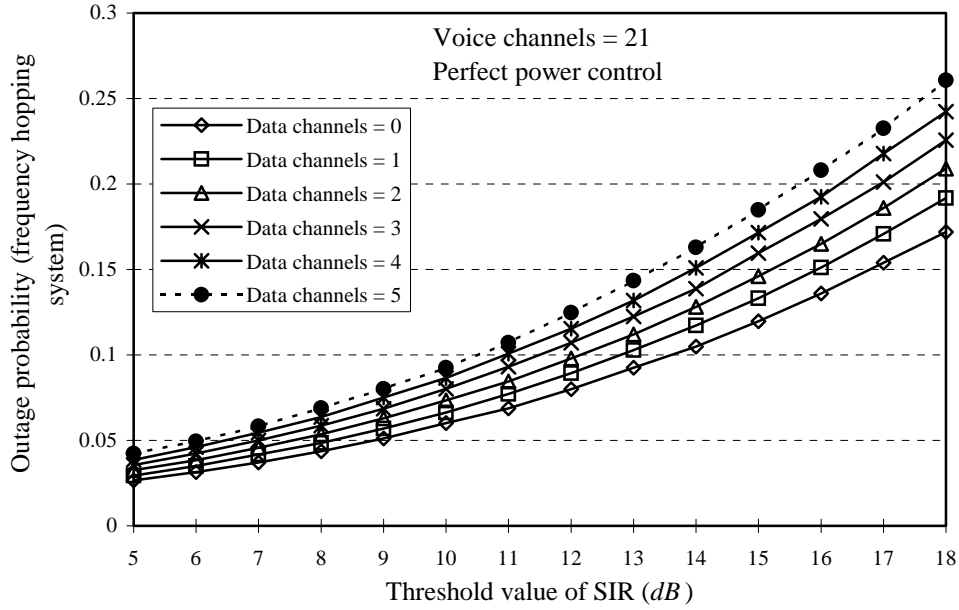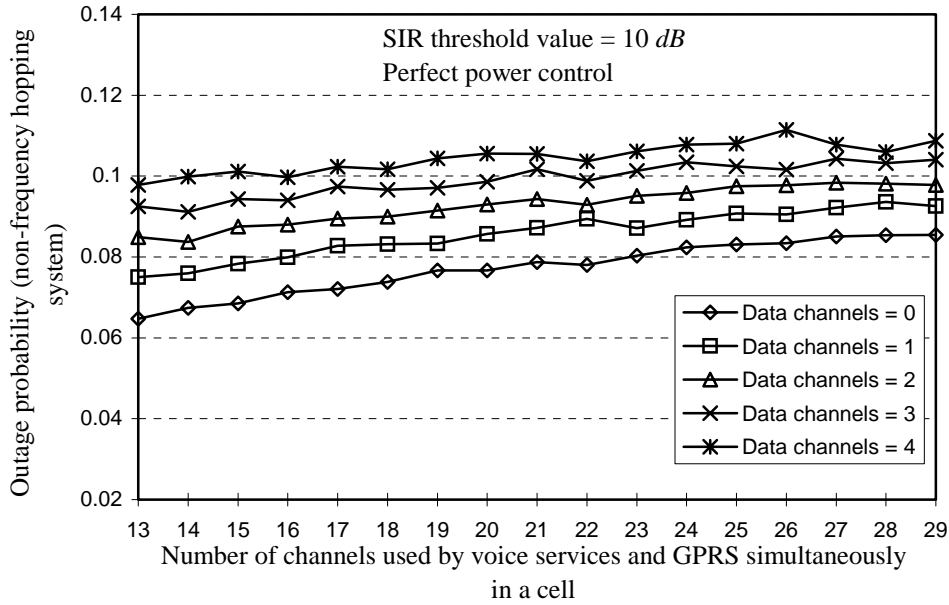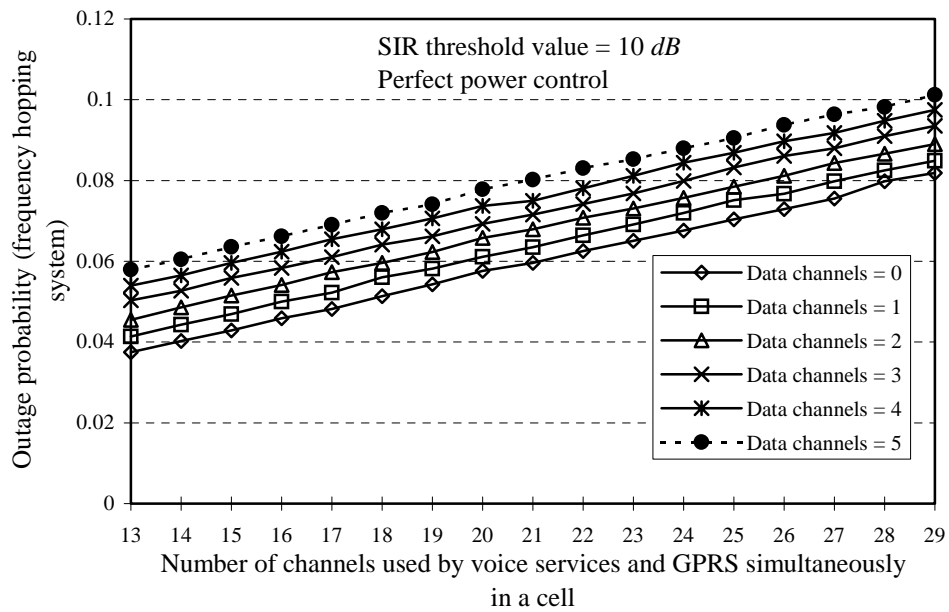
Figure 2a. Outage probability (90% worse case) of the frequency hopping system vs. SIR and different GPRS channel occupancy (21 channels occupied by voice services, perfect power control).
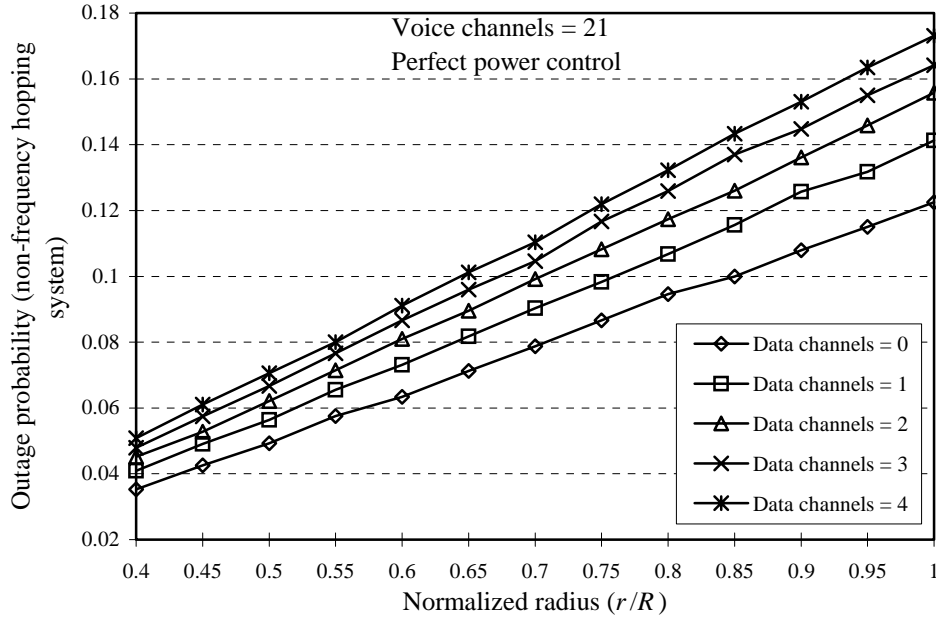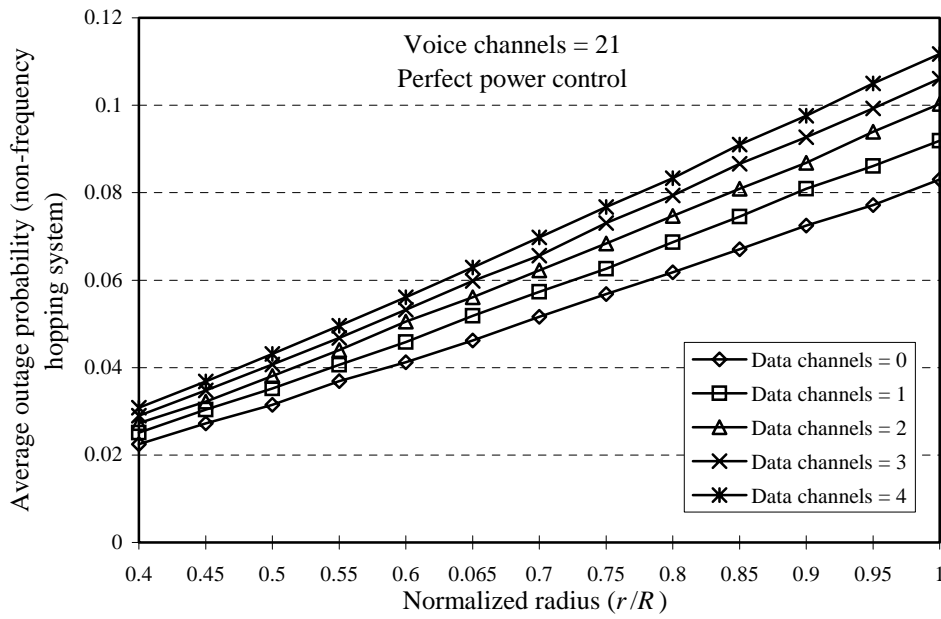

Figure 2b. Outage probability (90% worse case) of the frequency hopping system vs. SIR and different GPRS channel occupancy (21 channels occupied by voice services, standard deviation of the power control error = 2 *dB*).

Figure 3. Outage probability (90% worst case) of the non-frequency hopping system vs. different number of channels occupied by voice services and GPRS (SIR threshold value = 10 *dB*, perfect power control).
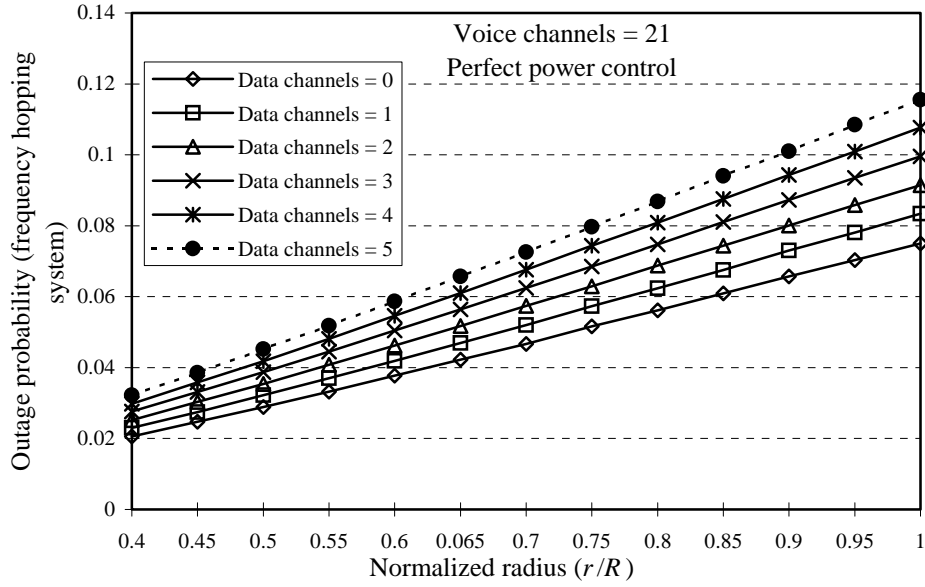

Figure 4. Outage probability (90% worst case) of the frequency hopping system vs. different number of channels occupied by voice services and GPRS (SIR threshold value = 10 *dB*, perfect power control).
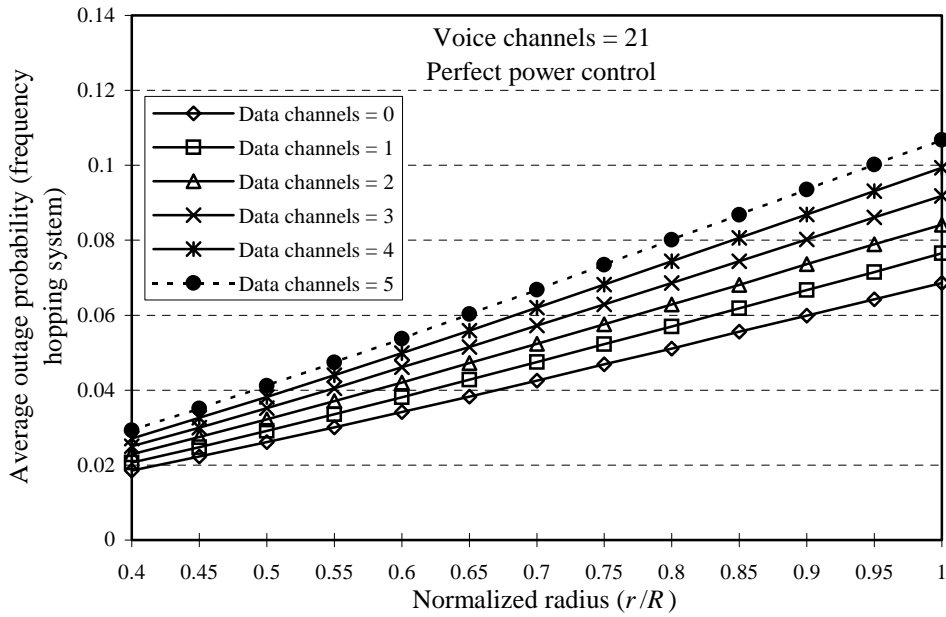
Figure 5a. Outage probability (90% worst case) of the non-frequency hopping system vs. normalized radius (*r/R*).

Figure 5b. Average outage probability of the non-frequency hopping system vs. normalized radius (*r/R*).

Figure 6a. Outage probability (90% worst case) of the frequency hopping system vs. normalized radius (*r/R*).

Figure 6b. Average outage probability of the frequency hopping system vs. normalized radius (*r/R*).