Second-Order Power Control with Asymptotically Fast Convergence

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Abstract

This paper proposes a distributed power control algorithm that uses power levels of both current and previous iterations for power update. The algorithm is developed by applying the *successive overrelaxation method* to the power control problem. The gain from such a *second-order* algorithm is in faster convergence. Convergence analysis of the algorithm in case of *feasible* systems is provided in this paper. Using the *distributed constrained power control* (DCPC) as a reference algorithm, we carried out computational experiments on a DS-CDMA system. The results indicate that our algorithm significantly enhances the convergence speed of power control. A practical version of the proposed algorithm is provided and compared with the *bang-bang* type algorithm used in the IS-95 and the WCDMA systems. The results show that our algorithm also has a high potential for increasing the radio network capacity. Our analysis assumes that the system is feasible in the sense that we can support every active user by an optimal power control. When the system becomes infeasible because of high traffic load, it calls for another actions such as *transmitter removal*, which is beyond the scope of the present paper.

Keywords: Cellular radio system, power control, convergence, successive overrelaxation.

1 Introduction

Effective transmitter power control is essential for high-capacity cellular radio systems. In particular, power control is a direct remedy against the *near-far problem* in a DS-CDMA system. During recent decades, many researchers have investigated power control from different perspectives (see [1] and [2]

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for some of the latest review on power control). Along with "*distributiveness*", convergence speed of power control is one of the most important criteria by which we can determine the practical applicability of a given power control algorithm. A good power control algorithm should quickly and distributively converge to the state where the system supports as many users as possible. Designing such a power control algorithm is the topic of this paper.

Power control in cellular radio systems has drawn much attention since Zander's works on centralized [3] and distributed [4] *CIR balancing.* CIR balancing was further investigated by Grandhi *et al.* [5], [6]. Foschini and Miljanic [7] considered a more general and realistic model in which a positive receiver noise and a respective target CIR were taken into account. The Foschini and Miljanic's distributed algorithm was shown to converge either synchronously [7] or asynchronously [8] to a fixed point of a *feasible* system. Based on the Foschini and Miljanic algorithm, Grandhi *et al.* [9] suggested *distributed constrained power control* (DCPC), in which a transmission upper limit was considered. DCPC has become one of the most widely accepted algorithms for many purposes. Meanwhile, a framework on convergence of the *generalized uplink power control* was provided by Yates [12] and has been recently extended by Huang and Yates [13]. The results in [12] and [13] have become a breakthrough in that they can be applied to algorithms in [1]-[11], providing guidelines for designing and analyzing new algorithms. Huang and Yates have reported that DCPC converges to a fixed point at a geometric rate. It was, however, pointed out that the convergence of DCPC becomes slow as it approaches the fixed point. In some situations, it takes a long time to reach this fixed point. In the worst case, none or only a few of the transmitters are supported before reaching the fixed point, which makes the *transmitter removal* decision difficult [14].

Our algorithm is different from the existing *first-order* power control [1]-[11] in the sense that, for power update, it requires power levels of current and previous iterations. Gain from such a *second-order* algorithm is in faster convergence. Convergence analysis of the algorithm is provided in this paper, and computational experiments have been carried out on a DS-CDMA system. Throughout this paper, we assume that a given system is feasible in the sense that we can support every active user by an optimal power control. When the system becomes infeasible because of high traffic load, it calls for another actions such as *transmitter removal* and *admission control*, which is beyond the scope of the present paper.

The theoretical roots of many power control algorithms, including the one we are proposing here, can be found in *numerical linear algebra*. With this in mind, we will take a look at the convergence properties of power control algorithms in the next section. In particular, we will review the Foschini and Miljanic algorithm. In order to present our algorithm, in Section 3, we start with an algorithm for the relaxed problem that has no constraint on maximum power levels. We develop it to a constrained algorithm in Section 4, where we show that the *constrained second-order power control* (CSOPC) is *asymptotically* faster than DCPC. Numerical comparison between CSOPC and DCPC is contained in Section 5. It indicates that CSOPC significantly enhances power control convergence speed, especially as it approaches the fixed point. In that section, we introduce a practical version of CSOPC and compare it with the "*bang-bang*" type algorithm that is used in IS-95 [15] and considered by WCDMA [16]. The computational results are also quite encouraging. The modified CSOPC has a high potential for increasing the radio network capacity of real CDMA systems. Section 6 concludes the paper with remarks on future research topics of interest.

2 Iterative Method and Its Convergence

2.1 System Model

Suppose a cellular radio system, in which Q mobiles share the same channel at a given instance. As in many other papers, we focus on the so called *snapshot* situation. Without loss of generality, we consider the uplink only and assume that mobile i is assigned to base i at that instant. Further, we assume that the signal of mobile i will be received correctly if the carrier-to-interference-plus-noise ratio (CIR) at base i is not less than a given target value γ_i^t. However, since the ideal situation is to make connection with the minimal transmission power, we have the following CIR constraint on mobile i:

$$\frac{g_{ii}p_i}{\sum_{j\neq i}^{Q} g_{ij}p_j + \nu_i} = \gamma_i^t \tag{1}$$

In the above, p_i denotes the transmission power of mobile i, g_{ij} is the link gain from mobile j to base i, and ν_i is the receiver noise at base i.

Let us define a $Q \times Q$ matrix $H = [h_{ij}]$ such that $h_{ij} = \gamma_i^t g_{ij}/g_{ii}$ for $i \neq j$ and $h_{ij} = 0$ for $i = j$. In addition, let us define a vector $\eta = (\gamma_i^t \nu_i / g_{ii})$ of a length Q. Then, converting (1) into a matrix from, we have the following *power control problem*:

$$AP = \eta, \tag{2}$$

where $A = I - H$ and $P = (p_i)$ is the *power vector* of a length Q. Since the transmission power of a mobile is limited, we will consider the following constraint on the power vector:

$$0 \leq P \leq \bar{P}, \tag{3}$$

where $\bar{P} = (\bar{p}_i)$ denotes the maximum transmission power of each mobile. Throughout this paper, we assume that the system is feasible at the given instant; there exists a unique power vector P^* that solves the problem (2) within the range of (3). Thus, by the feasible system, we mean that the matrix A is nonsingular and $0 \leq P^* = A^{-1}\eta \leq \bar{P}$.

Unfortunately, every element in the matrix A is hardly available in practical systems. Thus, we cannot utilize efficient methods, e.g., *Gaussian elimination* for solving the linear equation system (2). Only those *iterative methods* that can be executed with local measurement and signaling has drawn much attention, and our study is also in the line of this approach. An important question is how quickly we can obtain P^*.

2.2 Iterative Method

Now, consider the following *general iterative method* for solving (2):

$$P^{(n+1)} = M^{-1}NP^{(n)} + M^{-1}\eta, \; n = 0, 1, \ldots \tag{4}$$

where M and N are matrixes of appropriate sizes such that $P^* = M^{-1}NP^* + M^{-1}\eta$. The vector $P^{(n)} = (p_i^{(n)})$ denotes the power level at iteration n. Appropriately selecting M and N, we can make the above iterative method *convergent*, i.e., $\lim_{n\to\infty} P^{(n)} = P^*$.

The power control algorithm proposed by Foschini and Miljanic [7], which is equivalent to the *Jacobian overrelaxation iterative method* (JOR) [19], is a special case of (4), where $M^{-1} = \beta I$ and $N = \frac{1}{\beta}I - A$

$(0 < \beta \leq 1)$. In particular when $\beta = 1$, the Foschini and Miljanic algorithm is reduced to

$$P^{(n+1)} = (I - A)P^{(n)} + \eta = HP^{(n)} + \eta,\ n = 0, 1, \ldots \tag{5}$$

It is easy to see that, for each mobile i, the method becomes

$$p_i^{(n+1)} = \frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)},\ n = 0, 1, \ldots \tag{6}$$

where $\gamma_i^{(n)}$ denotes the received CIR of mobile i at iteration n. For simplicity, hereafter, we will name the Foschini and Miljanic algorithm with $\beta = 1$ as DPC (*Distributed Power Control*). Note that the distributed CIR balancing algorithm in [6] is also called DPC in the literature. However, considering that the two algorithms are similar, we will let DPC denote the Foschini and Miljanic algorithm with $\beta = 1$. DPC is used as the underlying framework for DCPC [9], where the constraint (3) is considered during power update:

$$p_i^{(n+1)} = \min\{\frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)}, \bar{p}_i\},\ n = 0, 1, \ldots \tag{7}$$

Our algorithm is also based on the framework of the general iterative method (4). Different from existing algorithms, however, it has a second-order iterative form given by

$$p_i^{(n+1)} = \min\{\bar{p}_i,\ \max\{0,\ \omega^{(n)} \frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)} + (1 - \omega^{(n)}) p_i^{(n-1)}\}\},\ n = 1, 2, \ldots \tag{8}$$

where $p_i^{(0)}$ and $p_i^{(1)}$ are arbitrarily chosen from the range of (3). The value $\omega^{(n)}$ is a non-increasing sequence of control parameters satisfying $1 < \omega^{(1)} < 2$, $\omega^{(1)} = \omega^{(2)} < \omega^{(3)} = \omega^{(4)} < \ldots < \omega^{(2n)} = \omega^{(2n+1)} < \ldots$, and $\lim_{n \to \infty} \omega^{(n)} = 1$. The algorithm can be interpreted as follows: The power update form within the operator "max" of (8) can be rewritten to $\frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)} + (\omega^{(n)} - 1)(\frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)} - p_i^{(n-1)})$. In calculating $p_i^{(n+1)}$, the algorithm first compares the result from DPC, $\frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)}$ with the previous power value $p_i^{(n-1)}$. If $\frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)} > p_i^{(n-1)}$, the algorithm tries to choose a power value $p_i^{(n+1)}$ such that $p_i^{(n+1)} > \frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)}$. The gap between $p_i^{(n+1)}$ and $\frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)}$ becomes smaller as either $\omega^{(n)} - 1$ or $\frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)} - p_i^{(n-1)}$ is approaching zero. With the same reasoning, we can see that $p_i^{(n+1)} < \frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)}$ if $\frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)} < p_i^{(n-1)}$. When $\frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)} = p_i^{(n-1)}$, it is easy to see $p_i^{(n+1)} = \frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)}$ (same as DPC). Of course, $p_i^{(n+1)}$ will be adjusted if it is out of the range of the transmit powers given by (3).

2.3 Convergence of the Iterative Method

In the general iterative method (4), let $\alpha_1, \alpha_2, \ldots$ be the eigenvalues of the *iteration matrix*, $M^{-1}N$ and define $\rho(M^{-1}N) = \max_k |\alpha_k|$. The constant $\rho(M^{-1}N)$ is called the *spectral radius* of $M^{-1}N$. We define the *error vector* $\epsilon^{(n)}$ as

$$\epsilon^{(n)} = P^{(n)} - P^*,\ n = 0, 1, \ldots \tag{9}$$

From (4), we can express the error vector $\epsilon^{(n)}$ as

$$\epsilon^{(n)} = M^{-1}N \cdot \epsilon^{(n-1)} = \ldots = (M^{-1}N)^n \cdot \epsilon^{(0)} \tag{10}$$

It is well known that the error vector $\epsilon^{(n)}$ of a feasible system tends to the zero vector for any $\epsilon^{(0)}$ if and only if the spectral radius $\rho(M^{-1}N)$ is less than one [18], [19]. Foschini and Miljanic showed, regarding their algorithm, that $\rho(M^{-1}N) = \rho(I - \beta A) < 1$ for any feasible system if and only if the parameter β belongs to the half open interval $(0, 1]$.

For discussing the convergence rate, let $||\ \ ||$ denote the Euclidean norm. From (10), we then have

$$||\epsilon^{(n)}|| = ||(M^{-1}N)^n \cdot \epsilon^{(0)}|| \le ||(M^{-1}N)^n|| \cdot ||\epsilon^{(0)}||,\ n = 0, 1, \ldots \tag{11}$$

The *asymptotic average rate of convergence* (Theorem 3.2 in [18] and Corollary 3-7.3 in [19]) of the general iterative method (4) is defined by

$$\lim_{n\to\infty} ||(M^{-1}N)^n||^{\frac{1}{n}} = \rho(M^{-1}N) < 1 \tag{12}$$

For a given number $\delta > 0$, let $n(\delta)$ denote the minimum number of iterations that give $||\epsilon^{(n)}|| \le \delta \cdot ||\epsilon^{(0)}||$. Then, as δ approaches zero, $n(\delta)$ satisfies (Equation 3-7.17 in [19]):

$$n(\delta) = \frac{\log \delta}{\log \rho(M^{-1}N)} \tag{13}$$

According to (13), we can say that an iterative method with a smaller spectral radius, fulfilling $\rho(M^{-1}N) < 1$, converges to P^*, asymptotically faster.

Foschini and Miljanic claimed that for their algorithm, DPC ($\beta = 1$) is the best universal choice in terms of convergence speed. In fact, the spectral radius of the iteration matrix of DPC becomes $\rho(H)$. When the given system is feasible, it is known that $\rho(H) < 1$ (Theorem 3.11 in [18]).

3 Unconstrained Second-Order Power Control

In this section, we develop a distributed algorithm that outperforms DPC in convergence speed. For the purpose, let us consider the general iterative method (4) with the matrixes M and N defined by

$$M = \frac{1}{\omega}(I - \omega L), \quad N = \frac{1}{\omega}((1-\omega)I + \omega U), \tag{14}$$

where ω is a given number, and L and U are strict lower and upper triangular parts of H respectively. The iterative method with such matrixes is known as the *successive overrelaxation iterative method* (SOR), in which the number ω is called the *relaxation factor* [18], [19].

With SOR, appropriately choosing ω, we can solve the power control problem faster than the Foschini and Miljanic algorithm which can be viewed as a Jacobian overrelaxation iterative method (JOR). However, applying SOR directly to the power control problem (2) does not lend itself to a fully distributed power control algorithm. The reason is that it will result in a "*round-robin*" power update, which may require a *center* for scheduling. Furthermore, one iteration of the round-robin update would take as long as it takes for DPC to make Q iterations, and thus its performance is expected to be poor. To cope with these drawbacks, we now define the *mirror vectors* P_1 and P_2 of the original power vector P, and consider the following *augmented power control problem*:

$$A'P' = \eta', \tag{15}$$

where $A' = \begin{bmatrix} I & -H \\ -H & I \end{bmatrix}$, $P' = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$ and $\eta' = \begin{bmatrix} \eta \\ \eta \end{bmatrix}$. The unique solution of the augmented problem is $P_1 = P_2 = P^*$.

Let us define $H' = I - A'$ and apply SOR to the augmented problem. When applying SOR, instead of H, we use H' for constructing L and U. That is, $L = \begin{bmatrix} 0 & 0 \\ H & 0 \end{bmatrix}$ and $U = \begin{bmatrix} 0 & H \\ 0 & 0 \end{bmatrix}$. Then from (4) and (14), we have the following iterative matrix equation:

$$\begin{bmatrix} P_1^{(n+1)} \\ P_2^{(n+1)} \end{bmatrix} = \omega \begin{bmatrix} I & 0 \\ -\omega H & I \end{bmatrix}^{-1} \left(\frac{1}{\omega} \begin{bmatrix} (1-\omega)I & \omega H \\ 0 & (1-\omega)I \end{bmatrix} \begin{bmatrix} P_1^{(n)} \\ P_2^{(n)} \end{bmatrix} + \begin{bmatrix} \eta \\ \eta \end{bmatrix} \right) \tag{16}$$

$$= \begin{bmatrix} (1-\omega)I & \omega H \\ \omega(1-\omega)H & \omega^2 H^2 + (1-\omega)I \end{bmatrix} \begin{bmatrix} P_1^{(n)} \\ P_2^{(n)} \end{bmatrix} + \begin{bmatrix} \omega\eta \\ \omega(\omega H + I)\eta \end{bmatrix} \tag{17}$$

$$= \begin{bmatrix} (1-\omega)P_1^{(n)} + \omega H P_2^{(n)} + \omega\eta \\ \omega H((1-\omega)P_1^{(n)} + \omega H P_2^{(n)} + \omega\eta) + (1-\omega)P_2^{(n)} + \omega\eta \end{bmatrix} \tag{18}$$

l			n
0	$\begin{bmatrix} P_1^{(0)} \\ P_2^{(0)} \end{bmatrix}$	$\begin{matrix} \to P^{(0)} \\ \to P^{(1)} \end{matrix}$	$\begin{matrix} 0 \\ 1 \end{matrix}$
1	$\begin{bmatrix} P_1^{(1)} \\ P_2^{(1)} \end{bmatrix}$	$\begin{matrix} \to P^{(2)} \\ \to P^{(3)} \end{matrix}$	$\begin{matrix} 2 \\ 3 \end{matrix}$
$\vdots$	$\vdots$		$\vdots$

Figure 1: Representation of $\{P_1^{(l)}\}$ and $\{P_2^{(l)}\}$ by $\{P^{(n)}\}$.

$$= \begin{bmatrix} \omega(HP_2^{(n)} + \eta - P_1^{(n)}) + P_1^{(n)} \\ \omega(HP_1^{(n+1)} + \eta - P_2^{(n)}) + P_2^{(n)} \end{bmatrix} \tag{19}$$

The iterative method (19) is interpreted in the following manner. As both sequences of vectors $\{P_1^{(n)}\}$ and $\{P_2^{(n)}\}$ converge to P^* for an appropriately chosen ω, we define a new sequence of vectors $\{P^{(n)}\}$ by means of

$$P^{(2l)} = P_1^{(l)}, \quad P^{(2l+1)} = P_2^{(l)}, \; l = 0, 1, \ldots \tag{20}$$

In terms of the vector $P^{(n)}$, we can rewrite (19) in the simpler form

$$P^{(n+1)} = \omega(HP^{(n)} + \eta - P^{(n-1)}) + P^{(n-1)}, \; n = 1, 2, \ldots \tag{21}$$

where $P^{(0)} = P_1^{(0)}$ and $P^{(1)} = P_2^{(0)}$ (see also Figure 1). Similar to DPC in (6), for each mobile i, the above iterative method becomes

$$p_i^{(n+1)} = \omega \frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)} + (1-\omega) p_i^{(n-1)}, \; n = 1, 2, \ldots \tag{22}$$

where $p_i^{(0)}$ and $p_i^{(1)}$ are arbitrary. Hereafter, we will call the algorithm (22) *unconstrained second-order power control* (USOPC). In particular when $\omega = 1$, USOPC is reduced to DPC.

In (16), let us denote the iteration matrix, $M^{-1}N = \begin{bmatrix} I & 0 \\ -\omega H & I \end{bmatrix}^{-1} \begin{bmatrix} (1-\omega)I & \omega H \\ 0 & (1-\omega)I \end{bmatrix}$ by ζ_ω for a given ω. When $\omega = 1$, we can see from (19) that ζ_ω corresponds to the DPC iteration matrix for the case where two DPC power updates are performed at each iteration. Therefore, we have $\rho(\zeta_1) = \rho(H)^2$. The question is under what range of ω does USOPC converge to the P^* of a feasible system asymptotically faster than DPC, i.e., $\rho(\zeta_\omega) < \rho(\zeta_1) = \rho(H)^2 < 1$. The rest of this section gives an answer to this.

Proposition 1. (Theorems 3.5 and 3.16 [18]) *If the diagonal entries of the nonnegative matrix H' are zero* (which is obvious in our problem),

i) $\rho(\zeta_w) \geq |\omega - 1|$

ii) $0 < \rho(\zeta_{\omega_1}) < \rho(\zeta_{\omega_2}) < 1$, for all $0 < \omega_2 < \omega_1 \leq 1$

Since we are interested in the optimum relaxation factor ω^* that minimizes $\rho(\zeta_\omega)$ and satisfies $\rho(\zeta_{\omega^*}) < 1$, from Proposition 1, we have the following optimization problem:

$$\min_{\omega} \rho(\zeta_\omega) = \min_{0<\omega<2} \rho(\zeta_\omega) = \min_{1\leq\omega<2} \rho(\zeta_\omega) \tag{23}$$

Because, of all $0 < \omega \leq 1$, $\omega = 1$ gives the best convergence rate (from Proposition 1-ii), we end up with the optimization over $1 \leq \omega < 2$ in the above. However, with the help of the following property, we can have the optimization over $1 < \omega < 2$.

Proposition 2. *If the system is feasible, then in USOPC, the optimal relaxation factor* ω^* *is greater than one, and thus* $\min_\omega \rho(\zeta_\omega) = \min_{1<\omega<2} \rho(\zeta_\omega)$.

Proof. Let us denote the complex valued eigenvalue $\lambda_i' = a_i + b_i \cdot i$ of H' as a point (a, b) in the two dimensional space. Assume $E(x, y)$ denotes the smallest-volume ellipse that contains, in the closed interior, all the pairs (a, b) corresponding to the eigenvalues of H'. Such an ellipse is given by

$$E(x, y) = (\frac{x}{\bar{a}})^2 + (\frac{y}{\bar{b}})^2 = 1, \tag{24}$$

where $\bar{a}$ and $\bar{b}$ are positive numbers. It is then derived in Chapter 6.4 of [19] that, for the class of matrices that A' belongs to (consistently ordered with non-vanishing diagonal elements and complex eigenvalues), the optimal relaxation factor ω^* is

$$\omega^* = \frac{2}{1 + \sqrt{1 + \bar{b}^2 - \bar{a}^2}} \tag{25}$$

From the structure of H', it is clear that if E is an eigenvector of H, then $E' = \begin{bmatrix} E \\ E \end{bmatrix}$ is an eigenvector of H'. Thus all eigenvalues of H are also eigenvalues of H' with multiplicity two. Since the system is feasible, Theorem 3.9 in [18] guarantee that H is an irreducible nonnegative matrix and $\rho(H) = \rho(H') < 1$. In addition, the Perron-Frobenius theorem [18], [19] implies that the number $\bar{a}$ in (24) and (25) is the dominant real eigenvalue of H and thereby dominant eigenvalue of H'. Therefore, $\bar{b} < \bar{a} = \rho(H) = \rho(H') < 1$, and it follows from (25) that $1 < \omega^* < 2$. $\square$

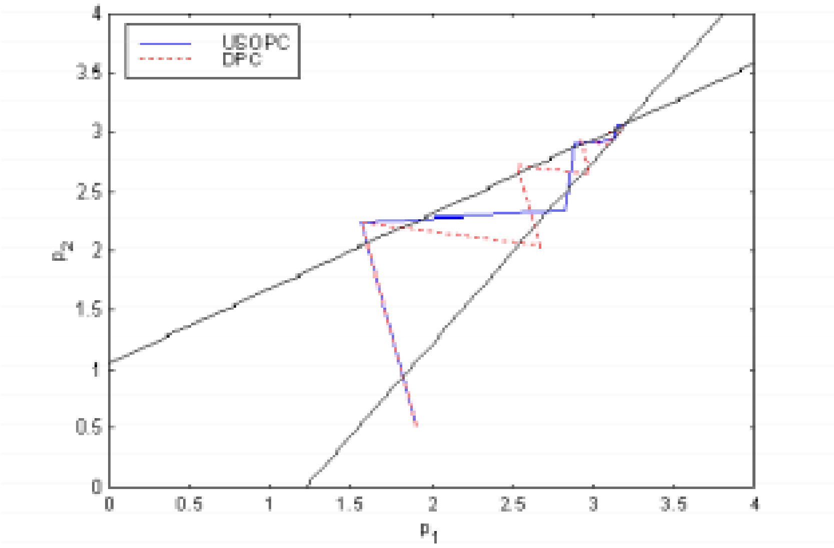

Figure 2: Traces of USOPC and DPC.

From Proposition 1-ii, USOPC converges to P^* if ω belongs to the open interval $(0, 1)$. It will be asymptotically faster as ω approaches one, but still slower than DPC ($\rho(\zeta_\omega) > \rho(\zeta_1)$). However, from Proposition 2, by selecting optimal ω^* from $(1, 2)$, we can make USOPC asymptotically faster than DPC. We will explain this by the following example.

Example 1. Consider a feasible system, where two mobiles use the same channel at the given instant. The link gain matrix is given by

$$G = \begin{bmatrix} 0.3288 & 0.0534 \\ 0.0602 & 0.3826 \end{bmatrix} \tag{26}$$

The receiver noise level is 0.1 and the target CIR is 6 dB. We then have the normalized link gain matrix given by

$$H = \begin{bmatrix} 0 & 0.6466 \\ 0.6264 & 0 \end{bmatrix} \tag{27}$$

The fixed point is known to be $P^* = (3.1657, 3.0235)$.

We will now apply both USOPC and DPC to the problem to illustrate difference in convergence. It is

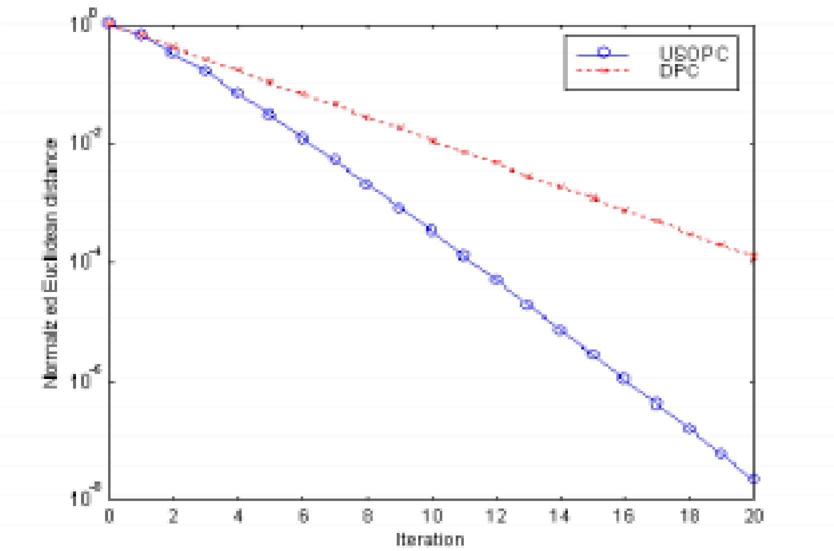

Figure 3: Normalized Euclidean distance between the current power vector and P^* as a function of iteration ($||\epsilon^{(n)}||/||\epsilon^{(0)}||$).

easy to obtain $\rho(H) = \rho(H') = 0.6364$, and from (25), we have the optimal relaxation factor $\omega^* = 1.1291$ for USOPC ($\bar{a} = 0.6364$ and $\bar{b} = 0$). A randomly chosen initial power vector $P^{(0)} = (1.8772, 0.5211)$ is used for both algorithms. In USOPC, the power vector $P^{(1)}$ is obtained by a one-step iteration of DPC on $P^{(0)}$. Figure 2 shows the traces of the two algorithms, where we can see that both algorithms converge to P^*. Figure 3 shows the Euclidean distance between the current power vector and P^*, normalized by $||\epsilon^{(0)}|| = ||P^{(0)} - P^*||$. It is clear that USOPC converges faster. The speed difference becomes bigger as USOPC approaches P^*.

Although USOPC is faster than DPC, applying USOPC to practical problems seems to be hopeless because of the following reasons. First, it is difficult to find ω^* since the matrix H (and thus H') is not available in real situations. Second, we have not considered the transmission range described in (3). Even if USOPC converges to P^* of a feasible system, it can generate power vectors that are out of the range (3), during power update. Furthermore, it is even possible that $p_i^{(n)} < 0$ at a certain iteration n.

In the next section, we show how to cope with such difficulties.

Remark 1. We could construct fourth-, sixth-,... order power control algorithms in the similar manner to that above, but unfortunately they would have the same performance as the second-order algorithm. The reason for this is that all even-order iteration matrixes have the same eigenvalues of different multiplicity. The odd-power control algorithms do not lend themselves to distributed implementation.

4 Constrained Second-Order Power Control

As the relaxation factor, we now consider a non-stationary number $\omega^{(n)}$ that varies according to the iteration. For this, we suggest the use of a non-increasing sequence $\omega^{(n)}$ that satisfies $\omega^{(1)} > 1$, $\omega^{(1)} = \omega^{(2)} < \omega^{(3)} = \omega^{(4)} < \ldots < \omega^{(2n)} = \omega^{(2n+1)} < \ldots$, and $\lim_{n\to\infty} \omega^{(n)} = 1$. As a result, considering the constraint (3), we confine USOPC to the following version, called CSOPC:

$$p_i^{(n+1)} = \min\{\bar{p}_i,\ \max\{0,\ \omega^{(n)} \frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)} + (1-\omega^{(n)}) p_i^{(n-1)}\}\},\ n = 1, 2, \ldots \tag{28}$$

where $p_i^{(0)}$ and $p_i^{(1)}$ are arbitrarily chosen from the range of (3).

Proposition 3. *If the system is feasible, CSOPC converges to P^*.*

Proof. Let us *sample* the sequence $\omega^{(n)}$ in (28) by taking every second element of it. Then the *sampled sequence* becomes a decreasing one. Now, we redefine $\omega^{(n)}$ to denote the elements of the sampled sequence so that the first element of the sampled sequence has index zero. With help of our sampled sequence, we can rewrite $\omega^{(n)} \frac{\gamma_i^t}{\gamma_i^{(n)}} p_i^{(n)} + (1-\omega^{(n)}) p_i^{(n-1)}$ of (28) into a matrix form as in (16):

$$P'^{(n+1)} = \zeta_{\omega^{(n)}} P'^{(n)} + \xi_{\omega^{(n)}},\ n = 0, 1, \ldots \tag{29}$$

where $\xi_{\omega^{(n)}} = \omega^{(n)}(I - \omega^{(n)} L)^{-1} \eta'$. For a given nonsingular matrix W with an appropriate size, define a vector norm as

$$||V||_\infty^W = ||W \cdot V||_\infty = \max_i |\sum_j w_{ij} v_j| \tag{30}$$

and the consistent matrix norm as

$$||M||_\infty^W = ||W \cdot M \cdot W^{-1}||_\infty \tag{31}$$

Let

$$T(P'^{(n)}) = \min\{\bar{P},\ \max\{0, \zeta_{\omega^{(n)}} P'^{(n)} + \xi_{\omega^{(n)}}\}\} \tag{32}$$

and $P'^* = \begin{bmatrix} P^* \\ P^* \end{bmatrix}$ be the solution to the problem (15). Then, it is clear $P'^* = \zeta_{\omega^{(n)}} P'^* + \xi_{\omega^{(n)}}$ for any n. For a given positive definite diagonal matrix W, we have

$$||T(P'^{(n)}) - P'^*||_\infty^W \leq ||\zeta_{\omega^{(n)}}(P'^{(n)} - P'^*)||_\infty^W \leq ||\zeta_{\omega^{(n)}}||_\infty^W \cdot ||P'^{(n)} - P'^*||_\infty^W \tag{33}$$

Repeating the iteration (33), we get

$$||T(P'^{(n)}) - P'^*||_\infty^W \leq (\prod_{k=0}^{n} ||\zeta_{\omega^{(k)}}||_\infty^W) \cdot ||P'^{(0)} - P'^*||_\infty^W \tag{34}$$

By Lemma 1 (Appendix), we can choose a positive definite diagonal matrix W and a number $\hat{\omega} > 1$ such that $||\zeta_\omega||_\infty^W < 1$ for $1 < \omega < \hat{\omega}$. Since the sampled sequence $\omega^{(n)}$ is a decreasing sequence and $\lim_{n\to\infty} \omega^{(n)} = 1$, it is guaranteed that there exists a number N such that $1 < \omega^{(n)} < \hat{\omega}$ for all $n > N$. Therefore,

$$\lim_{n\to\infty} \prod_{k=0}^{n} ||\zeta_{\omega^{(k)}}||_\infty^W = 0 \tag{35}$$

Thus the power vector P' converges to P'^*. □

Remark 2. From inequality (34), we can see that if the initial value $\omega^{(0)}$ of the sampled sequence is unnecessarily big, then the number of iterations needed to decrease $||\zeta_{\omega^{(k)}}||_\infty^W$ below one is going to be large. By $||\zeta_{\omega^{(n)}}||_\infty^W \geq \rho(\zeta_{\omega^{(n)}})$ (Theorem 2-3.4 in [19]) and $\rho(\zeta_{\omega^{(n)}}) \geq |\omega^{(n)} - 1|$ (Proposition 1-i), it is clear that we do not gain anything by choosing $\omega^{(0)} \geq 2$ of the sampled sequence. Therefore, it is recommendable to choose the initial value $\omega^{(1)}$ of (28) from the open interval $(1, 2)$.

We would now like to compare CSOPC with DCPC in terms of convergence speed.

Proposition 4. *If the system is feasible, CSOPC is asymptotically faster than DCPC.*

Proof. By Proposition 3, it is clear that CSOPC converges and thus there exists a number N such that $0 < T(P'^{(n)}) < \bar{P}$ for all $n > N$. Therefore, for any nonsingular matrix W, we have

$$||T(P'^{(n)}) - P'^*||_\infty^W \leq ||\zeta_{\omega^{(n)}}||_\infty^W \cdot ||P'^{(n)} - P'^*||_\infty^W, \text{ for all } n > N \tag{36}$$

Repeating the iteration (36), we get

$$\frac{||T(P'^{(n)}) - P'^{*}||_{\infty}^{W}}{||P'^{(0)} - P'^{*}||_{\infty}^{W}} \leq K \prod_{k=N+1}^{n} ||\zeta_{\omega^{(k)}}||_{\infty}^{W}, \text{ for all } n > N \tag{37}$$

where K is a positive constant. We will call the right hand side of (37) the *error reduction rate*. Consider now the ratio of the error reduction rates of CSOPC and DCPC described by

$$\kappa_n = K' \frac{\prod_{k=N+1}^{n} ||\zeta_{\omega^{(k)}}||_{\infty}^{W}}{(||\zeta_1||_{\infty}^{W})^{n-N}}, \text{ for all } n > N \tag{38}$$

where K' is a positive constant. By Lemma 2 (Appendix), we can choose a nonsingular matrix W and a number $\hat{\omega} > 1$ such that $||\zeta_{\omega}||_{\infty}^{W} < ||\zeta_1||_{\infty}^{W} < 1$ for $1 < \omega < \hat{\omega}$. Since the sampled sequence $\omega^{(n)}$ is a decreasing sequence and $\lim_{n\to\infty} \omega^{(n)} = 1$, it is guaranteed that there exists a number $N_1 > N$ such that $1 < \omega^{(n)} < \hat{\omega}$ for all $n > N_1$. Therefore, $\lim_{n\to\infty} \kappa^{(n)} = 0$, and we can conclude that CSOPC converges asymptotically faster than DCPC. $\square$

Remark 3. Like the asymptotic average rate of convergence defined in (12), the error reduction rate in (37) can be used for estimating the minimum number of iterations, $n'(\delta)$ that gives

$$K \prod_{k=N+1}^{n'(\delta)} ||\zeta_{\omega^{(k)}}||_{\infty}^{W} = \delta \tag{39}$$

In the case of DPC, $N = 0$ and $K = ||\zeta_1||_{\infty}^{W}$. Let us choose a positive definite diagonal matrix W such that the diagonal entries of W^{-1} are the components of the eigenvector of ζ_1 that corresponds to the dominant eigenvalue. Then, $||\zeta_1||_{\infty}^{W} = \rho(\zeta_1)$ and the equation (39) can be solved analytically. By noting that $\rho(\zeta_1) = \rho(H)^2$ and $n'(\delta) = 2n(\delta)$, we get the same result as in (13).

5 Computational Experiments

First, we investigate how quickly CSOPC converges to P^* of a feasible system. DCPC is used as a reference algorithm. The DS-CDMA system with 19 omni-bases located in the centers of 19 hexagonal cells is used as a test system (Figure 4). We consider an IS-95 example, where the spreading bandwidth is 1.2288 MHz and the data rate is 9.6 Kbps (*processing gain* = 21dB). For a given instance, a total of 190 mobiles are generated, the locations of which are uniformly distributed over the 19 hexagonal cells.

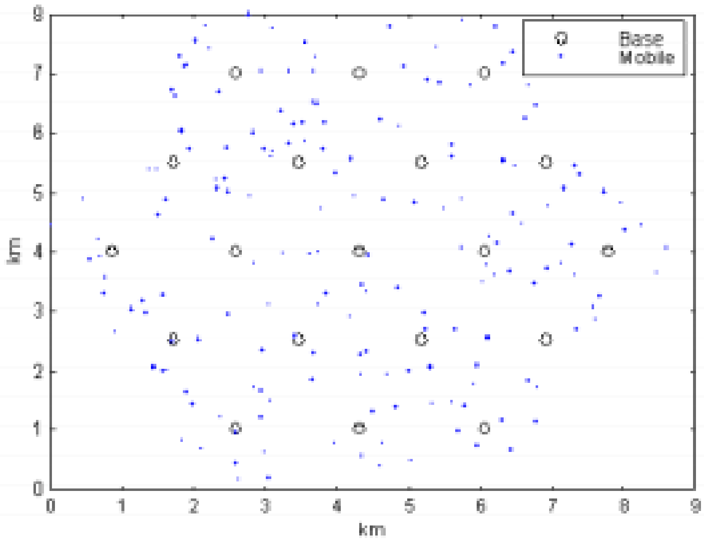

Figure 4: DS-CDMA cellular system with 19 omni-bases.

The link gain g_{ij} is modeled as $g_{ij} = s_{ij} \cdot d_{ij}^{-4}$, where s_{ij} is the shadow fading factor and d_{ij} is the distance between base i and mobile j. The log-normally distributed s_{ij} is generated according to the model in [17] (pp. 185-186, $E(s_{ij}) = 0$ dB, $\sigma(s_{ij}) = 8$ dB).

The base receiver noise is taken to be 10^{-12}. The relative maximum mobile power is set to one. The base that gives the lowest attenuation is assigned to each mobile. The received E_b/I_0 from mobile i at the corresponding base is calculated as follows:

$$\left(\frac{E_b}{I_0}\right)_i = \frac{g_{ii}p_i/9.6 \cdot 10^3}{\left(\sum_{j\neq i} g_{ij}p_j + 10^{-12}\right)/1.2288 \cdot 10^6} \tag{40}$$

The target E_b/I_0 is set to 8 dB for each mobile.

The *outage probability* is used as a performance measure. To evaluate this, we have taken 10000 independent "feasible" instances of mobile locations and shadow fadings. In each instance, we have performed twenty power control steps. The initial power for each mobile is randomly chosen from the interval [0,1]. Similar to Example 1, in CSOPC, the power vector $P^{(1)}$ is obtained by a one-step iteration

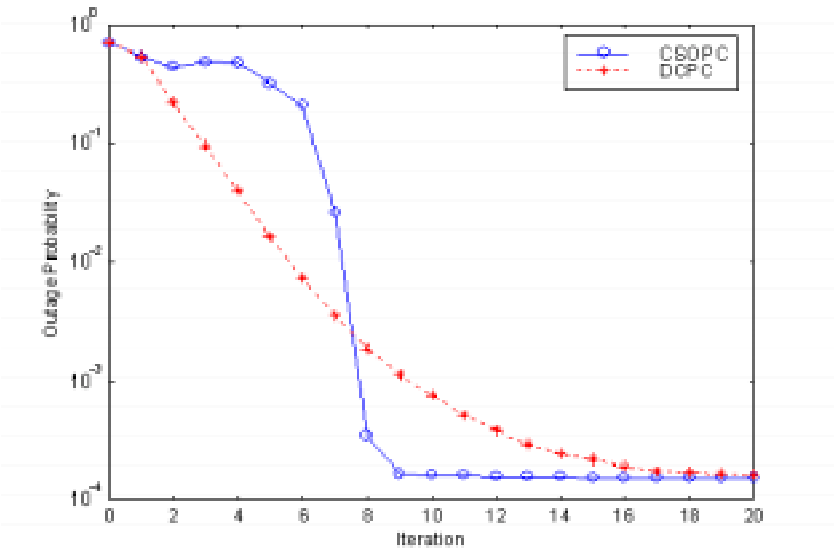

Figure 5: Outage probability as a function of iteration.

of DCPC on $P^{(0)}$. CSOPC uses a non-stationary relaxation factor given by

$$\omega^{(n)} = 1 + \frac{1}{1.5^n}, \; n = 1, 2, \ldots, \tag{41}$$

The outage probability at each iteration is computed over 10000 instances by counting the portion of the number of unsupported mobiles at the iteration. Figure 5 shows the outage probability of each algorithm as a function of iteration. CSOPC takes 9 iterations on average to reach the state with the outage probability of $1.9 \cdot 10^{-4}$ that we consider as almost the zero-outage. We can see that DCPC requires 20 iterations on average to reach the point. At the beginning, DCPC converges faster than CSOPC. However, DCPC becomes slower as approaching the fixed point P^*. In (41), we use the $\omega^{(n)}$ sequence of which property is slightly different from the one defined in (28). However, the results still agree with Propositions 3 and 4; It converges to P^*, asymptotically faster than DCPC. It indicates that Propositions 3 and 4 could be generalized to hold for any decreasing sequence of $\omega^{(n)}$.

In the case of CSOPC, the small increase in the outage level from iteration 2 to 3 is caused by the dynamics of the second-order power control. Consider the scenario where there exists a set of users at

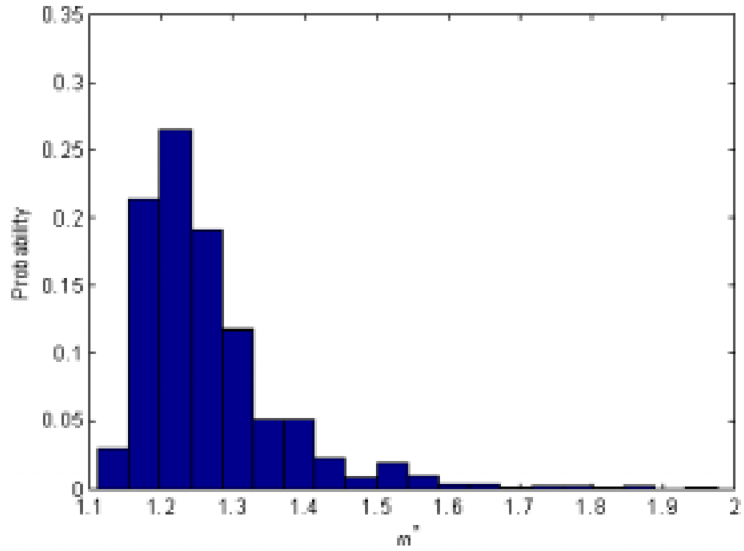

Figure 6: Probability density function of ω^*.

maximum power at iteration 1. The existence of such users is possible because of the random initial power of iteration 0. Then at iteration 2, the powers of the users belonging to that set are decreased because the overall interference level drops. Thus more users are supported and the outage decreases. At iteration 3, CSOPC remembers the maximum power values at iteration 1 and utilizes them for reducing the powers of those users further (see also interpretation of CSOPC in Section 2.2). This leads to a situation in which the powers of those users become unnecessarily low, many of those users can no longer be supported, and the outage probability increases. As iterations go on, the power vector rapidly approaches the fixed point solution and thus the power update steps become smaller and the effects of the overshoots to the outage become negligible.

There are two major restrictions in implementing power control algorithms in real systems. First, a narrow bandwidth is dedicated to power control commands in general. Second, the dynamic range on power up/down is limited in any implementation of transmitters. Therefore in practical systems, there is only a binary command, power up/down. Transmitters adjust their power levels by increasing/decreasing

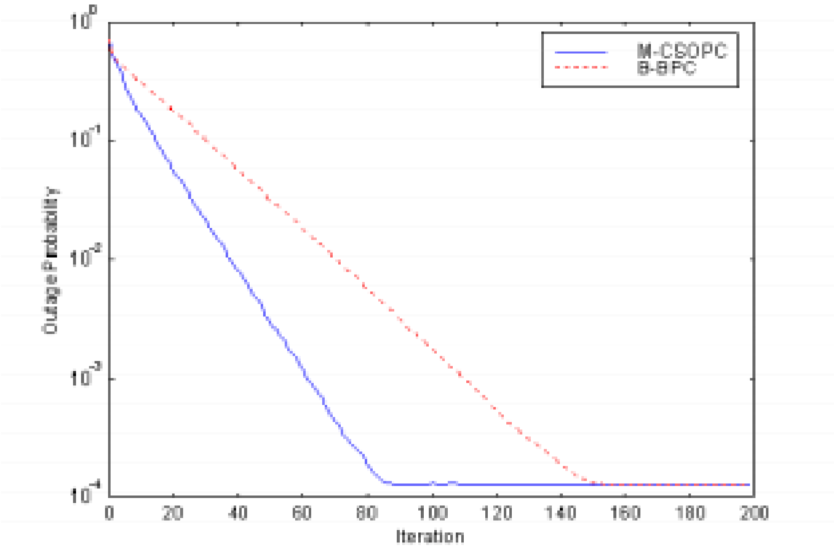

Figure 7: Outage probability as a function of iteration.

a fixed (or possibly a variable) amount according the received binary commands. These restrictions, along with measurement errors and loop delay, contribute performance gap between the theoretical and the practical algorithms. Furthermore, if we would like to implement CSOPC in a real system, we also have to consider that CSOPC resets the sequence $\omega^{(n)}$ when a change has occurred to H (thus H') due to mobile movement. Detecting such a change in any real environment seems to be hard. With these restrictions in mind, we modify CSOPC to a practical version as follows:

$$p_i^{(n+1)} = \min\{\bar{p}_i, \max\{0, \omega\Delta_i^{(n)} p_i^{(n)} + (1-\omega)p_i^{(n-1)}\}\}, \tag{42}$$

where $\Delta_i^{(n)} = \begin{cases} \Delta, & \gamma_i^{(n)} \leq \gamma_i^t \\ \frac{1}{\Delta}, & \gamma_i^{(n)} > \gamma_i^t \end{cases}$. If we choose $\omega = 1$, the above is reduced to the so called "bang-bang" type power control (B-BPC) that is used for uplink power control of the IS-95 system [15] and currently being considered for up- and downlinks by the WCDMA system [16].

We have compared the modified CSOPC (M-CSOPC) with B-BPC, using the same network configuration and assumptions as in Figure 5. To choose ω in M-CSOPC, we first investigate the probability density function of ω^* of (25) through 10000 instances (Figure 6). The probability that ω^* belongs to the interval $[1.156, 1.286]$ is highest. Accordingly, we set ω to 1.2 in the experiment. The target E_b/I_0 and the step size Δ are 8 dB and 0.5 dB, respectively. We run M-CSOPC and B-BPC until the iteration number reaches 200. The outage probability at each iteration is computed over 10000 instances by counting the portion of the number of mobiles whose received E_b/I_0 is less than 7 dB (1 dB margin to the target) at the iteration. Figure 7 shows a quite encouraging result. M-CSOPC converges to the stable state (outage probability of $1.9 \cdot 10^{-4}$) after 90 iterations, whereas B-BPC requires approximately 150 iterations. Furthermore, the number of mobiles below $E_b/I_0 = 7$ dB (represented by outage in the figure) is less than that of B-BPC over the whole the range of iterations. Thus, it is very likely that M-CSOPC increases the radio network capacity.

6 Concluding Remarks

A group of the minimal power assignment algorithms [13] including DCPC, converge to a fixed point of a feasible system with a geometric rate. In some cases, it takes a long time to reach this fixed point, and this process is especially slow when approaching the fixed point. The second-order power control algorithm will resolve such an undesirable phenomenon due to its asymptotically fast convergence. It would be possible to implement a switch in the power control mode between DCPC and CSOPC. For example, we can imagine a hybrid algorithm that starts with DCPC mode and continues with CSOPC mode as the power vector approaches P^*. This algorithm will receive advantages from both DCPC and CSOPC in terms of convergence speed.

From the encouraging computational results, we believe the practical algorithm, M-CSOPC, will significantly improve the network capacity of a real CDMA system, compared with the currently adopted power control scheme. There are, however, some issues requiring further investigation. First, selecting ω for M-CSOPC may need another optimization procedure. Second, M-CSOPC uses one more information $p_i^{(n-1)}$. Because of measurement errors and loop delay, realized power values might differ from the ideal values. Consequently, M-CSOPC uses one more, possibly erroneous value $p_i^{(n-1)}$. This could affect the performance of power control negatively. These problems constitute an interesting future research topic.

Acknowledgements

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Appendix

Lemma 1. *If the system is feasible, there exists a positive definite diagonal matrix W and a number $\hat{\omega} > 1$ such that $||\zeta_\omega||_\infty^W < 1$ for $1 < \omega < \hat{\omega}$.*

Proof. Let $T_{\hat{\omega}} = |\zeta_{\hat{\omega}}|$, where $|\ \ |$ is an element-wise absolute value operator. Since $T_{\hat{\omega}}$ is an irreducible nonnegative matrix, Perron-Frobenius Theorem [18], [19] guarantees that there exists a positive vector $E = (e_i)$ such that $T_{\hat{\omega}} E = \rho(T_{\hat{\omega}})E$. Let us choose a matrix $W = \mathrm{diag}\{1/e_i\}$. Clearly,

$$||\zeta_{\hat{\omega}}||_\infty^W \le ||T_{\hat{\omega}}||_\infty^W = \rho(T_{\hat{\omega}}) \tag{43}$$

By Theorem 4-5.9 in [19], $\rho(T_{\hat{\omega}}) < 1$ if $1 < \hat{\omega} \le \frac{2}{1+\rho(H)^2}$. From the definition of the matrix norm in (31), it is clear that if we decrease the absolute value of any element in the matrix, then the corresponding norm cannot increase and thus $||\zeta_\omega||_\infty^W < 1$ for $1 < \omega < \hat{\omega}$ must hold. This concludes the proof. □

Lemma 2. *If the system is feasible, there exists a nonsingular matrix W and a number $\hat{\omega} > 1$ such that $||\zeta_\omega||_\infty^W < ||\zeta_1||_\infty^W < 1$ for $1 < \omega < \hat{\omega}$.*

Proof. By Theorem 2-3.5 [19], there exits a nonsingular matrix W and a number $\hat{\omega} > 1$ such that

$$||\zeta_{\hat{\omega}}||_\infty^W \le \rho(\zeta_{\hat{\omega}}) + \varepsilon \tag{44}$$

where $\varepsilon > 0$ can be made arbitrarily small. Kahan [20] has proved that $\rho(\zeta_\omega)$ is decreasing for $0 < \omega \le \bar{\omega}$ and $\bar{\omega} > 1$. So, if we choose $\hat{\omega} \le \bar{\omega}$ and a sufficiently small ε, then

$$||\zeta_{\hat{\omega}}||_\infty^W \le \rho(\zeta_{\hat{\omega}}) + \varepsilon < \rho(\zeta_\omega) \le ||\zeta_\omega||_\infty^W, \quad 1 \le \omega < \hat{\omega} \tag{45}$$

If we further choose $\hat{\omega}$ arbitrarily close to one, then the norm $\Delta = ||\zeta_1 - \zeta_{\hat{\omega}}||_\infty^W$ can be made arbitrarily small. Thus for a small enough $\Delta > 0$ and $\varepsilon > 0$

$$||\zeta_1||_\infty^W \le ||\zeta_{\hat{\omega}}||_\infty^W + \Delta < 1 \tag{46}$$

holds. This concludes the proof. □

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