Power Control with Partially Known Link Gain Matrix

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Abstract

In power control, convergence rate is one of the most important criteria that can determine the practical applicability of a given algorithm. The convergence rate of power control is especially important when propagation and traffic conditions are changing rapidly. To track these changes, the power control algorithm must converge quickly. The purpose of this paper is to provide a new theoretic framework such that we can utilize partially known link gain information in improving the convergence speed. For the purpose, *block power control* (BPC) is suggested with its convergence properties. BPC is centralized within each *block* in the sense that it exchanges link gain information *within* the same block. However, it is distributed in a block-wise manner and no information is exchanged *between* different blocks. Depending on availability of link gain information, a block can be any set of users, and can even consist of a single user. Computational experiments are carried out on a DS-CDMA system, illustrating how BPC utilizes available link gain information in increasing the convergence speed of the power control.

Keywords: Cellular radio system, block power control, convergence, link gain matrix.

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1 Introduction

The scarce radio spectrum has been imposing hard limitations on the design of cellular radio systems. Providing wireless communication services with higher capacity as well as better quality necessitates powerful and robust methods for *sharing* the radio spectrum in the most efficient way. All sharing methods in practice introduce *interference* of one sort or another that is proportional to transmitter powers. In particular, when it comes to DS-CDMA systems [1], [2], handling the interference is a critical factor. The *transmitter power control* is a key technique to better balance between received *signal and interference* (SIR), which in turn enables more efficient resource sharing.

During recent decades, many researchers have investigated power control from different perspectives (see [3], [4] for some of the latest reviews on power control). Especially, power control in cellular radio systems has drawn much attention since Zander's works on centralized [5] and distributed [6] *SIR balancing*. SIR balancing was further investigated by Grandhi *et al.* [7], [8]. Foschini and Miljanic [9] considered a more general and realistic model, in which a positive receiver noise and a respective target SIR were taken into account. The Foschini and Miljanic's distributed algorithm was shown to converge either synchronously [9] or asynchronously [10] to a fixed point of a *feasible* system. Based on the Foschini and Miljanic algorithm, Grandhi *et al.* [11] suggested *distributed constrained power control* (DCPC), in which a transmission upper limit was considered. DCPC has become one of the most widely accepted algorithms by the academic community. Meanwhile, a framework on convergence of the *generalized uplink power control* was provided by Yates [12] and has been recently extended by Huang and Yates [13]. The results in [12] and [13] have become a breakthrough, providing guidelines for designing and analyzing new algorithms.

Along with *distributiveness*, the convergence rate is one of the most important criteria by which we can determine the practical applicability of a given power control algorithm. A good algorithm should quickly and distributively converge to the state where the system supports as many users as possible. The convergence rate of power control is especially important when propagation and traffic conditions are changing rapidly. It is expected that future wireless traffic will become much more bursty than today's voice dominated traffic. With bursty traffic, slow algorithms will perhaps not even be able to converge before the data burst ends. To track these changes, the power control algorithm must converge quickly. Huang and Yates have reported that DCPC converges to a fixed point at a geometric rate [13]. It was, however, pointed out that the convergence of DCPC becomes slow as it approaches the fixed point [14]. In some cases, it takes a long time to reach this fixed point, which makes the *transmitter removal* difficult [15], [16]. To cope with such drawbacks, a *second-order power control* algorithm (SOPC) has been recently suggested by Jäntti and Kim [14]. SOPC is different from the existing *first-order* power control [6], [8]-[13] in a way that, for power update, it requires power levels of both current and previous iterations. Gain from SOPC is in faster convergence and thus in increasing the radio network capacity.

The purpose of this paper is in the line of SOPC; designing a power control algorithm with fast convergence. However, we approach the problem from another perspective. If the link conditions (*link gain matrix* [5]) were partially known, how could we incorporate this additional information into the power control in a way that the convergence speed increases? In this paper, we will provide a theoretic framework on power control as an effort to answer this question.

The power control algorithm suggested in this paper, called *block power control* (BPC) is centralized within each *block* in the sense that it exchanges link gain information *within* the same block. However, it is distributed in a block-wise manner and no information is exchanged *between* different blocks. A block can be any set of users, and can even consist of a single user. BPC is "parameterized" so that the reliability of available link gain information and the dynamic range of transmitters can be taken into account. We provide convergence properties of BPC and illustrate by numerical examples how BPC improves the convergence speed. As a reference algorithm, we use DCPC [11] that is fully distributed and does not require any prior knowledge about link gains.

One possible application of our work is the *bunched* radio resource management scheme [17]. The basic assumption of the scheme is that the link gains within a bunch are, at least partially, known. A bunch is generally equivalent to a block of this paper. It may be argued that the bunch concept requires a lot of signaling and is rather impractical. However, we can imagine an example of "natural" bunches where the signaling can be done locally within one base station controller. In the Wideband CDMA system [2], dedicated pilot bits are associated with each traffic channel in both up- and downlinks, supporting the adaptive antennas. This property may enable the efficient estimation of link gains, required in the bunch concept.

In the next section, we provide our system model that will be used throughout this paper. To present BPC in Section 3, we start with an algorithm for the relaxed problem that has no constraint on maximum power levels. Next, we develop it to a constrained algorithm. Numerical comparison between BPC and DCPC is contained in Section 4. Finally, Section 5 concludes the paper.

2 System Model

Suppose a cellular radio system, in which M transmitters are accessing a common frequency channel. Each transmitter communicates with exactly one receiver. For the uplink case, the transmitters are the mobiles and the receivers are their corresponding base stations; and for the downlink case, their roles are reversed. We consider a time instant in which the link gain between every receiver i and every transmitter j is stationary and is given by g_{ij}. Without loss of generality, we will assume that transmitter i is communicating with receiver i. In a DS-CDMA system, many mobiles will communicate with the same base station through the same frequency channel. Thus, in our notation below, receivers i and j in the uplink may denote the same physical one if the transmitters (mobiles) i and j are assigned to the same base station. We will denote the power of transmitter i by p_i. In the uplink case, the value p_i means the transmission power of mobile i. However, in the downlink, it denotes the transmission power dedicated to mobile i by the base station to which mobile i belongs.

We assume that the signal of transmitter i will be received correctly if the carrier-to-interference-plus-noise ratio (CIR) at the receiver i is not less than a given target value γ_i^t. However, since the ideal situation is to make connection with the minimal transmission power, we have the following CIR constraint on transmitter i:

$$\frac{g_{ii}p_i}{\sum_{\substack{j=1\\j\neq i}}^{M} g_{ij}\theta_{ij}p_j + \nu_i} = \gamma_i^t, \quad i = 1, 2, \ldots, M \tag{1}$$

In the above ν_i is the thermal noise at receiver i. The quantity θ_{ij} is the normalized cross-correlation between p_i and p_j at receiver i. For instance, $\theta_{ij} = 1$ for both up- and downlinks in an F/TDMA system. In a DS-CDMA system, we assume $\theta_{ij} = 1$ for the uplink case whereas, in the downlink, $\theta_{ij} \in [0, 1]$ if mobiles i and j are assigned to the same base station; otherwise $\theta_{ij} = 1$.

Let us define an $M \times M$ matrix $\mathbf{H} = [h_{ij}]$ such that $h_{ij} = \gamma_i^t g_{ij}\theta_{ij}/g_{ii}$ for $i \neq j$ and $h_{ij} = 0$ for $i = j$. In addition, let us define a vector $\boldsymbol{\eta} = (\eta_i)$ such that $\eta_i = \gamma_i^t \nu_i / g_{ii}$. Then, converting (1) into a matrix from, we have the following *power control problem*:

$$\mathbf{A}\mathbf{p} = \boldsymbol{\eta}, \tag{2}$$

where $\mathbf{A} = \mathbf{I} - \mathbf{H}$ and $\mathbf{p} = (p_i)$ denotes the *power vector*. Since the power of a transmitter is limited, we will consider the following constraint on the power vector:

$$\mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}}, \tag{3}$$

where $\bar{\mathbf{p}} = (\bar{p}_i)$ denotes the maximum transmission power. Throughout this paper, unless otherwise noted, we assume that the system is *feasible* at the given instant. That is, there exists a unique power vector $\mathbf{p}^*$ that solves the problem (2) within the range of (3).

Let us assume that transmitters are grouped into N *blocks*, $\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_N$. A block can be any set of transmitters, and can even consist of a single transmitter. Detailed discussion about designing the block is contained in Section 3. For notational simplicity, we assume that the first $|\mathcal{B}_1|$ transmitters belong to $\mathcal{B}_1$ and the next $|\mathcal{B}_2|$ transmitters belong to $\mathcal{B}_2$, and so forth. Then, we can represent $\mathbf{H}$ as the following block matrix:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \cdots & \mathbf{H}_{1N} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \cdots & \mathbf{H}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{N1} & \mathbf{H}_{N2} & \cdots & \mathbf{H}_{NN} \end{bmatrix} \tag{4}$$

The submatrix $\mathbf{H}_{ij}$ has the size of $|\mathcal{B}_i| \times |\mathcal{B}_j|$. As the same manner, we can decompose:

$$\boldsymbol{\eta} = (\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \cdots, \boldsymbol{\eta}_N)' \tag{5}$$

$$\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_N)' \tag{6}$$

3 Block Power Control

3.1 Unconstrained Case

Let us consider the following iterative method for solving the power control problem (2):

$$\mathbf{p}(n+1) = \mathbf{M}^{-1}(\mathbf{N}\mathbf{p}(n) + \boldsymbol{\eta}), \tag{7}$$

where $\mathbf{M}$ and $\mathbf{N}$ are matrixes satisfying $\mathbf{A} = \mathbf{M} - \mathbf{N}$. The vector $\mathbf{p}(n)$ denotes the power level at iteration n. Let $\lambda_1, \lambda_2, \ldots$ be the eigenvalues of a given square matrix $\mathbf{B} = [b_{ij}]$, and define $\rho(\mathbf{B}) = \max_k |\lambda_k|$. The constant $\rho(\mathbf{B})$ is called the *spectral radius* of $\mathbf{B}$. It is well known that $\mathbf{p}(n)$ converges into $\mathbf{p}^*$ of a feasible system if and only if the spectral radius $\rho(\mathbf{M}^{-1}\mathbf{N})$ is less than one (Theorem 3.7 in [19]).

Let us consider matrixes $\mathbf{M}$ and $\mathbf{N}$ satisfying $\mathbf{A} = \mathbf{M} - \mathbf{N}$, $\mathbf{M}^{-1} \geq \mathbf{0}$ and $\mathbf{N} \geq \mathbf{0}$. Matrixes $\mathbf{M}$ and $\mathbf{N}$ that fulfill this condition are said to form a *regular splitting* of matrix $\mathbf{A}$.

Proposition 1 *(Theorem 3.13 in [19]) If the matrixes $\mathbf{M}$ and $\mathbf{N}$ form a regular splitting of $\mathbf{A}$ of a feasible system, then $\rho(\mathbf{M}^{-1}\mathbf{N}) < 1$ and thus the iterative method (7) converges to $\mathbf{p}^*$, starting from an arbitrary initial vector $\mathbf{p}(0)$.*

Proposition 2 *(Theorem 3.15 in [19]) Let $\mathbf{A} = \mathbf{M}_1 - \mathbf{N}_1 = \mathbf{M}_2 - \mathbf{N}_2$ be two regular splittings of $\mathbf{A}$ of a feasible system. If $\mathbf{N}_2 \geq \mathbf{N}_1 \geq \mathbf{0}$ (equality excluded), then $\rho(\mathbf{M}_1^{-1}\mathbf{N}_1) < \rho(\mathbf{M}_2^{-1}\mathbf{N}_2) < 1$.*

The *asymptotic average rate of convergence* (Theorem 3.2 in [19]) of the convergent iterative method (7) is defined as

$$R_\infty = \lim_{n\to\infty} -\frac{1}{n} \ln\left(||(\mathbf{M}^{-1}\mathbf{N})^n||_2\right) = -\ln\rho\left(\mathbf{M}^{-1}\mathbf{N}\right) \tag{8}$$

where $||\cdot||_2$ denotes the Euclidean norm. Proposition 2 gives us a hint about how to choose the matrixes $\mathbf{M}$ and $\mathbf{N}$ to obtain fast convergence; the smaller the spectral radius is, the higher (faster) the asymptotic average rate of convergence is.

Let us choose $\mathbf{M} = \boldsymbol{\Omega}^{-1}(\mathbf{I} - \boldsymbol{\Psi} \otimes \mathbf{H})$ and $\mathbf{N} = \boldsymbol{\Omega}^{-1} - \mathbf{I} + (\mathbf{1} - \boldsymbol{\Omega}^{-1}\boldsymbol{\Psi}) \otimes \mathbf{H}$, where $\otimes$ denotes element-wise multiplication and $\mathbf{1}$ is a matrix of an appropriate size, consisted of ones. The matrix $\boldsymbol{\Omega}$ has the form

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_{11} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega}_{22} & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Omega}_{NN} \end{bmatrix} \tag{9}$$

where $\mathbf{\Omega}_{ii}$ is a $|\mathcal{B}_i| \times |\mathcal{B}_i|$ matrix fulfilling

$$\mathbf{0} \leq \mathbf{\Omega}_{ii} \leq \mathbf{I} \tag{10}$$

Similarly, $\mathbf{\Psi}$ has the form

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{\Psi}_{11} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Psi}_{22} & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Psi}_{NN} \end{bmatrix} \tag{11}$$

where $\mathbf{\Psi}_{ii}$ is a $|\mathcal{B}_i| \times |\mathcal{B}_i|$ matrix fulfilling

$$\mathbf{0} \leq \mathbf{\Psi}_{ii} \leq \mathbf{\Omega}_{ii}\mathbf{1} \tag{12}$$

Then, using these $\mathbf{M}$ and $\mathbf{N}$, we can construct the following iterative power control algorithm, which we will call *unconstrained block power control* (UBPC).

$$\begin{aligned} &\mathbf{p}(n+1) = \mathcal{I}\big(\mathbf{p}(n)\big) \\ &\triangleq (\mathbf{I} - \mathbf{\Psi} \otimes \mathbf{H})^{-1}\mathbf{\Omega}\Big(\big(\mathbf{\Omega}^{-1} - \mathbf{I} + (\mathbf{1} - \mathbf{\Omega}^{-1}\mathbf{\Psi}) \otimes \mathbf{H}\big)\mathbf{p}(n) + \boldsymbol{\eta}\Big) \end{aligned} \tag{13}$$

And, by showing that the matrixes $\mathbf{M}$ and $\mathbf{N}$ form a regular splitting of $\mathbf{A}$, we have:

Proposition 3 *UBPC converges to* $\mathbf{p}^*$ *of a feasible system, starting from an arbitrary initial vector* $\mathbf{p}(0)$.

Proof. The feasibility condition implies that $\rho(\mathbf{H}) < 1$ (Theorem 3.9 in [19]). Since $\mathbf{H}_{ii}$ is a principal submatrix of $\mathbf{H}$, by Lemma 2.4 in [19], we have $\rho(\mathbf{H}_{ii}) < \rho(\mathbf{H}) < 1$. By definition, $\mathbf{\Psi}_{ii} \leq \mathbf{\Omega}_{ii}\mathbf{1} \leq \mathbf{1}$ holds, and Theorem 2.8 in [19] guarantees that $\rho(\mathbf{\Psi}_{ii} \otimes \mathbf{H}_{ii}) \leq \rho(\mathbf{H}_{ii})$. Thus, by Theorem 3.9 in [19], we have

$$\begin{gathered} (\mathbf{I} - \mathbf{\Psi} \otimes \mathbf{H})^{-1} = \\ \begin{bmatrix} (\mathbf{I} - \mathbf{\Psi}_{11} \otimes \mathbf{H}_{11})^{-1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & (\mathbf{I} - \mathbf{\Psi}_{22} \otimes \mathbf{H}_{22})^{-1} & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & (\mathbf{I} - \mathbf{\Psi}_{NN} \otimes \mathbf{H}_{NN})^{-1} \end{bmatrix} \geq \mathbf{0} \end{gathered} \tag{14}$$

From (4), (10), (12) and (14), it is clear that

$$\mathbf{M}^{-1} = (\mathbf{I} - \mathbf{\Psi} \otimes \mathbf{H})^{-1}\mathbf{\Omega} \geq \mathbf{0} \tag{15}$$

and

$$\mathbf{N} = \mathbf{\Omega}^{-1} - \mathbf{I} + (\mathbf{1} - \mathbf{\Omega}^{-1}\mathbf{\Psi}) \otimes \mathbf{H} \geq \mathbf{0} \tag{16}$$

Thus, by Proposition 1, UBPC converges. □

To give the notion of "block-wise" power control, let $\mathbf{\Gamma}(n) = \mathrm{diag}\{\frac{\gamma_i^t}{\gamma_i(n)}\}$, where $\gamma_i(n)$ denotes the received CIR of user i at iteration n (see (1) for the definition of CIR). This allows us to replace $\mathbf{H}\mathbf{p}(n) + \boldsymbol{\eta}$ in (13) by $\mathbf{\Gamma}(n)\mathbf{p}(n)$. Since $\mathbf{\Gamma}$ is of the same size as $\mathbf{\Omega}$ or $\mathbf{\Psi}$, we can decomposite it into blocks in the same manner:

$$\mathbf{\Gamma}(n) = \begin{bmatrix} \mathbf{\Gamma}_{11}(n) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma}_{22}(n) & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Gamma}_{NN}(n) \end{bmatrix} \tag{17}$$

By noting that $\mathbf{\Omega}$, $\mathbf{\Psi}$ and $\mathbf{\Gamma}(n)$ are block diagonal matrixes, we can rewrite (13) in a block-wise form as follows:

$$\mathbf{p}_i(n+1) = \mathcal{I}_i\big(\mathbf{p}(n)\big) \triangleq \Big(\mathbf{I} + (\mathbf{I} - \mathbf{\Psi}_{ii} \otimes \mathbf{H}_{ii})^{-1}\mathbf{\Omega}_{ii}\big(\mathbf{\Gamma}_{ii}(n) - \mathbf{I}\big)\Big)\mathbf{p}_i(n), \tag{18}$$

where $\mathbf{p}_i(n)$ denotes the power levels of the transmitters of the block i at iteration n. Equations (13) and (18) are mathematically equivalent. The only difference between them is that the latter utilizes measurable information, $\mathbf{H}_{ii}$ and $\Gamma_{ii}(n)$, within block i.

In UBPC given by the equation (18), the elements in $\mathbf{\Psi}_{ii}$ represent availability and reliability on the corresponding elements in the normalized link gain matrix, $\mathbf{H}_{ii}$. For example, if the users in block i have a full confidentiality on an element in $\mathbf{H}_{ii}$, then the corresponding element in $\mathbf{\Psi}_{ii}$ can be set to its maximum value given by the bound (12). However, the zero element in $\mathbf{\Psi}_{ii}$ corresponds to the opposite case, where the information is either not available or has poor reliability. When $\mathbf{\Psi}_{ii} = \mathbf{0}$, we can verify through (18) that the power update within block i becomes fully distributed, requiring only local CIR measurement. In fact, if we choose $\mathbf{\Psi} = \mathbf{0}$, then UBPC will be equivalent to the Foschini and Miljanic algorithm [9]. By the definition of $\mathbf{H}_{ii}$, if every block is composed of a single user, UBPC will be also reduced to the Foschini and Miljanic algorithm. The diagonal elements of $\mathbf{\Omega}_{ii}$ constitute a damping factor of the power control algorithm like the β-parameter in the Foschini and Miljanic algorithm. The damping factor can be used to increase the robustness of the power control algorithm by adjusting the step size of power update. For example, in case of power control errors, such as CIR measurement error, smaller fault can be done by decreasing the damping factor. The damping factor can be also used for taking into account the "sluggishness" of the transmitter, i.e., the power amount that can be varied in one update. Nevertheless, as will be stated in Proposition 4, largest $\mathbf{\Omega}_{ii}$ and $\mathbf{\Psi}_{ii}$ will generate the best convergence speed, provided that the link gains and the CIRs are measured accurately in block i.

Proposition 4 *Among those $\mathbf{\Omega}_{ii}$ and $\mathbf{\Psi}_{ii}$ that fulfill inequalities (10) and (12), the choice $\mathbf{\Omega}_{ii} = \mathbf{I}$ and $\mathbf{\Psi}_{ii} = \mathbf{1}$ is best with respect to asymptotic average rate of convergence.*

Proof. For notational convenience, let us denote the iteration matrix $\mathbf{M}^{-1}\mathbf{N} = (\mathbf{I} - \mathbf{\Psi} \otimes \mathbf{H})^{-1}\mathbf{\Omega}\big(\mathbf{\Omega}^{-1} - \mathbf{I} + (\mathbf{1} - \mathbf{\Omega}^{-1}\mathbf{\Psi}) \otimes \mathbf{H}\big)$ in (13) by $\mathbf{Z}(\mathbf{\Omega}, \mathbf{\Psi})$. Let $\bar{\mathbf{\Omega}}_{ii} = \mathbf{I}$ and $\bar{\mathbf{\Psi}}_{ii} = \mathbf{1}$. Assume that $\mathbf{\Omega}_{ii} \neq \bar{\mathbf{\Omega}}_{ii}$ and $\mathbf{\Psi}_{ii} \neq \bar{\mathbf{\Psi}}_{ii}$. It follows that $\mathbf{\Omega}^{-1} - \mathbf{I} + (\mathbf{1} - \mathbf{\Omega}^{-1}\mathbf{\Psi}) \otimes \mathbf{H} \geq \bar{\mathbf{\Omega}}^{-1} - \mathbf{I} + (\mathbf{1} - \mathbf{\Omega}^{-1}\mathbf{\Psi}) \otimes \mathbf{H} \geq \bar{\mathbf{\Omega}}^{-1} - \mathbf{I} + (\mathbf{1} - \bar{\mathbf{\Omega}}^{-1}\bar{\mathbf{\Psi}}) \otimes \mathbf{H}$, where the equality is excluded. Thus, by Proposition 2, we have $\rho\big(\mathbf{Z}(\mathbf{\Omega}, \mathbf{\Psi})\big) > \rho\big(\mathbf{Z}(\bar{\mathbf{\Omega}}, \bar{\mathbf{\Psi}})\big)$. □

The sizes and structures of submatrixes $\mathbf{\Psi}_{ii}$ (thus the size and structure of each block) define the computational complexity and distributiveness of UBPC. They also give a paramount effect on the convergence speed. If the block sizes are large and the elements in the corresponding $\mathbf{\Psi}_{ii}$-matrixes are also large, then the convergence speed is going to be fast. Actually, the best possible performance with respect to the convergence speed, assuming that the link gains could be measured accurately, is achieved by including all the users into one block. In this special case, UBPC becomes fully centralized. One drawback of using large block sizes with dense $\mathbf{\Psi}_{ii}$ is that the computational complexity is high, since we need to invert large $(\mathbf{I} - \mathbf{\Psi}_{ii} \otimes \mathbf{H}_{ii})$ at each iteration. Another drawback is that the degree of signaling is high, since a large amount of measurement information must be collected. To reduce the complexity, we can use sparse $\mathbf{\Psi}_{ii}$ or reduce the block size. However, this is done at the cost of reducing the convergence speed. In practice, the sizes and structures of $\mathbf{\Psi}_{ii}$ are upper limited by the amount of signaling, the availability of the link gain estimates and the computational complexity of the matrix inversion operation. However, it is generally difficult to compare between utilizing small blocks with reliable information and utilizing larger blocks with information of inferior quality. In Example 1, we will illustrate a practical way of choosing blocks such that the computational complexity is kept low.

Example 1. Consider a DS-CDMA system, where all the mobiles assigned to a particular base station constitute one block. The link gains between mobiles and the base station within the same block are assumed to be known. Further, in the downlink case, we assume that the normalized cross-correlation between different channels in the block k is uniform, i.e., $\theta_{ij} = \theta$ if $i, j \in \mathcal{B}_k$. If we choose $\mathbf{\Omega}_{kk} = \mathbf{I}$ and $\mathbf{\Psi}_{kk} = \mathbf{1}$, $k = 1, \ldots, N$; then UBPC becomes, in up- and downlink cases,

$$p_i(n+1) = \frac{\gamma_i^t}{\big(1 + \gamma_i^t\big)\big(1 - \sum_{j \in \mathcal{B}_k} \frac{\gamma_j^t}{1+\gamma_j^t}\big)} \frac{I_i(n)}{g_{ii}}, \quad i \in \mathcal{B}_k \tag{19}$$

and

$$p_i(n+1) = \frac{\gamma_i^t}{1 + \theta\gamma_i^t}\Big(\frac{\theta \sum_{j \in \mathcal{B}_k} \frac{\gamma_j^t}{1+\theta\gamma_j^t} \frac{I_j(n)}{g_{jj}}}{1 - \theta \sum_{j \in \mathcal{B}_k} \frac{\gamma_j^t}{1+\theta\gamma_j^t}} + \frac{I_i(n)}{g_{ii}}\Big), \quad i \in \mathcal{B}_k \tag{20}$$

respectively. For user i, the *external interference* from the outside of block k, is given by

$$I_i(n) = \frac{g_{ii}p_i(n)}{\gamma_i(n)} - \sum_{\substack{j \in \mathcal{B}_k \\ j \neq i}} g_{jj}p_j(n), \quad i \in \mathcal{B}_k \tag{21}$$

and

$$I_i(n) = \frac{g_{ii}p_i(n)}{\gamma_i(n)} - \theta g_{ii} \sum_{\substack{j \in \mathcal{B}_k \\ j \neq i}} p_j(n), \quad i \in \mathcal{B}_k \tag{22}$$

in up- and downlink cases, respectively. Note that, from our definition in Section 2, the link gains g_{jj} in (21) and g_{ii} in (22) can be replaced by g_{ij} (see Appendix A for the derivation of above results). Also, we can easily verify that if $\mathcal{B}_k = \{i\}$ and $\theta = 1$, then both (19) and (20) will be reduced to the fully distributive form, $p_i(n+1) = \frac{\gamma_i^t}{\gamma_i(n)}p_i(n)$.

3.2 Constrained Case

Let us consider a more realistic case, where we have an upper limit for transmission powers as given in (3). The constrained block power control (CBPC) algorithm is given by

$$\mathbf{p}(n+1) = \mathcal{T}\big(\mathbf{p}(n)\big) \triangleq \min\Big\{\bar{\mathbf{p}}, \mathcal{I}\big(\mathbf{p}(n)\big)\Big\} \tag{23}$$

For block i, the update rule can be expressed as

$$\mathbf{p}_i(n+1) = \mathcal{T}_i\big(\mathbf{p}(n)\big) \triangleq \min\Big\{\bar{\mathbf{p}}_i, \mathcal{I}_i\big(\mathbf{p}(n)\big)\Big\} \tag{24}$$

Proposition 5 *CBPC converges to* $\mathbf{p}^*$ *of a feasible system, starting from any power vector* $\mathbf{p}(0)$ *that is in the range of (3).*

Proof. First, for a given vector $\mathbf{x}$ and a nonsingular matrix $\mathbf{W}$ with an appropriate size, define the *weighted maximum norm* as

$$||\mathbf{x}||_\infty^{\mathbf{W}} = ||\mathbf{W}\mathbf{x}||_\infty = \max_i |\sum_j w_{ij}x_j| \tag{25}$$

and for a matrix $\mathbf{B}$, the consistent matrix norm as

$$||\mathbf{B}||_\infty^{\mathbf{W}} = ||\mathbf{W}\mathbf{B}\mathbf{W}^{-1}||_\infty \tag{26}$$

If the weight matrix $\mathbf{W}$ is diagonal then the above can be written as

$$||\mathbf{B}||_\infty^{\mathbf{W}} = \max_i |\sum_j b_{ij}\frac{w_{ii}}{w_{jj}}| \tag{27}$$

By Lemma 1 (see Appendix B), there exists a positive definite diagonal matrix $\mathbf{W}$ such that $||\mathbf{Z}(\mathbf{\Omega}, \mathbf{\Psi})||_\infty^{\mathbf{W}} < 1$, and by Proposition 3, $\mathbf{Z}(\mathbf{\Omega}, \mathbf{\Psi}) \geq \mathbf{0}$.

Therefore, we have

$$\begin{aligned} ||\mathcal{T}(\mathbf{p}) - \mathbf{p}^*||_\infty^{\mathbf{W}} &\leq ||\mathbf{Z}(\mathbf{\Omega}, \mathbf{\Psi})(\mathbf{p} - \mathbf{p}^*)||_\infty^{\mathbf{W}} && (28) \\ &\leq ||\mathbf{Z}(\mathbf{\Omega}, \mathbf{\Psi})||_\infty^{\mathbf{W}} ||\mathbf{p} - \mathbf{p}^*||_\infty^{\mathbf{W}} && (29) \\ &< ||\mathbf{p} - \mathbf{p}^*||_\infty^{\mathbf{W}} && (30) \end{aligned}$$

Thus, we can conclude that $\mathcal{T}(\mathbf{p})$ is a *pseudo-contraction mapping* with respect to the weighted maximum norm, and thus CBPC converges geometrically with the rate $||\mathbf{Z}(\mathbf{\Omega}, \mathbf{\Psi})||_\infty^{\mathbf{W}}$. □

Remark 1 *It is also possible to prove the convergence of CBPC by verifying that $\mathcal{T}(\mathbf{p})$ is a standard interference function (Theorem 2 in [12]).*

Remark 2 *Proposition 5 also provides a convergence proof for the constrained version of the Foschini and Miljanic algorithm. Note that if $\mathbf{\Omega} = \mathbf{I}$ and $\mathbf{\Psi} = \mathbf{0}$ then the corresponding constrained Foschini and Miljanic algorithm is equivalent to DCPC [11].*

Corollary 1 *Proposition 4 also holds for CBPC.*

Proof. By Proposition 5, CBPC is a pseudo-contraction mapping. Thus, there exits $0 < K < \infty$ such that $\mathcal{T}(\mathbf{p}(\mathbf{n})) < \bar{\mathbf{p}}$, for all $n > K$ and for all $\mathbf{\Omega}$, $\mathbf{\Psi}$ fulfilling (10) and (12). So for $n > K$, the dynamics are described by UBPC and thus Proposition 4 applies. □

Corollary 2 *If $\mathbf{\Omega}_{ii}(n)$ and $\mathbf{\Psi}_{ii}(n)$ fulfill (10) and (12) at every iteration, then the non-stationary CBPC converges to $\mathbf{p}^*$ of a feasible system, starting from any power vector $\mathbf{p}(0)$ that is in the range of (3).*

Proof. By Lemma 1 (see Appendix B), there exists a positive definite diagonal matrix $\mathbf{W}$ such that $||\mathbf{Z}(\mathbf{\Omega}, \mathbf{\Psi})||_\infty^{\mathbf{W}} < 1$ for all $\mathbf{\Omega}$ and $\mathbf{\Psi}$ that fulfill (10) and (12). Therefore $||\mathbf{Z}(\mathbf{\Omega}(n), \mathbf{\Psi}(n))||_\infty^{\mathbf{W}} < 1$ for all n and the equations (28)-(30) hold. Thus $\mathcal{T}(\mathbf{p})$ is a pseudo-contraction mapping even in the non-stationary case. □

Corollary 2 states that the damping factor and the amount of link gain information utilized by the power control can vary from iteration to another without causing any problem to the convergence. In addition, we have the following property that the algorithm converges even if the power updates are done in the asynchronous fashion.

Remark 3 *The asynchronous CBPC converges to $\mathbf{p}^*$ of a feasible system, starting from any power vector $\mathbf{p}(0)$ that is in the range of (3).*

This property can be easily proved by following the steps taken in the proof of Proposition 3.1 in [10].

As mentioned in Section 2, we have been focusing on the feasible system throughout the paper. However, it is noticeable that UBPC and CBPC may result in nonpositive power values when the system becomes infeasible; we cannot support every transmitter. In this case, the inequality (14) may no longer hold because $\rho\big(\mathbf{\Psi}_{ii}\mathbf{H}_{ii}\big) \geq 1$ (see also Theorem 3.9 in [19]). This means that the block i, and therefore the whole system, is overloaded and some of the users must be removed. Unfortunately, this condition alone is not enough to detect infeasibility since it may happen that the block i is feasible but the interference coming from other cells is too high ($\rho\big(\mathbf{H}_{ii}\big) \leq \rho\big(\mathbf{H}\big) < 1$ but $\mathbf{p}^*$ is not in the range of (3) or $\rho\big(\mathbf{H}_{ii}\big) < 1$ but $\rho\big(\mathbf{H}\big) \geq 1$) or it may even happen that $\mathbf{\Psi}_{ii}$ is chosen such a way that the inverse matrix is positive although the block is overloaded ($\rho\big(\mathbf{H}_{ii}\big) \geq 1$ but $\rho\big(\mathbf{\Psi}_{ii}\mathbf{H}_{ii}\big) < 1$). In case of negative powers, we could, if all the link gains in $\mathbf{H}_{ii}$ are known, utilize some advanced removal strategy like removing the worst interferer (the user that has the smallest link gain) first. Another approach that could be useful, especially if the link gain information has poor reliability, is to force the inverse of $\mathbf{I} - \mathbf{\Psi}_{ii} \otimes \mathbf{H}_{ii}$ matrix to become positive by decreasing some of the elements in $\mathbf{\Psi}_{ii}$. This effect can also be achieved by dividing the block into several smaller ones although this may require more signaling effort. After the inverse is made positive, the overall system can still be infeasible. If this is the case, the power vector will converge to a fixed point where some (or all) of the users are using the maximum power but are not supported. To cope with this situation, standard removal schemes like *gradual removal* [16] should be applied.

4 Simulation

We investigate how quickly CBPC converges to $\mathbf{p}^*$ of a feasible system. DCPC is used as a reference algorithm. The DS-CDMA system with 19 omni-bases located in the centers of 19 cells is used as a test system (Figure 1). The distance between two nearest base stations is 2 km. We consider both up- and downlink cases of an IS-95 example, where the *processing gain* is 21dB [1]. For a given instance, a total of 190 mobiles are generated in the uplink case, whereas 380 mobiles are considered in the downlink case. The reason for the difference in the number of users is to keep the relative load approximately the same for both up- and downlinks.

The locations of mobiles are uniformly distributed over the 19 cells. The link gain g_{ij} is modeled as $g_{ij} = s_{ij} \cdot d_{ij}^{-4}$, where s_{ij} is the shadow fading factor and d_{ij} is the distance between base i and mobile j. The log-normally distributed s_{ij} is generated according to the model in [18] (pp. 185-186, $E(s_{ij}) = 0$ dB, and $\sqrt{E(s_{ij}s_{kl})} = 8$ dB if $i = k$; $\sqrt{E(s_{ij}s_{kl})} = 4\sqrt{2}$ dB if $i \neq k$).

The receiver noise at both mobiles and base stations is taken to be 10^{-12}. The relative maximum power of a mobile, and in the downlink case the relative maximum power of a traffic channel assigned to a mobile is set to one. The base that gives the lowest attenuation is assigned to each mobile. The received E_b/I_0 is calculated by adding the processing gain to the corresponding CIR value (in dB). The target E_b/I_0 is set to 8 dB for both up- and downlinks of each mobile.

When applying CBPC, we have used the same assumption as in Example

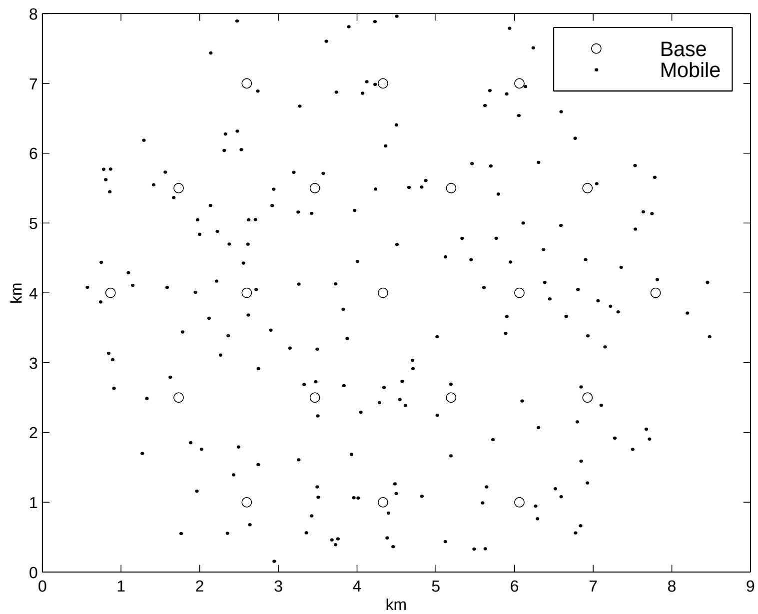

Figure 1: DS-CDMA cellular system with 19 omni-bases.

1. That is, all the mobiles assigned to a particular base station constitute one block. The link gains between mobiles and the base station within the same block are assumed to be known. We choose $\mathbf{\Omega}_{ii} = \mathbf{I}$ and $\mathbf{\Psi}_{ii} = \mathbf{1}$ and apply (19) and (20), considering the maximum power constraint. In the downlink case, we use the normalized cross-correlation $\theta_{ij} = 0.4$ if mobiles i and j belong to the same base station.

The *outage probability* is used as a performance measure. To evaluate this, we have taken 1000 independent "feasible" instances of mobile locations and shadow fadings. In each instance, we have performed thirty power control steps. The initial power for each mobile is randomly chosen from the interval [0,1]. The outage probability at each iteration is computed over 1000 instances by counting the portion of the number of non-supported mobiles at the iteration. Figures 2 and 3 show the outage probabilities of CBPC and DCPC as a function of iteration. In the uplink case (Figure 2), CBPC takes about 7 iterations on average to reach the state with the outage probability of 10^{-4}. However, we can see that DCPC requires more than 30 iterations on average to reach that point. In the downlink (Figure 3), CBPC requires 8 iterations, whereas DCPC does 20 iterations. The reason for the performance difference in CBPC between up- and downlinks is that the uplink interference within a cell is much larger than that in the downlink and that main contribution of CBPC is to efficiently mitigate the interference within the cell (block).

Figures 4 and 5 show the Euclidean distance between the current power vec-

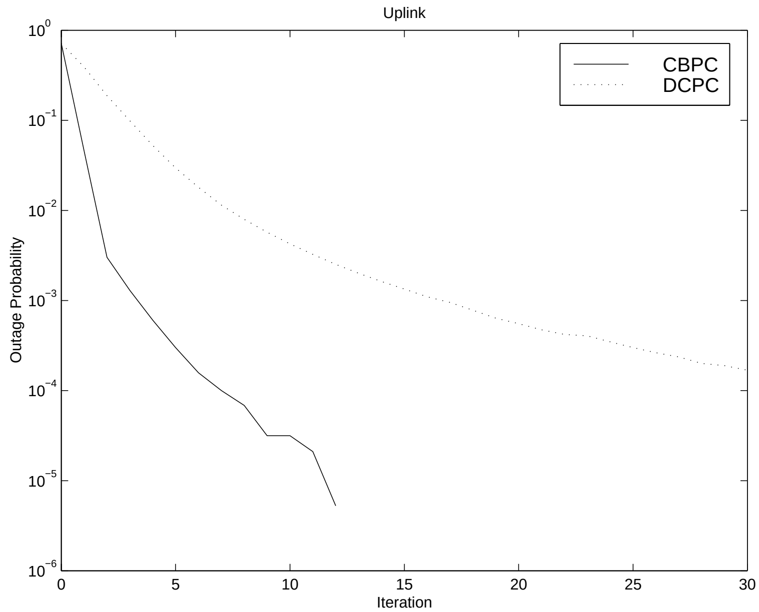

Figure 2: Uplink outage probability as a function of iteration.

tor and $\mathbf{p}^*$. The distance is computed by averaging over 1000 instances. It is clear that CBPC also converges faster in terms of the Euclidean distance. Figures 2-5 indicate that a significant improvement in the convergence speed has been obtained through utilizing link gain information. The speed difference becomes bigger as both algorithms approach $\mathbf{p}^*$. This coincides with the theoretic results of Corollary 1 on the asymptotic average rate of convergence.

5 Concluding Remarks

In this paper, we proposed a power control algorithm, which can incorporate available link gain information in a way that the convergence speed increases. Acceleration of convergence speed is based on the accurate measurement of link gains and received CIRs in each block. However, if those measurements were too erroneous, this would affect the power control negatively. To cope with the situation, we have introduced parameters $\mathbf{\Omega}$ and $\mathbf{\Psi}$ into our algorithm. As was discussed in Section 3.1, those parameters will determine the algorithm's key properties such as distributiveness, robustness and convergence speed.

Finally, we would like to mention that the proposed algorithm constitutes a *generalized framework* and that it contains existing algorithms [9], [11] as special cases. Also, our work opens possibility to have a power control algorithm that is between the fully distributed and the centralized ones.

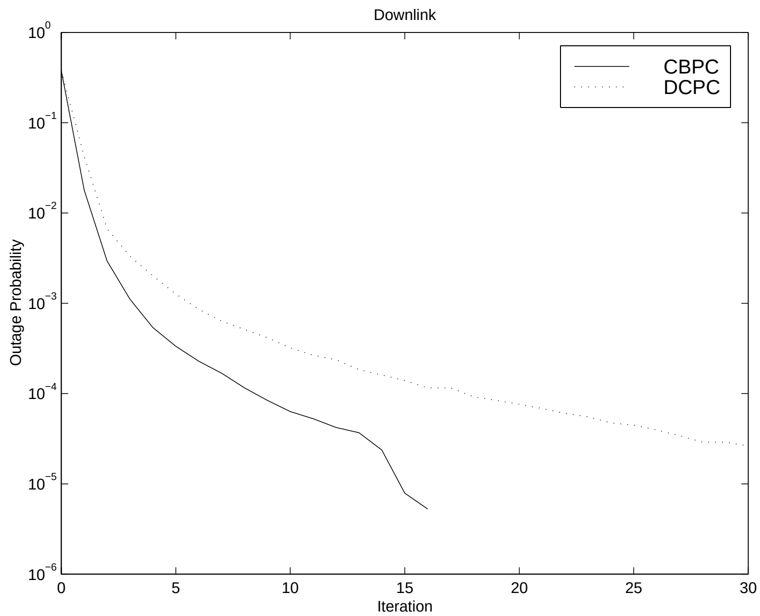

Figure 3: Downlink outage probability as a function of iteration.

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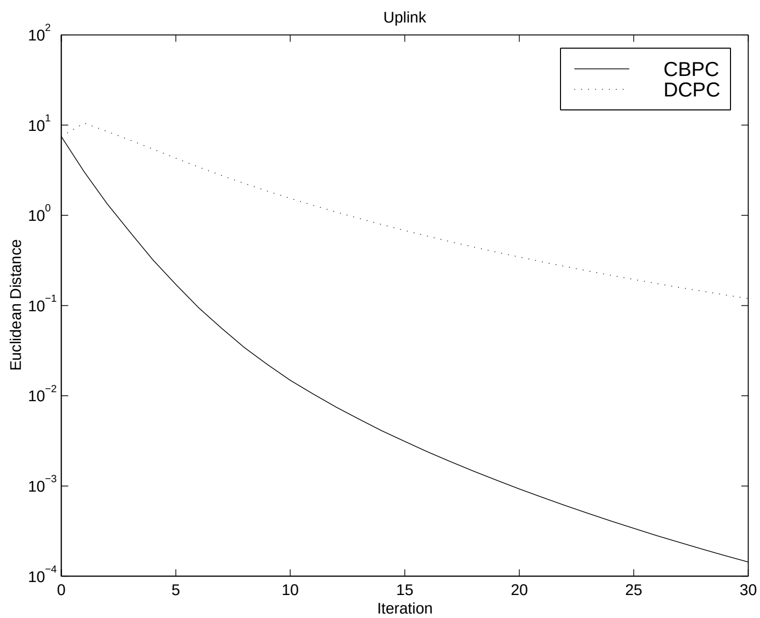

Figure 4: Uplink Euclidean distance between the current power vector and $\mathbf{p}^*$.

Appendix A: Derivation of (19) - (22)

Since $\mathbf{A} = \mathbf{M} - \mathbf{N}$, from (2) and the defintion of $\mathbf{M}$ and $\mathbf{N}$ in UBPC, we get

$$\mathbf{p} = \big(\boldsymbol{\Psi} \otimes \mathbf{H} + \mathbf{I} + \boldsymbol{\Omega}(\mathbf{1} - \boldsymbol{\Omega}^{-1}\boldsymbol{\Psi}) \otimes \mathbf{H}\big)\mathbf{p} + \boldsymbol{\Omega}\boldsymbol{\eta} \tag{31}$$

Substituting $\boldsymbol{\Omega}_{kk} = \mathbf{I}$ and $\boldsymbol{\Psi}_{kk} = \mathbf{1}$ into the above and writing the equation system row by row, we get

$$p_i = \gamma_i^t \Big(\sum_{\substack{j \in \mathcal{B}_k \\ j \neq i}} \frac{g_{ij}}{g_{ii}} \theta_{ij} p_j + \frac{I_i}{g_{ii}} \Big), \quad i \in \mathcal{B}_k, \tag{32}$$

where

$$I_i = \sum_{j \notin \mathcal{B}_k} g_{ij} p_j + \nu_i \quad i \in \mathcal{B}_k \tag{33}$$

is the total noise plus external interference experienced by user i.

Consider first the uplink case. Since in our example a block is equal to a cell, all the receivers are co-located and thus $g_{ij} = g_{jj}$ and $I_i = I_j$ for all $i, j \in \mathcal{B}_k$. Therefore, by noting that $\theta_{ij} = 1$ for all $i, j \in \mathcal{B}_k$, we can rewrite (32) as follows:

$$g_{ii} p_i = \frac{\gamma_i^t}{1 + \gamma_i^t} \Big(\sum_{j \in \mathcal{B}_k} g_{jj} p_j + I_i \Big), \quad i \in \mathcal{B}_k \tag{34}$$

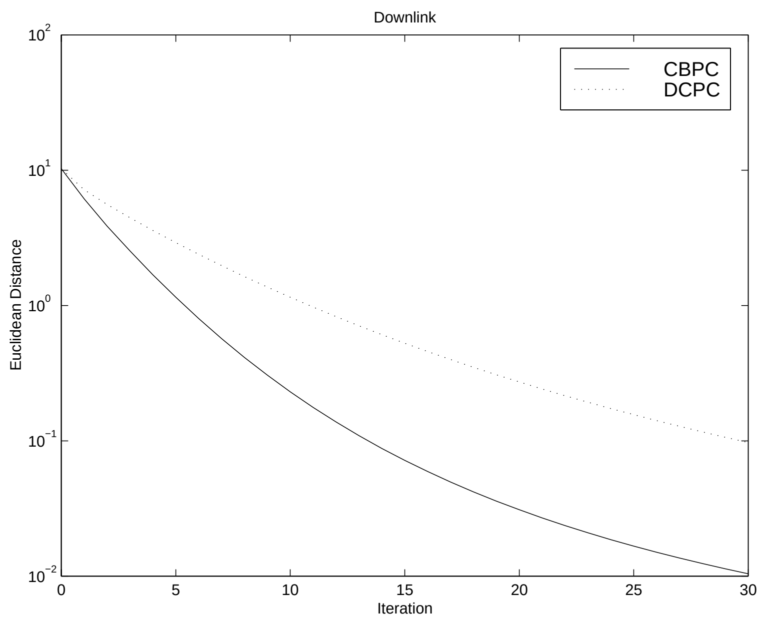

Figure 5: Downlink Euclidean distance between the current power vector and $\mathbf{p}^*$.

Summing (34) over $i \in \mathcal{B}_k$ yields

$$\sum_{i\in\mathcal{B}_k} g_{ii}p_i = \sum_{i\in\mathcal{B}_k} \Big(\frac{\gamma_i^t}{1+\gamma_i^t}\big(\sum_{j\in\mathcal{B}_k} g_{jj}p_j + I_i\big)\Big) \tag{35}$$

From the above, we get

$$\sum_{i\in\mathcal{B}_k} g_{ii}p_i = \frac{\sum_{i\in\mathcal{B}_k}\frac{\gamma_i^t}{1+\gamma_i^t}I_i}{1-\sum_{i\in\mathcal{B}_k}\frac{\gamma_i^t}{1+\gamma_i^t}} \tag{36}$$

Substituting (36) into (34) and dividing the result by g_{ii} yields

$$p_i = \frac{\gamma_i^t}{(1+\gamma_i^t)(1-\sum_{j\in\mathcal{B}_k}\frac{\gamma_j^t}{1+\gamma_j^t})}\frac{I_i}{g_{ii}} \tag{37}$$

In the downlink case, all the intra-block interference is coming from the same source and therefore we have $g_{ij} = g_{ii}$ for all $i, j \in \mathcal{B}_k$. It was assumed that $\theta_{ij} = \theta$ for all $i, j \in \mathcal{B}_k$. So, the equation (32) becomes

$$p_i = \gamma_i^t\big(\theta \sum_{\substack{j\in\mathcal{B}_k\\ j\neq i}} p_j + \frac{I_i}{g_{ii}}\big), \quad i \in \mathcal{B}_k \tag{38}$$

By following the same steps as in (35)-(37) but keeping in mind that $I_i \neq I_j$, we get

$$p_i = \frac{\gamma_i^t}{1+\theta\gamma_i^t}\Big(\frac{\theta\sum_{j\in\mathcal{B}_k}\frac{\gamma_j^t}{1+\theta\gamma_j^t}\frac{I_j}{g_{jj}}}{1-\theta\sum_{j\in\mathcal{B}_k}\frac{\gamma_j^t}{1+\theta\gamma_j^t}}+\frac{I_i}{g_{ii}}\Big), \quad i\in\mathcal{B}_k \tag{39}$$

Appendix B

Lemma 1 *If the system is feasible, there exists a positive definite diagonal matrix* $\mathbf{W}$ *such that* $||\mathbf{Z}(\mathbf{\Omega},\mathbf{\Psi})||_\infty^{\mathbf{W}} < 1$ *for all* $\mathbf{\Omega}$ *and* $\mathbf{\Psi}$ *that fulfill inequalities (10) and (12).*

Proof. The feasibility condition implies that $\mathbf{H}$ is a positive irreducible matrix (Theorem 3.11 in [19]). Therefore, the *Perron-Frobenius Theorem* (Theorem 2.1 in [19]) guarantees that there exits a positive vector $\mathbf{e} = (e_1 e_2 \cdots e_M)'$, called the *Perron eigenvector*, such that

$$\mathbf{He} = \rho(\mathbf{H})\mathbf{e} \tag{40}$$

It follows that

$$\begin{aligned}
\mathbf{Z}(\mathbf{\Omega},\mathbf{\Psi})\mathbf{e} &= (\mathbf{I}-\mathbf{\Psi}\otimes\mathbf{H})^{-1}\mathbf{\Omega}\big(\mathbf{\Omega}^{-1}-\mathbf{I}+\mathbf{H}-\mathbf{\Omega}^{-1}\mathbf{\Psi}\otimes\mathbf{H}\big)\mathbf{e} && (41)\\
&= (\mathbf{I}-\mathbf{\Psi}\otimes\mathbf{H})^{-1}\big(\mathbf{I}+\mathbf{\Omega}(\rho(\mathbf{H})-1)-\mathbf{\Psi}\otimes\mathbf{H}\big)\mathbf{e} && (42)\\
&= \big(\mathbf{I}+(\mathbf{I}-\mathbf{\Psi}\otimes\mathbf{H})^{-1}\mathbf{\Omega}(\rho(\mathbf{H})-1)\big)\mathbf{e} && (43)
\end{aligned}$$

Since the system is feasible, by Theorem 3.9 in [19], we have $\rho\big(\mathbf{H}\big) < 1$ and by Proposition 3, we have $\mathbf{Z}(\mathbf{\Omega},\mathbf{\Psi}) \geq \mathbf{0}$, equality excluded. It is also clear that $\mathbf{Z}(\mathbf{\Omega},\mathbf{\Psi})$ has a full rank. Therefore,

$$\mathbf{0} < \mathbf{Z}(\mathbf{\Omega},\mathbf{\Psi})\mathbf{e} < \mathbf{e} \tag{44}$$

If we choose $\mathbf{W} = \mathrm{diag}(\frac{1}{e_i})$, which is clearly a positive definite diagonal matrix, then using (27) we have

$$||\mathbf{Z}(\mathbf{\Omega},\mathbf{\Psi})||_\infty^{\mathbf{W}} < 1 \tag{45}$$

Thus by constructing a matrix $\mathbf{W}$, we have shown the existence of it. □

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