B. Varone, J. M. A. Tanskanen, and S. J. Ovaska, "Response analysis of feed-forward neural network predictors," in *Proc. 1997 International Conference on Acoustics, Speech, and Signal Processing*, Munich, Germany, April 1997, pp. 3309–3312.

RESPONSE ANALYSIS OF FEED-FORWARD NEURAL NETWORK PREDICTORS

Barbara Varone[1]*, Jarno M. A. Tanskanen*[2]*, and Seppo J. Ovaska*[3]

[1]Politecnico di Milano
20133 Milan, Italy
E-mail: bva@wooster.hut.fi

[2]Laboratory of Signal Processing and Computer Technology
Institute of Radio Communications
Helsinki University of Technology
FIN-02150 Espoo, Finland
E-mail: jarno.tanskanen@hut.fi

[3]Institute of Intelligent Power Electronics
Helsinki University of Technology
FIN-02150 Espoo, Finland
E-mail: seppo.ovaska@hut.fi

ABSTRACT

In this paper, we investigate the characteristics of some one-step-ahead nonlinear predictors based on a two-layer feed-forward neural network (2LFNN). The behavior of neural networks (NN) is investigated in the frequency domain using two frequency response estimation techniques, and in the time domain, by analyzing the unit step and triangular pulse responses. Some of the estimated frequency responses of these NNs resemble those of corresponding linear polynomial predictors, revealing the nearly polynomial nature of the applied training signals. Similarity of the two frequency response estimates is an indication of good generalization properties.

1. INTRODUCTION

A. General

Even though nonlinear systems do not have unique frequency responses, it is possible to observe their responses to known input signals. A class of non-linear filters, FIR-median hybrid filters, has been analyzed by Neejärvi et al. using sinusoidal and pulse input signals [1], [2]. In this paper, the same sinusoidal input method is used, along with white Gaussian noise (WGN) inputs, to estimate *input dependent* frequency responses of a set of NNs. In certain cases, the two frequency response estimation methods are found to yield similar results, while for some networks the frequency response estimates are greatly input dependent. For the time domain analysis, unit step and triangular pulse inputs are used. Understanding NN's frequency and time domain characteristics will give NN designers valuable additional insight to the NNs, and be of great help in deciding the system parameters, like the number of neurons in each layer or the initial weights, and evaluating the applicability of the designed NNs.

In this paper, a single-output 2LFNN predictor is analysed. The prediction $\hat{x}(n+1)$ is given by

$$\hat{x}(n+1) = \mathrm{f}\left(\sum_{m=0}^{M-1} w_{out}(m)\,\mathrm{g}\left(\sum_{l=0}^{L-1} w_{hid}(l,m)\,x(n-l)\right)\right) \quad (1)$$

where $x(n)$ is the NN input signal sample with time index n, $\mathrm{f}(\cdot)$ and $\mathrm{g}(\cdot)$ are the output and hidden layer activation functions, respectively, $w_{out}(m)$ is the mth connection weight of the output layer neuron, and $w_{hid}(l,m)$ is the lth weight of the mth hidden layer neuron. The input layer is fed from a delay line with L taps. The network is illustrated in Figure 1.

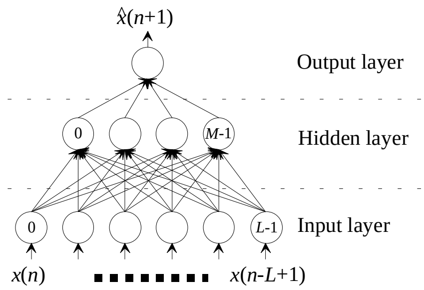

Figure 1: A predictive 2LFNN with L inputs, M hidden layer neurons, and a single output.

B. Application

The presented NN setup is motivated by power control needs of code-division multiple-access (CDMA) mobile communications systems. The user capacity of a CDMA system crucially depends on functionality of the power control. To enhance the power control system, predictive filtering has been proposed [3] to be used within the closed power control loop to lessen loop delay effects, reduce noise and interference in received power measurements, and smoothen out fast fading. This work is not concerned with the power control system itself. In this paper, we analyze a set of feed-forward NNs that have originally been selected for the power control application. In [3], an adaptive NN approach is proposed for the power estimation problem. Considering the greatly time-varying and environment-dependent characteristics of natural radio channels, the adaptive approach is naturally more effective but also computationally more demanding as compared to a fixed predictor approach. The analysis methods presented in this paper can be used in designing adaptive networks as well.

B. Varone, J. M. A. Tanskanen, and S. J. Ovaska, “Response analysis of feed-forward neural network predictors,” in *Proc. 1997 International Conference on Acoustics, Speech, and Signal Processing*, Munich, Germany, April 1997, pp. 3309–3312.

2. NEURAL NETWORKS

NNs with biases, a hidden layer of nodes with *tangent-sigmoid* activation functions, and an output layer with *linear* activation functions, are capable of approximating any function with a finite number of discontinuities [4]. A 2LFNN topology with 15 input nodes, 4 hidden nodes with tangent-sigmoid activation functions, and 1 output node with a linear activation function, was selected for analysis. This topology was found to be potential for processing received power samples in noisy Rayleigh-fading radio channel conditions at urban mobile speeds. Initial values of the weights were chosen randomly between -2 and 2. The Levenberg-Marquardt (L-M) learning rule [5] is used to minimize the sum of squared-error (SSE) function.

The predictor training input signal is a *sweeping sinusoid*, corrupted by zero-mean additive white Gaussian noise (AWGN) with unit variance, and the desired target signal is the corresponding one-step-ahead predicted noiseless signal. A sweeping sinusoidal signal is a time-varying sinusoid whose frequency varies linearly from zero to a fixed maximum frequency. Four zero-mean sweeping sinusoids with unit maximum amplitudes, and maximum frequencies 10 Hz, 25 Hz, 50 Hz, and 100 Hz, sampled at 1 kHz, were used to train four NNs, resulting in predictors NN1, NN2, NN3, and NN4, respectively. Such maximum Doppler shift frequencies correspond to the mobile speeds of approximately 6 km/h, 15 km/h, 30 km/h and 60 km/h, respectively, with the mobile system parameters defined in [3]. In nature, received power signals resemble sweeping sinusoids which are used as generalized presentations of fading in a mobile radio channel with different Doppler shift frequency ranges. NNs trained with actual fading models would naturally have given unfairly good results when tested with the same channel model, and sweeping sinusoids lead to better generalization properties. Also, the statistics of the channel are generally not known. It is furthermore to be noted that a feed-forward NN of a described type is necessarily stable, which is a fundamental requirement for application in a closed control loop.

A. Signal Preprocessing

The NN input signal being power of the received signal, is all positive and does not have a unique maximum amplitude, whereas the NN training signals are zero-mean sinusoidal signals with unit maximum amplitude. Without preprocessing this results in a destructive clipping effect. After local mean removal preprocessing, the predictor is able to function within its natural input signal conditions. The local mean is calculated over the last 15 samples, and is restored after the prediction. All the results in this paper are achieved employing local mean removal preprocessing and corresponding reverse postprocessing.

B. Prediction Performance

The NNs were tested with sweeping sinusoids, alike the training signals, except with different AWGN. In Table 1, the achieved SNR gains are shown when the trained NNs are presented with their training signals, and test signals which are the same sweeping sinusoids with new AWGN sequences added. From Table 1 it is seen that NN4 has not been able to learn its training signal, whereas NN1 operates seemingly well in differing noise conditions. The NN1 output for predicting a test signal with SNR of 10 dB is shown in Figure 2. SNR gain is the difference between the predictor output and input signal SNRs.

Table 1: SNR gains for the training and test sweeping sinusoids with input SNR 10 dB and 0 dB.

Signal	NN1	NN2	NN3	NN4
training	9.8 dB	4.8 dB	2.6 dB	-0.5 dB
test, 10 dB	5.4 dB	3.0 dB	1.4 dB	-0.3 dB
test, 0 dB	7.9 dB	3.2 dB	2.9 dB	1.5 dB

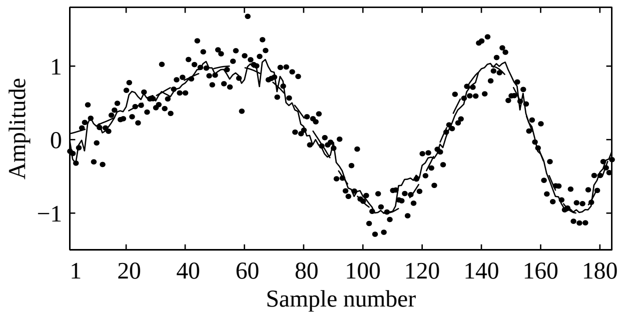

Figure 2: Test input (dotted) with SNR of 10 dB and maximum frequency of 10 Hz, and target output (dashed) sweeping sinusoidal signal, along with the NN1 output signal (solid).

3. FREQUENCY DOMAIN PROPERTIES

A. Sinusoidal Response

NN responses to sinusoidal input signals give an idea of NN behavior when the input signal contains smooth components [1], [2]. Given the sinusoidal input $x_{i,j}(n)$ where n is the time index, i is the fundamental frequency index, and j is the phase index, the discrete spectrum $Y_{i,j}(k)$ of the filter output $y_{i,j}(n) = \hat{x}_{i,j}(n)$ can be computed by the fast Fourier transform (FFT). The estimated transfer function $\hat{H}_s(k, j)$ at the frequency k corresponds to the value of the magnitude spectrum at the fundamental frequency $i = k$:

$$\hat{H}_s(k, j) = \sum_{i=0}^{N_f - 1} Y_{i,j}(i)\delta(i-k) \tag{2}$$

where N_f (here 256) is the number of fundamental frequency components and $\delta(\cdot)$ is the Dirac delta function. More reliable estimates are obtained by considering different input signal phases j, and averaging the estimated transfer functions. This approach was successfully used in [1] and [2] to analyze median-type filters.

All the frequency response estimates exhibit lowpass behavior, and the passband bandwidth becomes larger, as the maximum frequency of the training signal increases from NN1 to NN4. Also, an interesting result can be seen comparing NN1 with the linear polynomial Heinonen-Neuvo (H-N) predictor of the first degree and length 15 [6]: the two transfer functions closely resemble each other (Figure 3). From Figure 3 it can also be seen that NN1 exhibits lower passband gain peak than the corresponding H-N predictor.

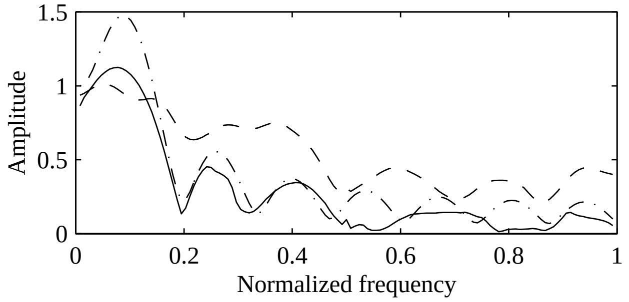

Figure 3: Sinusoidal frequency response estimates of NN1 (solid) and NN4 (dashed) along with the frequency response of the first degree H-N polynomial predictor of length 15 (dash-dot).

B. Power Spectrum

When the input signal is a sequence of AWGN samples with mean μ=0 and variance σ^2=1, the frequency behavior can be characterized by the power spectrum. The Welch method of averaging modified periodograms is particularly well suited for direct computation of the nonparametric power spectrum estimate [7]. The data record of length Q (here 2048) is sectioned into $L=Q/M$ segments of M samples (here 256). Each segment is Fourier transformed, in order to have L modified periodograms, and accumulated to obtain the power spectrum estimate. The estimated NN transfer function can be computed as the square root of the output-to-input power spectrum ratio, given in the discrete frequency domain by

$$\hat{H}_p(k) = \sqrt{\frac{B_{yy}(k)}{B_{xx}(k)}} \qquad (3)$$

where $B_{yy}(k)$ is the NN output power spectrum, and $B_{xx}(k)$ is the NN input power spectrum.

The results show clear lowpass behavior for NN1, and its frequency response estimate, shown in Figure 4, closely resembles that illustrated in Figure 3 for NN1.

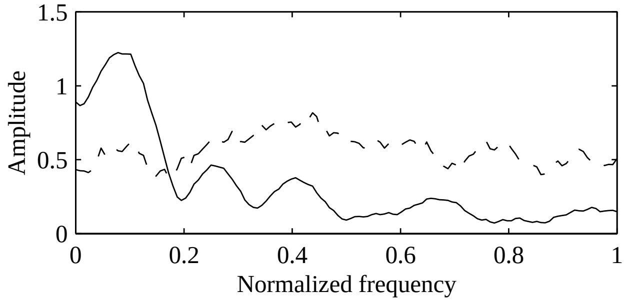

Figure 4: Welch method frequency response estimates of the NN1 (solid) and NN4 (dashed).

As the training signal bandwidth increases, the lowpass property becomes less pronounced, and is not any more present in the frequency response estimate for NN4 (Figure 4). Also, comparing the frequency response estimates for NN4 in Figures 3 and 4, it is noted that the frequency response estimates obtained by the two methods reveal input signal-dependent response characteristics.

C. Noise Gains

From transfer function estimates $\hat{H}_s(k)$ and $\hat{H}_p(k)$, it is possible to estimate the noise gains (NG) of the NNs by

$$NG = \frac{1}{N}\sum_{k=1}^{N}\left|\hat{H}(k)\right|^2 . \qquad (4)$$

The noise gain estimates are given in Table 2.

Table 2: Estimated noise gains of the NNs using the two transfer function estimates.

Estimate used	NN1	NN2	NN3	NN4
$\hat{H}_s(k)$	0.19	0.35	0.33	0.36
$\hat{H}_p(k)$	0.22	0.33	0.31	0.33

4. TIME DOMAIN BEHAVIOR

Since new harmonics and subharmonics of the input frequencies can be generated by a non-linear system, the nature of the time response is generally dependent on the input. To get insight to the time domain behavior, NN responses to unit step and triangular pulse signals are analyzed.

Our trained NNs usually seem to overestimate the constant zero level input, and underestimate unity input. Only NN3 exhibits underestimation also with constant zero input while NN2 outputs constant zero level quite accurately.

A. Step Response

The unit step responses reveal that the output settling time is the same as the number of input nodes, as natural, but during this period the number of oscillations in the output increase from NN1 to NN4. Furthermore, it can be highlighted that the prediction is biased, partly due to the non-unity DC gain, also visible from Figure 3 for NN1. The unit step response for NN1 is shown in Figure 5. Biasing effect could be eliminated by employing training signals which include segments of constant zero and unity levels.

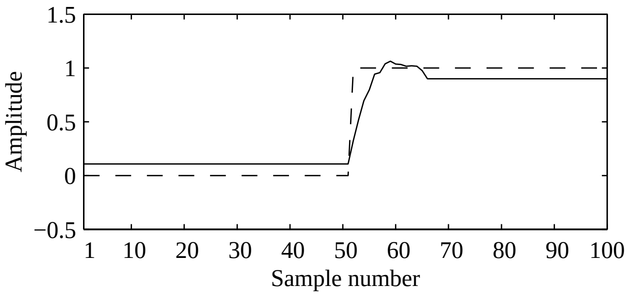

Figure 5: NN1 unit step response (solid) along with the unit step excitation (dashed).

B. Triangular Pulse Response

A triangular pulse can be seen as a simplified model for a slowly varying signal. Depending on the slope of the rising (setting) ramp, the signal has a different bandwidth. Our triangular pulse has unity maximum amplitude. The prediction accuracy depends on the slope, and gives clearly better results during the rising input. The results show that these NNs are actual predictors, as seen in Figure 6 for NN1, and the prediction is biased, as in the case of a unit step input (Figure 5).

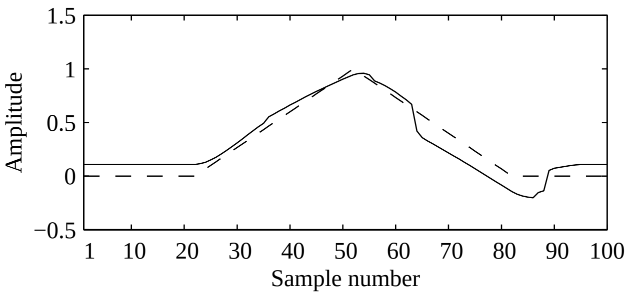

Figure 6: NN1 triangular pulse response (solid) along with the triangular pulse (dashed).

5. CONCLUSIONS

The frequency response characteristics of the predictors based on neural networks are naturally related to the frequency contents of the training signals. Transfer function estimates computed with the two different methods are generally different because NNs are nonlinear systems and their responses depend on the input signals. Nevertheless, if an NN has good generalization capabilities, as NN1, the two estimated transfer functions closely resemble each other, and in this case also the polynomial predictor transfer function, as expected because of the nearly piece-wise polynomial nature of the training signals.

To have the best prediction performance, the training signal should resemble the frequency contents of the signal to be predicted, and also the input signal should be preprocessed, in order to match the training signal amplitude characteristics. After prediction, reverse postprocessing has to be applied.

By computing the frequency response and time domain behavior estimates, an NN designer gains valuable insight into the NNs designed, and can use this additional information in neural network parameter selection and applicability considerations.

ACKNOWLEDGMENTS

The work of J. Tanskanen has been funded by the Technology Development Centre of Finland, Nokia Corporation, Telecom Finland, and Helsinki Telephone Company, and has also been supported by the Päijät-Häme Fund under the Auspices of the Finnish Cultural Foundation, and by Foundation of Technology, Finland.

REFERENCES

[1] J. Neejärvi, P. Heinonen, and Y. Neuvo, “Sine wave responses of median type filters” in *Proc. 1988 IEEE International Symposium on Circuits and Systems*, Espoo, Finland, June 1988, pp. 1503-1506.

[2] J. Neejärvi and Y. Neuvo, “Sinusoidal and pulse responses of FIR-median hybrid filters,” *IEEE Trans. Circuits and Systems*, vol. 37, pp. 1552-1556, Dec. 1990.

[3] X. M. Gao, J. M. A. Tanskanen, and S. J. Ovaska, “Comparison of linear and neural network-based power prediction schemes for mobile DS/CDMA system,” in *Proc. IEEE 46th Vehicular Tech. Conf.*, Atlanta, GA, Apr. 1996, pp. 61-65.

[4] D. E. Rumelhart and J. L. McClelland (Eds.), *Parallel Distributed Processing: Exploration in the Microstructure of Cognition*, Vols. 1 & 2. Cambridge, MA: MIT Press, 1986.

[5] H. Demuth and M. Beale, *Neural Network Toolbox for Use with Matlab*. South Natic, MA: the Math Works, May 1995.

[6] P. Heinonen and Y. Neuvo, “FIR-median hybrid filters with predictive FIR substructures,” *IEEE Trans. Acoustics, Speach, and Signal Processing*, vol. 36, pp. 892-899, June 1988.

[7] J. G. Proakis and D. G. Manolakis, *Digital Signal Processing: Principles, Algorithms, and Applications* (2nd ed.). New York, NY: Macmillan Publishing Company, 1988.