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Polynomial Prediction of Noise Shaping Rayleigh Fading

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ABSTRACT

In this paper, polynomial prediction of Rayleigh fading power signal produced by noise shaping is investigated. We focus on the processing of the measured input signal for power control purposes of a Code Division Multiple Access (CDMA) mobile station receiver. Predictive filtering based on a polynomial signal model is proposed, and a class of FIR-type polynomial predictors is investigated with simulations. Two alternative schemes are examined; direct prediction of the squared power signal, and prediction of the in-phase and quadrature signal components separately. The simulations show that finite impulse response (FIR) polynomial predictors can provide smoothed signal power samples with the signal-to-noise ratio (SNR) improved by ca. 5 dB, without any delaying of the signal. The results show that polynomial prediction is a highly potential tool for delayless filtering of additive noise and smoothing of fast fading of the power signal.

1. INTRODUCTION

The capacity, i.e., the number of simultateous users, of a Code Division Multiple Access (CDMA) system crucially depends on interference from other users. This interference can be reduced by employing a power control scheme to keep the transmission power of each user as low as possible thus increasing the capacity of the system [1], [2]. As power estimation and other processing of the signal power estimate introduce delay into the closed power control loop, a compensating predictive power estimation is proposed in this paper. A CDMA power control loop with predictive power level estimation is shown in Fig. 1.

The function of predictive filtering is twofold: to predict future values of the power signal, and to reduce the additive noise and interferences corrupting the power signal. Usually, the latter function is more important, i.e., *delayless smoothing* of the power signal. Our paper does not consider the power control problem itself but addresses the prediction approach of the received signal power as seen by the base station without power control.

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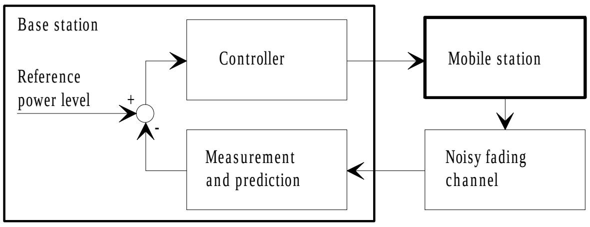

Fig. 1. *Power control loop in a CDMA system.*

2. CHANNEL MODEL

A single path propagation Rayleigh fading channel power response model with vertical monopole antenna geometry [3] was chosen to be analyzed. Simulated channel power response at 5 km/h is shown in Fig. 2. The fading was generated by a noise shaping method. Our power response simulation consists of a sum of two independent zero mean white Gaussian noise processes (WGN), which are independently shaped according to the antenna geometry by a corresponding noise shaping filter (NSF). The power signal components are contaminated by additive zero mean Gaussian noise (AWGN) before squaring and summing the components to produce SNRs of 10 dB and 0 dB in the components. The sum of two independent shaped Gaussian noise processes is guaranteed to be Rayleigh distributed. The simulator is illustrated in Fig. 3. Our carrier frequency was 1800 MHz, the sampling rate of the unmodulated in-phase and quadrature components was 1 kHz, and the applied vehicle speeds were 5 km/h and 50 km/h.

These power signal simulations were modeled by a polynomial signal model. With reference to the

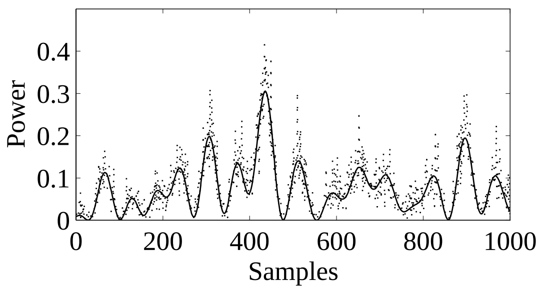

Fig. 2. *Noisy (dotted) and noiseless (solid) power signal simulations of one second at 5 km/h.*

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noiseless power signal in Fig. 2, it is easy to see that the piecewise polynomial model is expected to suit well for this signal.

3. OVERVIEW OF POLYNOMIAL PREDICTORS

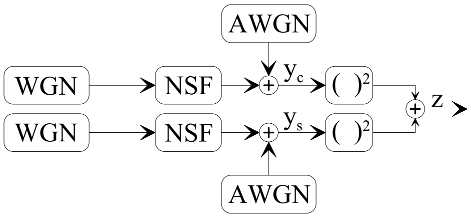

Fig. 3. *Block diagram of the noise shaping Rayleigh fading simulator. WGN is a white Gaussian noise process, NSF is a noise shaping filter and AWGN is an additive white Gaussian noise process.*

As the signals at hand closely resemble low degree polynomials, it was natural to use polynomial predictors to obtain predictive power estimates. The polynomial signal model of degree L is given by

$$\tilde{y}(n) = \nu_0 + \nu_1 n + \ldots + \nu_{L-1} n^{L-1} + \nu_L n^L + e(n) \quad (1)$$

where $\tilde{y}(n)$ is the value of the polynomial at n which is the index of a discrete-time sequence, $e(n)$ is an additive noise term and ν_i are the weighting coefficients with $i=0, 1, ..., L$. Generally, a predictor approximates future values of a signal from earlier signal samples, i.e., the prediction is a sum of weighted past signal samples y given by

$$\hat{y}(n) = \sum_{i=1}^{k} h(i) y(n-i) \quad (2)$$

with weights $h(i)$. The noise gain of this predictor is defined in both time and frequency domains as

$$NG = \sum_{i=-\infty}^{\infty} |h(i)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| H\left(e^{j\omega}\right) \right|^2 d\omega \quad (3)$$

where $H\left(e^{j\omega}\right)$ is the predictor transfer function.

A. Heinonen-Neuvo predictors

Heinonen-Neuvo (H-N) polynomial predictors [4] were chosen because they are derived from the conditions for unbiased prediction using the polynomial signal model. Furthermore, there exist closed form expressions for the predictor coefficients. The H-N predictors are derived to minimize the noise gain of the predictor in case of polynomial signals corrupted by zero mean white noise. In [4], the closed form predictor coefficients are given for predicting polynomials up to degree three, along with the generalization to the prediction of higher degree polynomials. The coefficients for the first and second degree predictors are given by

$$h_1(i) = \frac{4k - 6i + 2}{k(k-1)} \quad (4)$$

$$h_2(i) = \frac{9k^2 + (9 - 36i)k + 30i^2 - 18i + 6}{k^3 - 3k^2 + 2k}, \quad (5)$$

respectively, where the degree L of the predictor is denoted by the subscript of h, i=1, ..., k, and k is the length of the predictor. H-N predictors are of lowpass type with the passband bandwidth and passband gain decreasing with the increasing predictor length. Magnitude responses of first and second degree H-N predictors of lengths 20 and 50 are presented in Fig. 4.

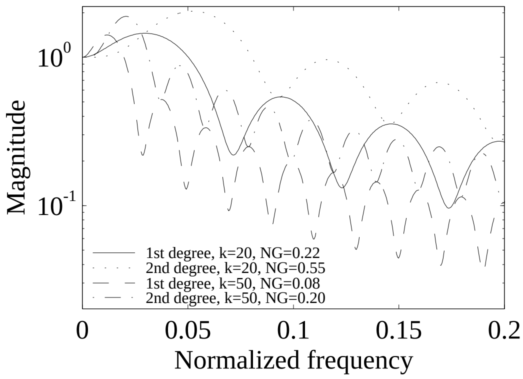

Fig. 4. *Magnitude responses of some H-N predictors.*

Although H-N predictors are intuitive for the power prediction purpose, some other predictor types might be perhaps even more applicable on the grounds of lower noise gains and better stopband attenuation. For example a system with an IIR prefilter with optimized FIR postpredictor [6], or recursive linear smoothed Newton predictor [7], could be considered as good predictor candidates.

4. POWER PREDICTION

The received high-frequency real-valued band-pass signal is transformed to equivalent complex-valued low-pass signal by demodulation at the receiver. The simulator, Fig. 3, creates the in-phase and quadrature components of the signal separately before squaring and summing them to produce the power signal. This signal construction suggests two different schemes for power prediction: 1) direct prediction of the noisy power signal which has been calculated from the noisy components, Fig. 5(a), and 2) getting the predictive estimate of the power signal as the sum of squared predictions of the components, Fig 5(b). In Fig. 5, y_c and y_s are the noisy in-phase and quadrature components, respectively. Detailed statistical analyzes of the two approaches are presented in [5].

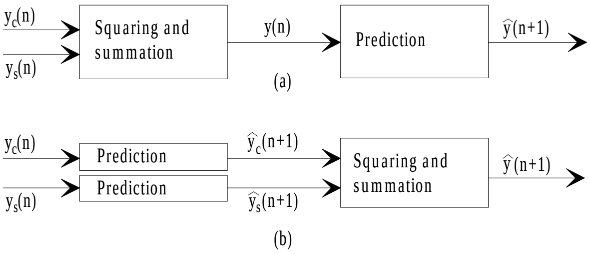

Fig. 5. *Block diagrams of the two prediction schemes.*

In the first approach, although the noisy power signal is always positive, the values of its estimate might be negative when a predictor with a long impulse response is used and the noisy input signal approaches zero. This is unnatural for a power control feedback loop, like in Fig. 1. On the other hand, the predictive power estimate is guaranteed to be strictly positive in the second approach, because of the final squaring operations. Noticing the variation of signal bandwidths before and after squaring, the second approach provides a possibility of reducing the prediction error by using narrower-band predictors and removing most of the noise as early as possible. The computational costs of the first and second approach may be different since they demand one or two predictors, respectively.

Simulation results presented in this paper are based on one-step-ahead predictions of both the power signal, and its in-phase and quadrature components.

5. SIMULATION RESULTS

Several H-N predictors were used with the noise shaping channel model with estimated SNRs 10 and 0 dB in the in-phase and quadrature components. The most interesting results are plotted in Figs. 6 - 9. In these figures the improvement in power signal SNR, i.e., the SNR gain, is shown.

A. Performance measure

The input and output SNRs were defined as

$$SNR_{in} = \frac{\sum_n \left[\left(x_c(n)^2 + x_s(n)^2\right)^2\right]}{\sum_n \left[\left(y_c(n)^2 + y_s(n)^2 - x_c(n)^2 - x_s(n)^2\right)^2\right]}, \quad (6)$$

$$SNR_{out} = \frac{\sum_n \left[\left(x_c(n+1)^2 + x_s(n+1)^2\right)^2\right]}{\sum_n \left[\left(\hat{y}(n) - x_c(n+1)^2 - x_s(n+1)^2\right)^2\right]}, \quad (7)$$

respectively, where $\hat{y}(n)$ is an output power sample, n spans over the sample sequence used for computing the SNRs, and $x_c(n)$ and $x_s(n)$ are noiseless input samples. The definitions resulted from the requirement of equal input and output SNRs when no filtering was applied. The SNR gain, shown in Figs. 6 - 9, was calculated by (8) subtracting the input SNR from the output SNR .

$$\text{SNR gain } (dB) = SNR_{out}(dB) - SNR_{in}(dB) \quad (8)$$

Least square (LS) optimal FIRs [10] were designed for each case and used to filter exactly the same sample sequence as used for designing the filters. This provides an 'upper bound' for the fixed-coefficient predictor performance.

B. Results

From Fig. 6, it can be seen that in low noise conditions at slow speeds the prediction in components requires longer predictors to reach the same SNR as the direct prediction of the power signal. In such cases, the prediction should be done directly from the power signal. The highest SNRs reached by both the 1st and 2nd degree power signal predictions are nearly equal. Under high noise conditions at low speeds, Fig. 7, prediction in components exhibits clearly better noise attenuation than the direct prediction of the power signal. In these cases the prediction should be done in components.

From Fig. 8, it can be concluded again that in the low noise conditions the direct prediction of the power signal performs better than the prediction of components. In this case no SNR improvement was evident. Because of the wide bandwidth of the signal, the predictors cannot predict the high frequency fast fading evident in the signal. As it is usually not desirable for the power controller to follow the fast fading, this behavior is actually desirable. At 50 km/h the SNR curves exhibit some non-monotony behavior, especially when the 2nd degree prediction in components is used. This is due to the fact that at this speed some predictors are not able to follow all characteristics of the noisy fading power signal. In Fig. 9 at 50 km/h, the results are similar to those of Fig. 7, i.e., under high noise conditions the maximum noise attenuation is achieved by using prediction in components. As the high frequency noise power in the case presented in Fig. 9 is greatly reduced by predictive filtering, the results are better than in Fig. 8 where there is less noise power to begin with, although also in the high speed high noise power case some fast fading is filtered out.

Similar simulations were also run using Jakes' Rayleigh fader [11] consisting of a sum of sinusoids at appropriately chosen Doppler shift frequencies. Also the effect of predicting only a low frequency part of the channel response was studied, as the power control system might be designed to utilize this 'slow' fading information only. Qualitatively the results were strikingly similar to those presented here. In the band-limited prediction case the results were better than presented here.

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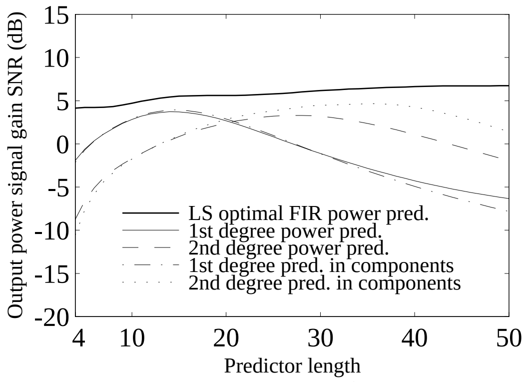

Fig. 6. *Output SNR gains of H-N predictors as functions of the predictor length.* (*Estimated component input SNR 10 dB, speed 5 km/h.*)

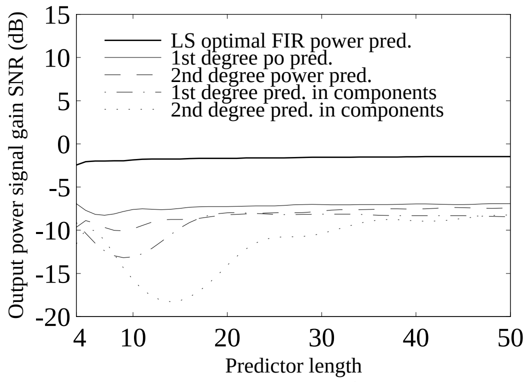

Fig. 8. *Output SNR gains of H-N predictors as functions of the predictor length.* (*Estimated component input SNR 10 dB, speed 50 km/h.*)

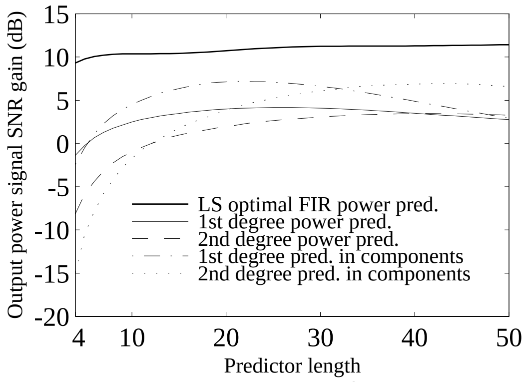

Fig. 7. *Output SNR gains of H-N predictors as functions of the predictor length.* (*Estimated component input SNR 0 dB, speed 5 km/h.*)

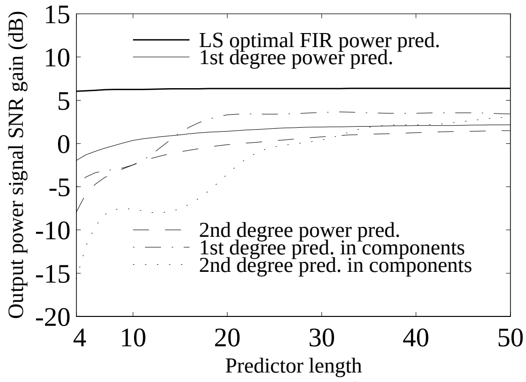

Fig. 9. *Output SNR gains of H-N predictors as functions of the predictor length.* (*Estimated component input SNR 0 dB, speed 50 km/h.*)

6. CONCLUSIONS

We find predictive filtering a highly potential tool for power control in the uplink transmission of CDMA systems. The results encourage to continue the work by applying predictors to power control loop simulations. Apart from the advantageous prediction and smoothing for short-term power control, predictive techniques may also be useful in forecasting the longer-term power level needed for the control of handovers from one base station to another.

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