A PI- Power Control Algorithm for Cellular Radio Systems

Zekeriya Uykan, Riku Jäntti and Heikki N. Koivo

The authors are with Control Eng. Lab., Helsinki University of Technology, Finland. E-mail: {zekeriya.uykan,riku.jantti,heikki.koivo}@hut.fi .

Abstract

Along with "*distributiveness*", convergence speed of power control is one of the most important criteria by which we can determine the practical applicability of a given power control algorithm. A good power control algorithm should quickly and distributively converge to the state where the system supports as many users as possible. This paper proposes a fast and distributed power control algorithm based on the well-known PI-controller. As in the paper by Foschini and Miljanic, we start with differential equation form of the controller and analyze its convergence properties in the case of *feasible* systems. The actual power control algorithm is then derived by discretization of the continuous time version. Using the *distributed constrained power control* (DCPC) as a reference algorithm, we carried out computational experiments on a CDMA system. The results indicate that our algorithm significantly enhances the convergence speed of power control.

Keywords

Power Control Algorithms, DS-CDMA Radio Systems.

I. Introduction

Effective transmitter power control is essential for high-capacity cellular radio systems. Power Control (PC) problem has drawn much attention since Zander's works on centralized [2] and distributed [3] *SIR balancing*. SIR balancing was further investigated by Grandhi *et al.* [5], [7]. In [4], Foschini and Miljanic considered a more general and realistic model, in which a positive receiver noise and a respective target SIR were taken into account. The Foschini and Miljanic's distributed algorithm (FMA) was shown to converge either synchronously [4] or asynchronously [6] to a fixed point of a *feasible* system. Based on the FMA, Grandhi *et al.* [9] suggested *distributed constrained power control* (DCPC), in which a transmission upper limit was considered. DCPC has become one of the most widely accepted algorithms by the academic community. Meanwhile, a framework on convergence of the *generalized uplink power control* was provided by Yates [11] and has been recently extended by Huang and Yates [12]. The results in [11] and [12] have become a breakthrough, providing guidelines for designing and analyzing new algorithms.

So far most of the power control algorithms suggested in literature have had first order structure. That is, they have made power updates based only on the the current power and SIR values. Recently in [10], a second-order power control algorithm (SOPC) requiring power levels of current and previous iterations was suggested. The algorith presented here is on the line of the SOPC. Unlike many of the PC algorithms, e.g. DPC and SOPC, our algorithm does not have roots in the numerical linear algebra, but in the linear control theory and we end up with the PI-controller, one of the most widely utilized controllers in the process industry because of its simplicity and robustness [13]. The use of PI-control in the context of transmission power control was firstly suggested by Blom and Gunnarsson [1]. The algorithm suggested here differs from theirs by the fact that our controller is linear while theirs is log-linear. The advantage of using linear controller is that we can relate the controller parameters to the parameter determining the feasibility (existence of positive solution vector) of the system, namely to the spectral radius of the normalized link gain matrix.

As in many other papers (e.g [3]), we consider *snapshot* analysis in which link gains are frozen until the PC algorithm converges and assume that the transmission quality is measured in terms of Carrier-to-Interference+noise ratio (CIR). Since our focus is on the performance of power control algorithm, the effect of admission control, base station assignment etc. are neglected from the analysis.

This paper is organized as follows: PC problem is described in Section II. Section III presents the differential and difference equation forms of the proposed algorithm. Simulation results are presented in Section IV followed by Concluding Remarks in Section V.

II. CDMA Power Control Problem

Suppose a cellular radio system, in which N mobiles share the same channel at a given instance. Without loss of generality, we consider the uplink only and assume that mobile i is assigned to base i at that instant. If several mobiles are assigned to the same base station, say e.g. mobiles i and j, then in our notation i and j refer to the same physical base station. Further, we assume that the signal of mobile i will be received correctly if the CIR at base i is not less than a given target value γ_i^{tgt}. However, since the ideal situation is to make connection with the minimal transmission power, we have the following CIR constraint on mobile i:

$$\gamma = \frac{a_{ii}p_i}{\sum_{j=1,j\neq i}^{N} a_{ij}p_j + \nu_i} \geq \gamma_i^{tgt}, \quad i = 1, \ldots, N \tag{1}$$

where p_i is the transmission power of mobile i, a_{ij} is the link gain from mobile j to base i and ν_i is the receiver noise at base station i. It is assumed that adjacent channel interference is negligible.

Defining $H_{ij} = [\mathbf{H}]_{ij} = \gamma^i_{tgt} a_{ii}/a_{ij}$ and $H_{ii} = 0$ and $\eta_i = \gamma^i_{tgt}\nu_i/a_{ij}$ one obtaines (1) in matrix form as follows

$$(\mathbf{I} - \mathbf{H})\mathbf{p} = \boldsymbol{\eta} \tag{2}$$

where $\mathbf{p}$ is the power vector, $\mathbf{H}$ is the normalized link gain matrix and $\boldsymbol{\eta}$ is the noise vector.

The problem in a fully distributed PC algorithm is how to change the power vector entries p_i using only the local measurements $\gamma^i_{tgt}, \gamma_i, p_i$ (possibly together with some of their history values) so that the power vector $\mathbf{p}$ converges to the optimal value $\mathbf{p^{opt}}$ as fast as possible.

$$\mathbf{p}^{opt} = (\mathbf{I} - \mathbf{H})^{-1}\eta \tag{3}$$

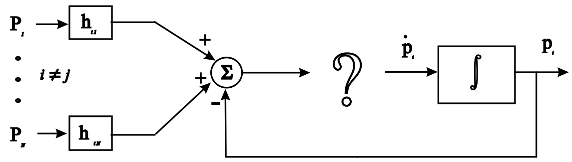
Throughout the paper we assume that the system is feasible. That is, the equation (2) has positive solution. The positivity condition is equivalent to requiring that the spectral radius of $\mathbf{H}$-matrix $\rho(\mathbf{H})$ must be less than one (see e.g. Theorem 3.9 in [14]).

III. Algorithms

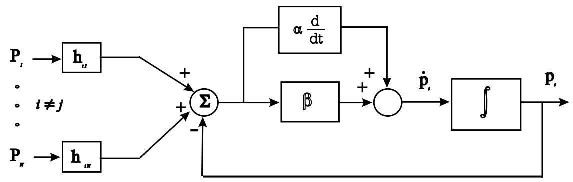
In this section, we offer a continuous model for PC algorithm, and then discretize it as done in [4]. The approach taken here is quite general and other linear controllers could be treated in the similar manner.

A. Differatial Equation Form of the Algorithm

Our starting point is shown in Fig. 1: How to design a Multiple Input Multiple Output (MIMO) system whose equilibrium point $p_i^{opt} = \sum_{j=1, j\neq i}^{N} h_{ij} p_j^{opt} + \eta_i$ is achieved as fast as possible. In addition, we have limitations on the availability of information based on which we should make the power updates: The power p_i must be updated using only the local measurements $\gamma^i_{tgt}, \gamma_i, p_i$ (possibly together with some of their history values).



(a)



(b)

Fig. 1. Controller design problem for fast and distributed PC.

From Fig. 1.b,

$$\dot{p}_i = -\beta(p_i - \sum_{j\neq i}^{N} h_{ij}p_j - \eta_i) - \alpha\frac{d}{dt}(p_i - \sum_{j\neq i}^{N} h_{ij}p_j - \eta_i) \tag{4}$$

which can equivalently be written as

$$\dot{p}_i = -\beta\Big(p_i - \frac{\gamma_i^{tgt}}{\gamma_i}p_i\Big) - \alpha\frac{d}{dt}\Big(p_i - \frac{\gamma_i^{tgt}}{\gamma_i}p_i\Big) \quad i = 1, \ldots, N \tag{5}$$

Eq.5 can be seen as a standart PI controller [13] in which the error is the difference between current power p_i and the optimal power given that all the other users would keep their powers constant $\frac{\gamma_i^{tgt}}{\gamma_i}p_i$.

Let us define the matrices $\mathbf{B}$ and $\mathbf{C}$ as follows

$$\mathbf{B} = \mathbf{I} - \mathbf{H} \quad and \quad \mathbf{C} = (\mathbf{I} + \alpha\mathbf{B})^{-1} \tag{6}$$

Then (4) can be written in matrix form as

$$\dot{\mathbf{p}} = -\beta\mathbf{C}\mathbf{B}\mathbf{p} + \beta\mathbf{C}\boldsymbol{\eta} \tag{7}$$

By setting $\alpha = 0$, equation (7) becomes equivalent to the differential form of the FMA. Now we would like to compare the PIPC and FMA in terms of converge speed.

Proposition 1: PIPC converges to $\mathbf{p^{opt}}$ starting from arbitrary initial power $\mathbf{p(0)}$ if the system is feasible (i.e. $\rho(\mathbf{H}) < \mathbf{1}$), $-\frac{1}{1-\rho(\mathbf{H})} < \alpha < \frac{1}{1-\rho(\mathbf{H})}$ and $\beta > 0$. Furthermore, PIPC converges faster than FMA if $-\frac{1}{1-\rho(\mathbf{H})} < \alpha < 0$.

Proof: Consider the weighted maximum norm of the difference between the current and the optimal power vectors $||\mathbf{p(t)} - \mathbf{p^{opt}}||_\infty^{\mathbf{W}}$. The solution to differential equation (7) $\mathbf{p(t)}$ can be written as follows

$$\mathbf{p(t)} = \exp\left(-\beta\mathbf{CBt}\right)\mathbf{p(0)} + \int_{\mathbf{0}}^{\mathbf{t}} \exp\left(-\beta\mathbf{CB(t-s)}\right)\beta\mathbf{C}\boldsymbol{\eta}\mathbf{ds} \tag{8}$$

Therefore the norm of error is upper bounded as follows

$$||\mathbf{p(t)}-\mathbf{p^{opt}}||_\infty^{\mathbf{W}} \leq ||\exp\left(-\beta\mathbf{CBt}\right)||_\infty^{\mathbf{W}}||\mathbf{p(0)}||_\infty^{\mathbf{W}} + \left|\left|\int_{\mathbf{0}}^{\mathbf{t}} \exp\left(-\beta\mathbf{CB(t-s)}\right)\ \beta\mathbf{Cds} - \mathbf{B^{-1}}\right|\right|_\infty^{\mathbf{W}} ||\boldsymbol{\eta}||_\infty^{\mathbf{W}} \tag{9}$$

Since $\mathbf{H}$ is a nonnegative and irreducible matrix (by Lemma 2.1 in [8]), the Perron-Frobenius theorem (see e.q Theorem 2.1 in [14]) guarantees that there exists a positive vector $\mathbf{e}$ such that $\mathbf{He} = \rho(\mathbf{H})\mathbf{e}$. Thus by choosing $\mathbf{W} = \mathrm{diag}\left\{\frac{\mathbf{1}}{\mathbf{e_i}}\right\}$, the inequality (9) can be written as

$$||\mathbf{p(t)} - \mathbf{p^{opt}}||_\infty^{\mathbf{W}} \leq \exp\left(-\frac{\beta(\mathbf{1}-\rho(\mathbf{H}))}{\mathbf{1}+\alpha(\mathbf{1}-\rho(\mathbf{H}))}\mathbf{t}\right)\left(||\mathbf{p(0)}||_\infty^{\mathbf{W}} + \frac{\mathbf{1}}{\mathbf{1}-\rho(\mathbf{H})}||\boldsymbol{\eta}||_\infty^{\mathbf{W}}\right) \tag{10}$$

¿From the above, we conclude that as long as $\frac{\beta(1-\rho(\mathbf{H}))}{1+\alpha(1-\rho(\mathbf{H}))} > 0$ the PIPC converges and for $\alpha < 0$ the convergence is faster than for $\alpha = 0$ corresponding to the FMA algorithm. This concludes the proof. ∎

B. Difference Equation Form of the Algorithms

Difference equations may be obtained in different ways. By normalizing the time coordinate so that step interval corresponds to the consecutive power vector iterations and applying Forward Euler method to (7), one obtains

$$\mathbf{p}(k+1) = \Big(\mathbf{I} - \beta\mathbf{CB}\Big)\mathbf{p}(k) + \beta\mathbf{C}\boldsymbol{\eta} \tag{11}$$

By considering the same norm as was used in the proof of proposition 1. We can easily show that

$$||\mathbf{p(k+1)} - \mathbf{p^{opt}}||_\infty^{\mathbf{W}} \leq ||\mathbf{I} - \mathbf{CB}||_\infty^{\mathbf{W}} ||\mathbf{p(k+1)} - \mathbf{p^{opt}}||_\infty^{\mathbf{W}} \tag{12}$$

$$\leq \left|1 - \frac{\beta(1-\rho(\mathbf{H}))}{1+\alpha(1-\rho(\mathbf{H}))}\right| ||\mathbf{p(k+1)} - \mathbf{p^{opt}}||_\infty^{\mathbf{W}} \tag{13}$$

$$\tag{14}$$

¿From the above we see that the discretized PIPC is a pseudo-contraction mapping and thus convergent if $0 < \beta < 2(\frac{1}{1-\rho(\mathbf{H})} + \alpha)$ and $-\frac{1}{1-\rho(\mathbf{H})} < \alpha < \frac{1}{1-\rho(\mathbf{H})}$. Furthermore PIPC converges faster than FMA with respect to the weighted maximum norm if $0 < \beta < \frac{1}{1-\rho(\mathbf{H})} \frac{2+2\alpha(1-\rho(\mathbf{H}))}{2+\alpha(1-\rho(\mathbf{H}))}$.

Unfortunately, the algorithm given by (11) can not be implemented in distributed fashion if $\alpha \neq 0$. However, it gives valuable insight to the convergence properties.

By applying Forward Euler method to the derivatives of p_i and Backward Euler to the derivatived of γ_i in (5), we get

$$p_i(k+1) = \left(1 + \frac{-\beta\left(1 - \frac{\gamma_i^{tgt}}{\gamma_i}(k)\right) - \alpha\frac{\gamma_i^{tgt}}{\gamma_i^2(k)}(\gamma_i(k) - \gamma_i(k-1))}{1 + \alpha\left(1 - \frac{\gamma_i^{tgt}}{\gamma_i(k)}\right)}\right) p_i(k) \tag{15}$$

We call this version of PIPC as PIPC-I.

Alternatively, approximating the derivative part using two previous steps, the following difference equation may be obtained

$$p_i(k+1) = \left(1 - (\alpha+\beta)\left(1 - \frac{\gamma_i^{tgt}}{\gamma_i(k)}\right)\right) p_i(k) + \alpha\left(1 - \frac{\gamma_i^{tgt}}{\gamma_i(k-1)}\right) p_i(k-1) \tag{16}$$

This form, called PIPC-II, is similar to the standard "textbook" PI-controller.

The PIPC-II form is linear and can be written in the matrix form as

$$\begin{bmatrix} \mathbf{p(k+1)} \\ \mathbf{p(k)} \end{bmatrix} = \begin{bmatrix} \mathbf{I} - (\alpha+\beta)\mathbf{B} & \alpha\mathbf{B} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{p(k)} \\ \mathbf{p(k-1)} \end{bmatrix} + \begin{bmatrix} \beta\boldsymbol{\eta} \\ \mathbf{0} \end{bmatrix} \triangleq \mathcal{I}\left(\begin{bmatrix} \mathbf{p(k)} \\ \mathbf{p(k-1)} \end{bmatrix}\right) \tag{17}$$

Depending on initial condition and used parameters, it is possible that at some iterations the PIPC-II results in negative power values that are physically impossible to execute. In practical systems, the transmission powers are also upper bounded. To take these constrains into account, we suggest a confined version of the algorithm called constrained PIPC (CPIPC):

$$p_i(k+1) = \max\left\{0, \min\left\{\bar{p}_i, \left(1-(\alpha+\beta)\left(1-\frac{\gamma_i^{tgt}}{\gamma_i(k)}\right)\right)p_i(k)+\alpha\left(1-\frac{\gamma_i^{tgt}}{\gamma_i(k-1)}\right)p_i(k-1)\right\}\right\} \tag{18}$$

where $\bar{p}_i$ denotes the upper bound for power. It is notable that if $\beta = 1$ and $\alpha = 0$ the CPIPC is equivalent to DCPC.

Proposition 2: CPIPC converges to $\mathbf{p^{opt}}$ starting from any initial vectors $\mathbf{p(0)}$, $\mathbf{p(-1)}$ if the system is feasible (i.e. $\rho(\mathbf{H}) < \mathbf{1}$), $-\frac{1}{1-\rho(\mathbf{H})} < \alpha < \frac{1}{1-\rho(\mathbf{H})}$ and $\epsilon\alpha < \beta < \frac{2}{1-\rho(\mathbf{H})} + \alpha\epsilon$, for arbitrary small $\epsilon > 0$.

Proof: Let us denote the iteration matrix in (17) by $\mathbf{Z}$ and let us define the following vectors

$$\mathbf{z(k)} = \begin{bmatrix} \mathbf{p(k)} \\ \mathbf{p(k-1)} \end{bmatrix}, \quad \bar{\mathbf{z}} = \begin{bmatrix} \bar{\mathbf{p}} \\ \bar{\mathbf{p}} \end{bmatrix}, \quad \mathbf{z^{opt}} = \begin{bmatrix} \mathbf{p^{opt}} \\ \mathbf{p^{opt}} \end{bmatrix} \tag{19}$$

In terms of $\mathbf{z}$ and $\bar{\mathbf{z}}$ we can now write the CPIPC mapping as

$$\mathcal{T}(\mathbf{z(k)}) = \max\{\mathbf{0}, \min\{\bar{\mathbf{z}}, \mathcal{I}(\mathbf{z(k)})\}\} \tag{20}$$

Let us choose a new weight matrix

$$\mathbf{X} = \begin{bmatrix} \mathbf{W} & \mathbf{0} \\ \mathbf{0} & \frac{1}{1+\epsilon}\mathbf{W} \end{bmatrix} \tag{21}$$

It follows that

$$\begin{aligned} ||\mathcal{T}(\mathbf{z(k)}) - \mathbf{z^{opt}}||_\infty^{\mathbf{X}} &\leq ||\mathcal{I}(\mathbf{z(k)}) - \mathbf{z^{opt}}||_\infty^{\mathbf{X}} && (22) \\ &\leq ||\mathbf{Z}||_\infty ||\mathbf{z(k)} - \mathbf{z^{opt}}||_\infty^{\mathbf{X}} && (23) \\ &\leq |1+(\epsilon\alpha-\beta)(1-\rho(\mathbf{H}))| \cdot ||\mathbf{z(k)} - \mathbf{z^{opt}}||_\infty^{\mathbf{X}} && (24) \end{aligned}$$

Thus we may conclude that CPIPC is pseudo-contraction mapping and thus convergent. This concludes the proof. ■

IV. Simulation Results

A DS-CDMA system with 19 omni-bases located in the centers of 19 hexagonal cells is used as a test system. As in [10], we consider an IS-95 example, where the spreading bandwidth is 1.2288 MHz and the data rate is 9.6 Kbps (*processing gain* = 21dB). For a given instance, a total of 190 mobiles are generated, the locations of which are uniformly distributed over the 19 hexagonal cells (see Fig. 2). The link gain g_{ij} is modeled as $g_{ij} = s_{ij} \cdot d_{ij}^{-4}$, where s_{ij} is the shadow fading factor and d_{ij} is the distance between base i and mobile j. The log-normally distributed s_{ij} is generated according to the model in [17] (pp. 185-186, $E(s_{ij}) = 0$ dB, and $\sqrt{E(s_{ij}s_{kl})} = 8$ dB if $i = k$; $\sqrt{E(s_{ij}s_{kl})} = 4\sqrt{2}$ dB if $i \neq k$). The power level is confined between zero and one.

CPIPC with $\alpha = -0.2$ and $\beta = 1.2$ is compared with DCPC. The initial powers $p_i(0)$ were randomly chosen form the closed interval [0,1] and the initial powers $p_i(-1)$ were set to zero. The outage probability at each iteration is computed over 1000 feasible instances by counting the portion of the number of users whose CIR is more than 2% below their target CIR at the iteration. It was observed that CPIPC takes 16 iterations on average to reach the state with the outage probability of 10^{-4} that we consider as almost the zero-outage while DCPC takes 21 iteration on the average. The fact that CPIPC has higher convergence speed can be clearly seen from Fig. 3 in which the Euclidean distance between the current power vector and $\mathbf{p}^{\mathbf{opt}}$ is shown as a function of iterations.

V. Concluding Remarks

This paper proposes a fast and distributed power control algorithm which is obtained by discretization of its continuous model. The range of parameters for which the algorithm is stable and faster than FMA is investigated in both differatial and difference forms. The convergence speed is significantly improved in comparison with the FMA with $\beta = 1$. Simple computer simulations verified the effectiveness of the algorithm.

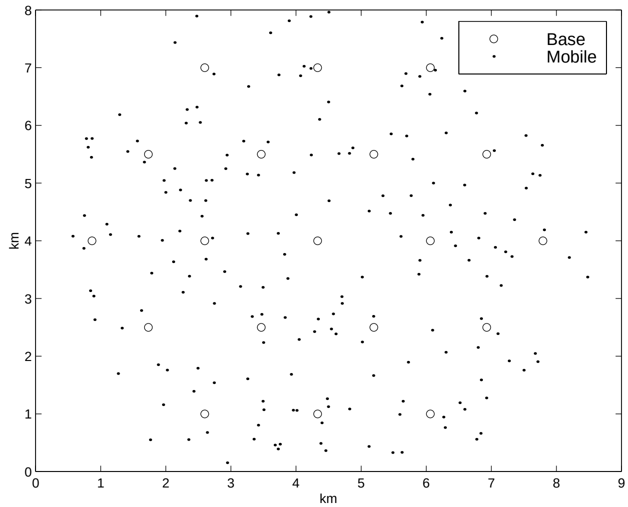


Fig. 2. DS-CDMA cellular system with 19 omni-bases.

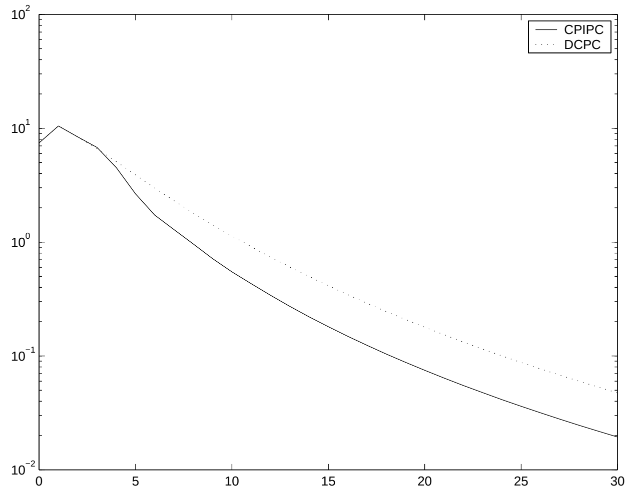


Fig. 3. Euclidean distance between the current power vector and $\mathbf{p}^{\mathbf{opt}}$ with respect to iteration.

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