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PROPERTIES OF THE MULTIPLICATIVE GENERAL PARAMETER ADAPTIVE ALGORITHM

Olli Vainio[1] *and Seppo J. Ovaska*[2]

[1]Tampere University of Technology, Department of Information Technology,
P. O. Box 553, FIN-33101 Tampere, Finland.
Tel. +358 3 3115 2928, Fax. +358 3 3115 3095, E-mail olli.vainio@tut.fi
URL: http://www.cs.tut.fi/ vainio/
[2]Helsinki University of Technology, Institute of Intelligent Power Electronics,
FIN-02150 Espoo, Finland.

ABSTRACT

The multiplicative general parameter (MGP) algorithm is a computationally efficient reduced-rank adaptive filter algorithm which typically uses only a few adaptive parameters. In this paper, we derive the stability condition of the MGP algorithm. Furthermore, use of the algorithm in polyphase filters is discussed. Examples of both single-rate and multirate adaptive polyphase filters are shown.

1. INTRODUCTION

Polyphase filter structures are an alternative to direct-form transversal filter structures, and are often used in efficient implementations of decimators and interpolators. The polyphase decomposition is also an important basis for many kinds of filter banks [1].

Several uses of adaptive algorithms in polyphase filters have been described in the literature. Iyer et al. [2] describe a polyphase adaptive structure, which is further generalized further in [3]. Since it is a transform domain adaptive algorithm, their technique is mostly suitable for applications where the impulse response can be very long, as in echo cancellation in telecommunication systems. Polyphase infinite impulse response (IIR) adaptive filters have been discussed, e.g., in [4] and [5], focusing primarily on identification of IIR systems, especially those with highly underdamped poles. Gerek and Cetin [6],[7] have investigated adaptive polyphase filter banks with perfect reconstruction property in image compression applications.

In this paper, we discuss the use of the MGP adaptive algorithm in polyphase finite impulse response (FIR) filters. The proposed algorithm facilitates reduced-rank adaptive filtering [8] in the sense that the number of adaptive parameters is typically smaller than the overall number of filter coefficients. Moreover, the algorithm is computationally simple, since no effort is devoted to statistically optimizing the direction of adaptation by using additional algorithmic complexity.

We first derive the stability condition of the MGP algorithm. Use of the algorithm in polyphase filter structures is then discussed with examples of system identification and adaptive prediction.

2. GENERALIZATION OF THE MGP ADAPTIVE ALGORITHM

In the original dual-parameter MGP-FIR filter [9],[10], the output is computed as:

$$y(n) = g_0(n) \sum_{k=0}^{N/2-1} h(k)x(n-k) + g_1(n) \sum_{k=N/2}^{N-1} h(k)x(n-k), \tag{1}$$

where $g_0(n)$ and $g_1(n)$ are the adaptive parameters, and the $h(k)$s are the coefficients of a fixed FIR basis filter. Thus, the coefficients of the composite filter are $\theta(k) = g_0(n)h(k)$ for $k = 0, 1, \cdots, N/2-1$, and $\theta(k) = g_1(n)h(k)$ for $k = N/2, \cdots, N-1$.

Here we consider a generalization of the algorithm with M parameters. Furthermore, the filter blocks can be interlaced and of arbitrary size. In the generalized form, the output is computed as:

$$y(n) = \sum_{j=0}^{M-1} g_j(n) \sum_{k \in G_j} h(k)x(n-k), \tag{2}$$

where G_j is the set of integers containing the tap indices related to parameter $g_j(n)$. Combined, these sets constitute the entire set $\{0, 1, \cdots, N-1\}$ and there is no overlap between the sets. The adaptation formula is:

$$g_j(n+1) = g_j(n) + \gamma e(n) \sum_{k \in G_j} h(k)x(n-k), \tag{3}$$

where γ is the gain factor and $e(n) = r(n) - y(n)$ is the output error or the difference between a reference signal $r(n)$ and the filter output.

The stability analysis can be carried out using a nearly similar approach as that used for the least mean squares (LMS) algorithm in [11]. Let us assume that the reference data sequence is generated by the linear time varying model

$$r(n) = \mathbf{x}(n)\mathbf{\Theta}_0(n) + v(n), \tag{4}$$

where $\mathbf{x}(n) = [x(n)\ x(n-1)\ \cdots\ x(n-N+1)]$. The true parameter $\mathbf{\Theta}_0(n)$ has a model of the form

$$\mathbf{\Theta}_0(n+1) = \mathbf{\Theta}_0(n) + \xi(n). \tag{5}$$

The variables $v(n)$ and $\xi(n)$ are considered to be noise or disturbances. The adaptation error is

$$\hat{\mathbf{\Theta}}(n) = \mathbf{\Theta}_0(n) - \mathbf{\Theta}(n), \tag{6}$$

where

$$\mathbf{\Theta}(n) = [\theta(0)\ \theta(1)\ \cdots\ \theta(N-1)]^{\mathrm{T}} \tag{7}$$

is the composite coefficient vector. Using (3),

$$\mathbf{\Theta}(n+1) = \mathbf{\Theta}(n) + \gamma \mathbf{S}(n)[r(n) - \mathbf{x}(n)\mathbf{\Theta}(n)]\mathbf{h}, \tag{8}$$

where

$$\mathbf{h} = [h(0)\ \ h(1)\ \cdots\ h(N-1)]^{\mathrm{T}}$$

is the vector of the fixed coefficients,

$$\mathbf{S}(n) = \begin{pmatrix} S_0(n) & 0 & 0 & \cdots & 0 & 0 \\ 0 & S_1(n) & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & S_{N-2}(n) & 0 \\ 0 & 0 & 0 & \cdots & 0 & S_{N-1}(n) \end{pmatrix},$$

and

$$S_j(n) = \sum_{j,k\, \in\, G_m} h(k)x(n-k),$$

$j = 0, 1, \cdots, N-1$. Here m is the index of the coefficient group G_m which is involved with parameter g_m and includes the jth tap. Therefore,

$$\hat{\mathbf{\Theta}}(n+1) = \hat{\mathbf{\Theta}}(n) + \xi(n)$$

$$-\gamma \mathbf{S}(n)[\mathbf{x}(n)\hat{\mathbf{\Theta}}(n) + v(n)]\mathbf{h}. \tag{9}$$

This can be written in the form

$$\hat{\mathbf{\Theta}}(n+1) = [\mathbf{I} - \gamma \mathbf{S}(n)\mathbf{h}\mathbf{x}(n)]\hat{\mathbf{\Theta}}(n)$$

$$+\xi(n) - \gamma \mathbf{S}\mathbf{h}v(n), \tag{10}$$

where $\mathbf{I}$ is a unit matrix of size $N \times N$. The stability of the algorithm, therefore, depends on the eigenvalues of the matrix $\mathbf{I} - \gamma \mathbf{S}(n)\mathbf{h}\mathbf{x}(n)$, which are equal to one except that given by

$$\lambda_c = 1 - \gamma \mathbf{x}(n)\mathbf{S}(n)\mathbf{h}. \tag{11}$$

Therefore, the algorithm converges when $-1 < \lambda_c < 1$, and γ should be chosen accordingly.

3. THE MGP ADAPTIVE ALGORITHM IN POLYPHASE FILTERS

Consider a polyphase implementation of a single-rate FIR filter, where the output is computed as

$$y(n) = \sum_{j=0}^{M-1} g_j(n) \sum_{k=0}^{L-1} h_j(k)x(n-j-kM), \tag{12}$$

where M is the number of branches and L is the number of coefficients in each subfilter. The fixed coefficients of the branch j are denoted by $h_j(k)$, $k = 0, 1, \cdots, L-1$. Thus, MGP adaptation is implemented so that the output of each fixed subfilter is multiplied by the adaptive parameter $g_j(n)$. Such a structure is computationally efficient because the branch filter outputs can be directly used in the parameter update formulae. It should be noticed, however, that this selection is often not the optimum from convergence point of view.

Compared with a fixed polyphase filter implementation, the additional computations required by the MGP algorithm consist of $2M+1$ multiplications and $M+1$ additions/subtractions.

The stability condition (11) is still valid, and we observe that

$$\mathbf{S}(n) = \begin{pmatrix} S_B(n) & 0 & 0 & \cdots & 0 & 0 \\ 0 & S_B(n) & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & S_B(n) & 0 \\ 0 & 0 & 0 & \cdots & 0 & S_B(n) \end{pmatrix},$$

where

$$\mathbf{S}_B(n) = \begin{pmatrix} S_0(n) & 0 & 0 & \cdots & 0 & 0 \\ 0 & S_1(n) & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & S_{M-2}(n) & 0 \\ 0 & 0 & 0 & \cdots & 0 & S_{M-1}(n) \end{pmatrix}$$

and

$$S_j(n) = \sum_{k=0}^{L-1} h_j(k)x(n-j-kM),$$

$j = 0, 1, \cdots, M-1$.

4. EXAMPLE 1

Consider a case where $M = 4$ and $L = 5$. Thus, there are four adaptive parameters and altogether 20 coefficients. The MGP algorithm is used in a system identification configuration [11]. The fixed coefficient vector $\mathbf{h}$ is designed as a least-squares bandpass filter with the passband at $0.25\pi \ldots 0.5\pi$. There are two systems to be modeled, one being a 10th-order Butterworth IIR filter with the passband at $0.25\pi \ldots 0.5\pi$, and the other is an FIR filter which is identical to $\mathbf{h}$. The input excitation is a dual-frequency signal ($\omega_1 = 0.25\pi$, $\omega_2 = 0.5\pi$).

A trace of the adaptive parameters is shown in Fig. 1, when the plant to be modeled is switched between the FIR filter (between sample numbers 0 to 200 and 401 to 600), and the IIR filter (sample numbers 201 to 400). For the FIR case, all the MGPs settle to unity value, as one would expect in the case of multi-frequency input. For the IIR case, the parameters settle to such values that the amplitude and phase responses of the polyphase filter match those of the plant for both ω_1 and ω_2. Here $\gamma = 3.6$, which sets the average value of λ_c to zero, providing the maximum speed of convergence.

The effect of the gain value on the speed of convergence is demonstrated in Fig. 2, where the step response of parameter $g_0(n)$ is plotted for $\gamma = 1.8$, 3.6, and 5.4. These gain values result in $\lambda_c = 0.5$, 0.0, and -0.5, respectively. As one would expect, for $\lambda_c = 0.5$ the response is slow, and for $\lambda_c = -0.5$ there are decaying oscillations. The fastest settling is achieved with $\lambda_c = 0$.

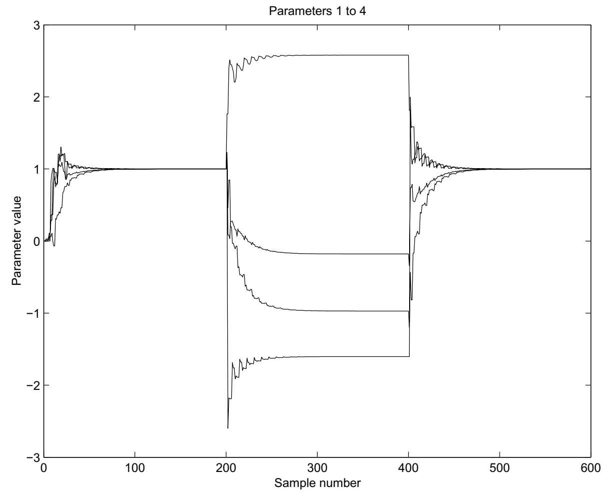

Fig. 1. Trace of the four adaptive parameters in system identification.

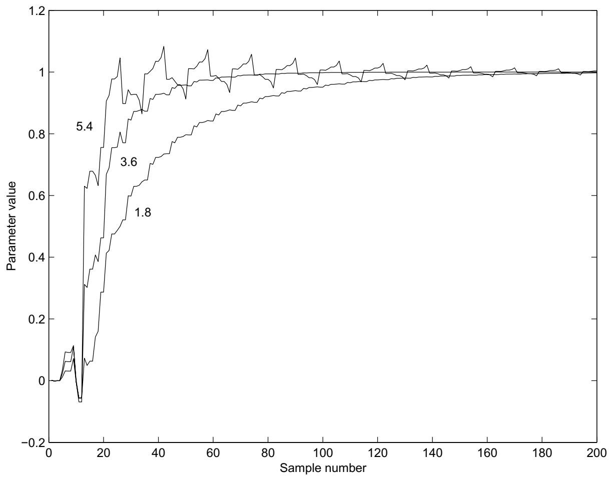

Fig. 2. Parameter step response for $\gamma = 1.8$, 3.6, and 5.4.

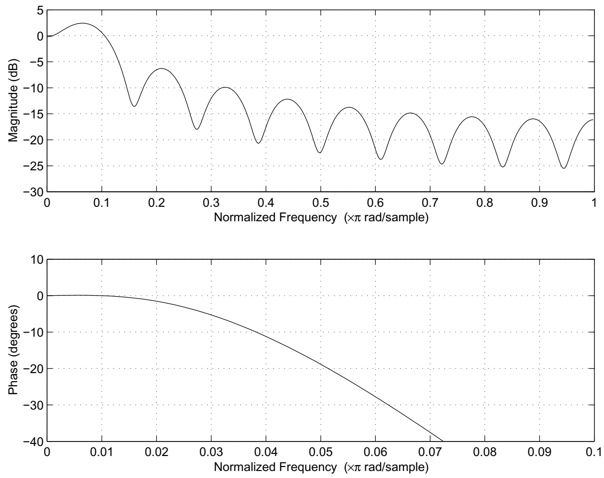

Fig. 3. Amplitude and phase (zoomed) responses of the predictive antialiasing filter.

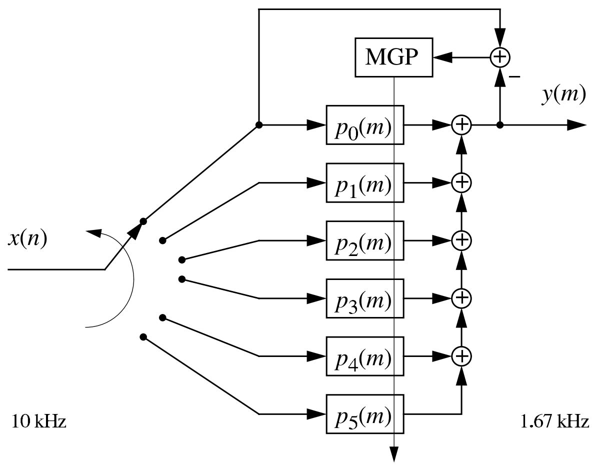

Fig. 4. Adaptive polyphase decimator.

5. EXAMPLE 2

Let us consider a polyphase decimator to be used for reducing the sampling rate from 10 kHz to 1.67 kHz when the primary signal of interest is a sine wave of the 50 Hz power line frequency. Decimation by six and antialias filtering are implemented using a polyphase filter with six branches. For synchronization purposes, decimation should be delayless in the sense that the output sample ideally tracks the latest input sample. Thus, a sine predictor [12] with a prediction step of zero is chosen as the basis filter. The frequency response of the predictor of order 17 is shown in Fig. 3. The phase delay is zero at $\omega = 0.01\pi$, corresponding to 50 Hz on the input side.

Although the 50 Hz line frequency is very stable on the average, there can be slight temporary variations, which is why the MGP adaptive scheme is proposed for fine tuning of the synchronous decimator. The structure of the adaptive polyphase decimator is shown in Fig. 4. The counterclockwise rotating commutator distributes each set of six input samples to the branch filters $p_0(m)$ to $p_5(m)$ [1]. The branch outputs are then computed and added together.

The reference signal for adaptation is the input of the uppermost branch, i.e, the latest input sample. All the arithmetic operations in the structure are done at the lower sampling rate. Since the primary signal is a sinusoid, it is sufficient to use two adaptive parameters. Overall, the filter has 18 basis coefficients, of which 12 are positive and six negative. We find it advantageous for convergence to group the positive coefficients into one group and the remaining six negative ones into the other group.

Fig. 5 shows the settling of the two parameters in an example case, where the input signal first is a sinusoid of the nominal frequency 50 Hz. Then the frequency changes abruptly -10% (at sample number 1666) and $+10\%$ (at sample number 3333) from the nominal. The parameters adapt so that the prediction error settles to zero. Since the variations in parameter values are small, the frequency response of the filter remains practically unchanged. Here the gain factor $\gamma = 1.15$. The stability limit for a sine wave of unity amplitude is at $\gamma = 3.2$.

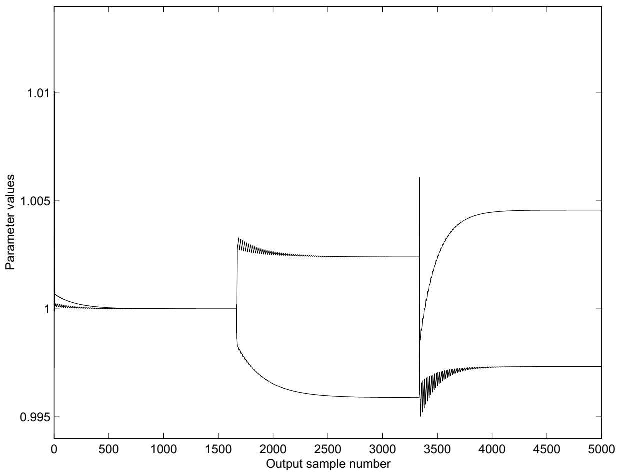

Fig. 5. Trace of the two parameters in sine wave decimation.

In the beginning of the experiment, the two parameters are initialized to unity values. This is not essential for the operation, but a reasonable thing to do, as the basis filter has been designed so that the assumed nominal values of the parameters are equal to one.

6. CONCLUSIONS

The stability condition of the polyphase MGP algorithm has been found. This algorithm is computationally simple, and it actually has features of both time-domain and transform-domain adaptive algorithms, which makes it attractive for use in polyphase filters. The algorithm can be generalized to other kinds of tap coefficient groupings than the regular polyphase structure which was the main topic of this paper. Design strategies for choosing the coefficient groups in an optimum manner will be a topic of our future publications.

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