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Design of Minimum Phase Digital IIR Filters by Using Genetic Algorithm

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ABSTRACT

Genetic Algorithm (GA) based design techniques are widely proposed for IIR filters. However, the unstability problem and the phase distortion occurring during the design process are important disadvantages for IIR filters. In this work, an effective design method using GA for minimum phase and stable digital IIR filters is presented.

1. INTRODUCTION

In the design process of optimal digital IIR filters, besides the unstability problem the phase response of the designed system also forms an important problem. Minimum phase filters have two main advantages: reduced filter length and minimum group delay. Minimum phase filters can simultaneously meet delay and magnitude response constraints, yet generally require fewer computations and less memory than linear phase filters[1,2].

Optimization algorithms have been used effectively in the design of digital IIR filters which are able to provide the desired characteristics. Because of error surfaces being quadratic and uni-modal according to the filter coefficients, gradient based algorithms have been commonly used in FIR filter design. However, gradient based algorithms are not so efficient in the IIR filter design. The reason is that IIR filters may have multi-modal error surfaces and standard gradient techniques do not have the ability of getting out from local minima[3]. Therefore, in the IIR filter design random-based algorithms are preferred. When considering global optimization methods for digital IIR filter design, the GA seems to have attracted considerable attention. Filters designed by GA have the potential of obtaining near global optimum solution[4,5].

In this work, an effective design method using GA for minimum phase stable digital IIR filters is presented. This paper is organized as follows. Section 2 contains a brief review of basic GA. Section 3 describes the application of GA to the design of stable minimum phase IIR digital filters. Section 4 presents the simulation results.

2. BASIC GENETIC ALGORITHM

The genetic algorithm is an artificial genetic system based on the process of natural selection and genetic operators. It is also a heuristic algorithm which tries to find the optimal results by decreasing the value of objective function (error function) continuously. A simplified GA cycle is shown in Fig.1.

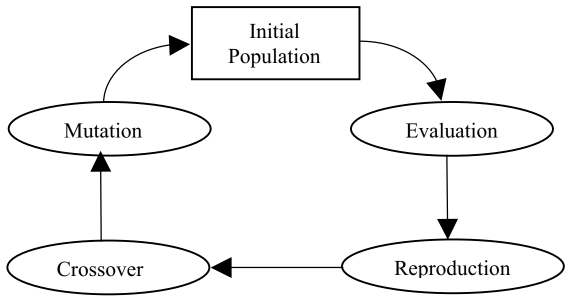


Fig. 1. A simplified GA cycle

Initial population consists of a collection of chromosomes[6]. In practice these chromosomes represent a set of solutions for the problem. The chromosome which produces the minimum error function value represents the best solution. The chromosomes which represent the better solutions are selected by the reproduction operator and then sent to the crossover operation. In this operation, two new chromosomes are produced from two chromosomes existing in the population. A common point in the selected chromosomes is randomly chosen and their corresponding digits are exchanged. Thus, new chromosomes which represent the new solutions are produced. The process in a simple crossover (single-point) operation is shown in Fig.2.

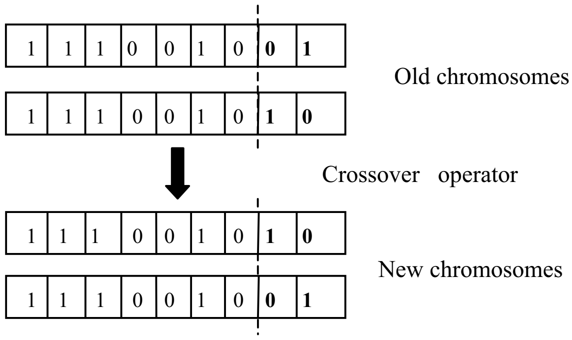


Fig. 2. Crossover process

The next operator is mutation. Generally, over a period of several generations, the genes tend to become more and more homogenous. Therefore, many chromosomes can not continue to evolve before they reach their optimal state. In the mutation process, some bits of the chromosomes mutate randomly. Namely, certain digits will be altered from either '0' to '1' or '1' to '0' in binary encoding[7].

The GA used in this study has been written with MATLAB programming language and in addition to the operators mentioned above GA also contains 'Elite' operator. By means of Elite operator, the best solution is always kept. In the evaluation process, the solutions in the population are evaluated and a fitness value associated with each solution is calculated. These fitness values are used by the selection operator. Roulette Wheel method is employed for the selection process.

3. APPLICATION OF GENETIC ALGORITHM TO THE PROBLEM

In order to evaluate the chromosomes representing possible IIR filters in the population basic error functions are usually used. The chromosomes which have higher fitness values represent the better filters[8]. In the filter design the following error functions can be used : Mean Squared Error (MSE), Least Mean Squared Error (LMS), Minimax Error or Mean Absolute Error (MAE). The expressions of these functions are given below:

a.) Mean Squared Error (MSE)

$$MSE = \sum_f [\ |H_I(f)| - |H(f)|\ ]^2 \qquad (1)$$

b.) Least - Mean Squared Error (LMS)

$$LMS = \left\{ \sum_f [\ |H_I(f)| - |H(f)|\ ]^2 \right\}^{\frac{1}{2}} \qquad (2)$$

c.) Mean Absolute Error (MAE)

$$MAE = \sum_f [\ |H_I(f) - H(f)|\ ] \qquad (3)$$

$H_I(f)$: Magnitude response of the ideal filter

$H(f)$: Magnitude response of the designed filter

When the error functions were directly used as they are in Equations (1-3), from the simulation results it was seen that magnitude response could be efficiently optimized, but the unstability problem and non-minimum phase problem appeared. In order to overcome this problem the basic error functions given above were modified. It is assumed that the number of poles that causes the unstability is q_p and the number of zeros that causes the non-minimum phase is q_z and the error function used is $e(f)$, then the objective function which is able to provide optimal magnitude response, stability and minimum-phase can be defined as,

$$\psi(f) = e(f) + w_p.q_p + w_z.q_z \qquad (4)$$

w_p , w_z = weight parameters

In expression (4), w_p and w_z have to be chosen appropriately. As the number of poles that cause the unstability increases, the effect of these poles on the error function will increase proportionally. Hence, by means of objective function the poles which are located out of the unit circle are pulled into to the inside of the unit circle. Similarly, as the number of zeros that cause the non-minimum phase increases, the effect of these zeros on the error function will increase proportionally. Hence, by means of objective function these zeros which are located out of the unit circle are pulled into to the inside of the unit circle. When all the poles and zeros are pulled inside the unit circle, the error function will be equal to the objective function since q_p=0 and q_z=0.

The fitness function used in this work is given by Equation (5).

$$Fitness = \frac{1}{\psi(f)} \qquad (5)$$

After several trials, it is seen that the most appropriate value for the parameter w_p in Equation (4) is 100. When the value of w_p is chosen too high, the value of the pole term becomes dominant on objective function and therefore, the GA might have difficulties with converging and finally could not reach the optimum solution. When the value of w_p is chosen too low, the influence of the pole term on the objective function becomes too small and hence the GA ignores this pole term and the designed filter might become unstable[8].

Besides, after several trials, it is seen that the most appropriate value for the parameter w_z in Equation (4) changes for each band of the magnitude response. In the simulations, the best results are obtained when the value of w_z is chosen as 10 in the passband region and as 50 in the stopband and transition band regions.

In conventional applications of GA, only one of the error functions is usually used as the error function. However, as mentioned above to optimize the magnitude response and to provide the stability and minimum phase we need to use all of these error functions, simultaneously. In order to realize this, LMS error function is employed in the passband region, MAE error function is employed in the transition band region and MSE error function is employed in the stopband region simultaneously during the design process. This is shown in Fig.3.

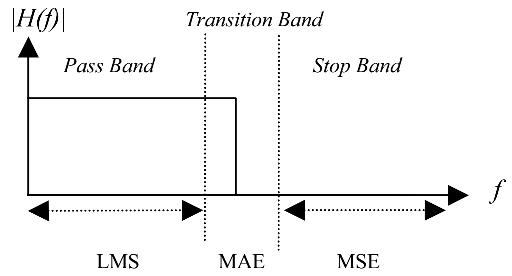


Fig. 3. Objective function structure used

4. SIMULATION RESULTS

The transfer function of an IIR filter is given by Equation (6).

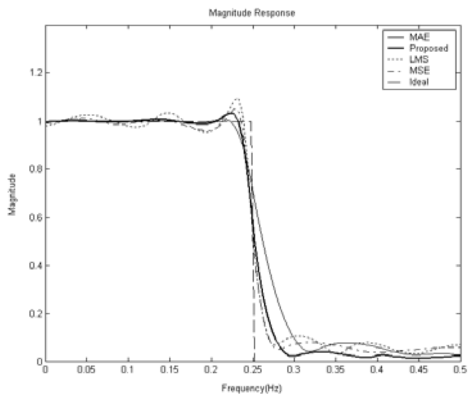
$$H(z) = \frac{\sum_{i=1}^{L} b_i z^{-i}}{\sum_{i=1}^{M} a_i z^{-i}} \tag{6}$$

a_i and b_i represent the filter parameters to be determined in the design process. The order of the minimum phase lowpass filter used in the simulations is 9, namely, M=L=10. Hence, each chromosome in the population of GA represents 20 filter parameters.

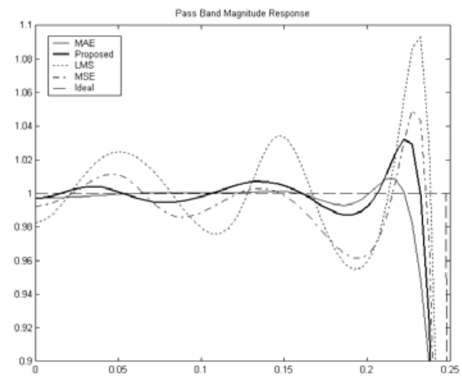
In the simulations, the sampling frequency was chosen as $f_s = 1Hz$. The control parameters of GA used in this work are as the following:

Generation Number	: 3.000
Population Size	: 100
Crossover Rate	: 0,6
Mutation Rate	: 0.01

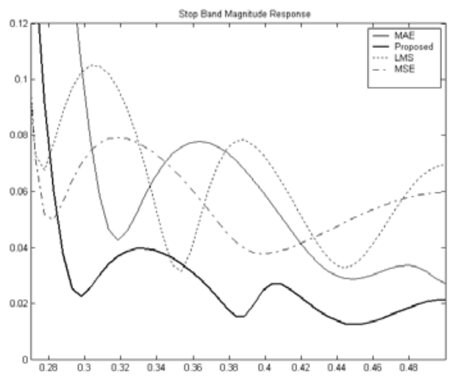
As mentioned above, in the first step, the objective function is improved such that it pulls all the poles and zeros which are located out of the unit circle into the inside of the circle. In the second step, the algorithm is modified such that it uses three different error functions simultaneously to evaluate the designed filters. As seen from the Figure 4 , the filters designed by employing MSE and LMS error functions have sharper transition band responses than that produced when only MAE is used. The proposed objective function produces similar transition band response to that obtained when MSE and LMS are used. On the other hand, as shown in the Figure 4.c. and Figure 5, the use of three error functions simultaneously in the objective function causes a better filter design in terms of attenuation for stop band region. For the passband region the proposed method produces better response then those obtained when the LMS and MSE functions are used. Consequently, when the proposed objective function given in Equation (4) and the error function given in Figure.3 are used, the minimum phase stable IIR filter can be efficiently designed.



(a)



(b)



(c)

Fig. 4. Magnitude responses obtained by using MSE, LMS, MAE and the proposed error function.

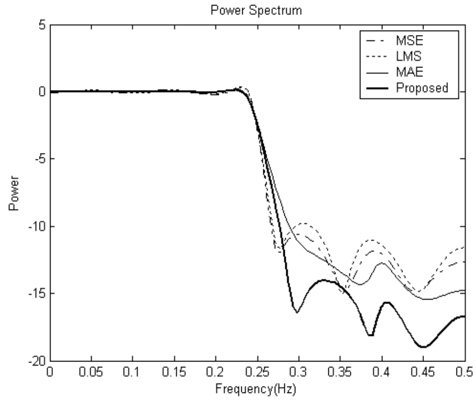
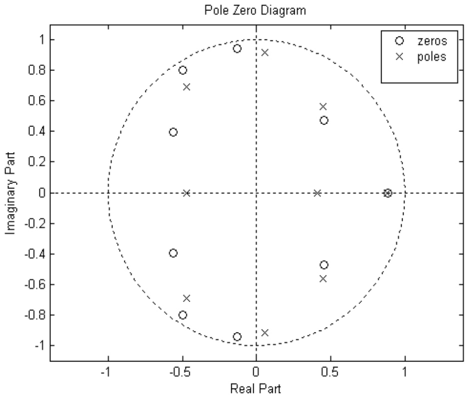
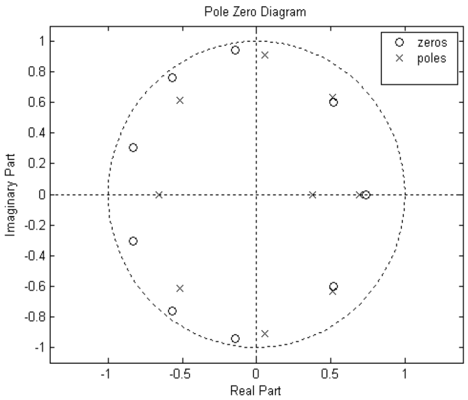


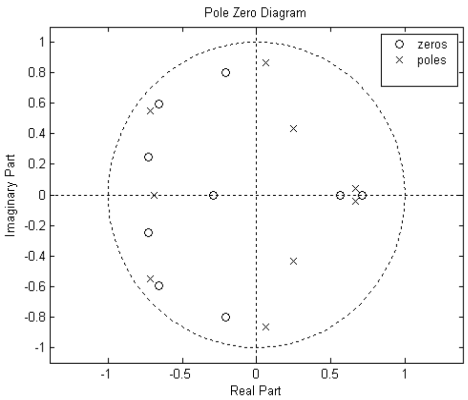
Fig. 5. Power spectrums obtained by using MSE, LMS, MAE and the proposed error function.



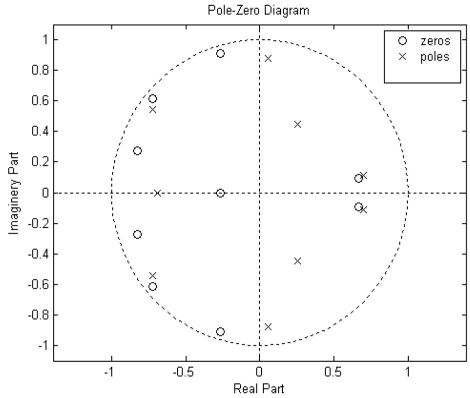
a.) MSE error function



b.) LMS error function



c.) MAE error function



d.) Proposed error function

Fig. 6. Pole-zero diagrams produced by using different error functions

5. CONCLUSIONS

In this paper, GA has been applied to the design of a high-order minimum phase IIR digital filter. Using three different error functions simultaneously to evaluate the filters, the desired characteristics have been obtained.

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