Multifrequency bio-impedance measurement:
undersampling approach

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ABSTRACT

For digital measurement of slowly changing modulation waveforms, e.g. electrical bio-impedance (EBI) variations of several higher frequency carriers simultaneously, the undersampling approach allows to extend the range of carrier frequencies significantly beyond the Nyquist limit, and to reduce the computational power at the same time. Basic equations for choosing of signal and sampling frequencies are found. An example solution is proposed, and analyzed by MATLAB simulation for further implementations on some real DSP-platform. In the simulated example, the 1 Hz EBI variation and 15,059 ks/s sampling rate in the case of 4 kHz and 64 kHz carriers were used.

1. INTRODUCTION

Bio-impedance measurement is an important field of physiological measurement using electronics [1]. Bio-impedance measurement devices can be realized by using of the according analog conversion (modulation/demodulation) circuits, and micro-controller [2], or can be realized as analog and digital mixed-signal ASIC [3].

Such devices, including simultaneous multifrequency units, can be efficiently realised also by using of digital signal processors (DSP) [4, 5]. DSP-based solutions, compared with other more-or-less “hard-wired” analog or digital solutions, are cost-effective, precise, flexible and can include much functionality of measuring, correction, pre- and post-conditioning of measured signals.

But drawback of classical signal processing [6], requiring sampling of the signals with at least twice rate of the highest frequency component (Nyquist limit), requiring much DSP and analog front-end resources to support such sampling and processing rates. Cost of these resources will be seen in price, board space, power consumption, and still in the limited possible carrier frequencies.

Solution could be the development of undersampling algorithms for bio-impedance measurements [7]. Basic idea behind the using of undersampling for bio-impedance measurements is, that relatively high frequency carrier/modulation frequencies are used for measurement (from fractions of kHz to tens or hundreds of MHz), but physiological modulation of the bio-impedance values is going on by relatively low frequency (0.1 to 10 Hz). So not all carrier periods must be necessarily sampled, and some undersampling methods can be used.

An optimal solution for the DSP-based device for the simultaneous multifrequency bio-impedance measurement is often using of the uniform undersampling approach, in spite of some limitations of such method, related to the selection of the actual frequencies of carrier signals, sampling frequency, and measurement period.

2. SOLUTION

2.1 General idea

The most common idea for undersampling method is to skip one or more full-periods of the “lowest harmonic component” of the signal (frequency, for what all other frequencies are integer times higher). This “lowest harmonic component” itself could be missing from the signal. This method is similar, used e.g. for AC power line harmonic measurements [10].
2.2. Frequency domain relationships

Let’s assume that excitation signal consists of the some of the following frequency components \( f_1, 2 \times f_1, 3 \times f_1, \ldots, n \times f_1 \), where \( n \times f_1 \) is the highest frequency. So the “primary” signal frequency is \( f_1 \) and it’s period is \( T_1 \):

\[
T_1 = \frac{1}{f_1}, \quad (1)
\]

And let’s assume that for every next sample- \( M \) periods of \( f_1 \) are skipped, plus “effective sampling step” \( AT \), defining the equivalent frequency resolution \( f_{\text{NYQUIST}} \):

\[
AT = \frac{1}{f_{\text{NYQUIST}}}, \quad (2)
\]

So the sampling period \( T_s \) and frequency \( f_s \) could be expressed as

\[
T_s = \frac{1}{f_s} = M \times T_1 + AT, \quad (3)
\]

and combining (1), (2) and (3) we’ll get:

\[
f_{\text{NYQUIST}} = \frac{f_s}{(1 - M \times f_s / f_1)} \quad (4)
\]

Let’s assume that the fractional “effective sampling step” \( AT \) is got as one period of \( f_1 \) divided to \( L \) samples:

\[
AT = \frac{T_1}{L}, \quad (5)
\]

or

\[
L = \frac{f_{\text{NYQUIST}}}{f_1} \quad (6)
\]

From the other hand, let’s assume, that total measuring (“integration” time \( T_o \) and (frequency \( f_o \)) could be expressed as \( T_o = (L \times M + 1) \times T_o \), or in other words:

\[
f_o = f_0 \times (L \times M + 1) \quad (7)
\]

or combining with (6)

\[
f_o = f_0 / (M \times (f_{\text{NYQUIST}} / f_s) + 1) \quad (8)
\]

So, defining \( M \) (“undersampling factor”), \( f_1 \) (“basic harmonic carrier frequency”) and \( f_s \) (sampling frequency), the values of \( f_{\text{NYQUIST}} \) (frequency resolution), and \( f_o \) (frequency of total measurements) can be calculated, according to formulas (4) and (8).

2.3. Hardware considerations

Hardware solution (Fig.1) for bio-impedance (Zx) measurement could be a typical DSP-based system, with classical analog front-end containing an analog-to-digital converter (ADC), including sample-and-hold circuit (S/H), a digital-to-analog converter (DAC), with a low-pass filter (LPF), and programmable gain amplifies (PGA) at the analog input of the system, as measured signals are small. Typically, the analog output signal is the current from a U/I converter, or as a voltage through relatively high resistor (e.g. from 10 to 100 kΩ). Circuitry should probably include some electrical isolation between measurement signals and output (e.g. connection to PC). Circuitry can include also multiplexing circuits at the analog input and output to switch output excitation and to measure the bio-impedance between different electrodes. In some cases, generation of the output analog signal can be more efficient, if done original by DSP-DAC, but by Direct Digital Synthesis (DDS) [11], it is synchronized with DSP.

2.3. Software and algorithm considerations

The proposed signal processing algorithm is actually very similar to one, proposed in [4], containing the multi-frequency waveform generator using a look-up table of waveform coefficients (programmed into the DSP memory) and a digital-to-analogue converter DAC with a LPF used for the output waveform reconstruction, and a U/I converter (or resistor) for determination of the excitation current \( I_{\text{exc}} \), directed through the complex impedance \( Z_x \) to be measured.

Variations \( Z(t) \) of the complex impedance \( Z \) causes both, the amplitude and phase modulations of the voltage response \( V_z \) as the sum of carrier signals. After amplification in a programmable gain amplifier PGA the voltage response will be digitized by the aid of a sample-and-hold circuit S/H and an ADC. Then the signals will be demodulated digitally using a synchronous (phase sensitive quadrature) detection [4], giving the real and imaginary parts of the demodulated impedance variations separately for each carrier.

Synchronous detection is actually multiplication of the sampled input signal to quadrature (sine and cosine) reference waveforms for the every frequency used, and then filtering (averaging) of these multiplication products.

The filtering can be done e.g. by first order IIR filter. Or alternatively, averaging can be done by accumulation of the multiplication products over some time window (over defined number of samples).

2.4. MATLAB Simulation

The proposed solution was simulated using MATLAB. The model of the device under test (DUT) is given on the figure 2. The circuit consists of the variable resistor \( R(t) = 500 \pm 200 \Omega \), modulated by a sawtooth waveform with 1 Hz frequency (Fig.3), and fixed \( r=200 \Omega \) and C. The
excitation current is generated using a resistor $R_0 = 100 \, k\Omega$. Simulations have been done for $C = 1.75 \, nF$ and for $C = 7 \, nF$ cases.

**Fig. 2.** Model of the device under test ($Z_x$)

![Diagram](image)

Fig. 3. Modulation of the resistance $R(t)$

The number of simultaneously used frequencies is two - 4 kHz and 64 kHz - see Fig.4 for a compound excitation signal. Sampling period $T_s$ equals to 4 1/4 periods of 64 kHz (or 1 1/16 periods of 4 kHz). So, $T_s = 66,406 \, \mu s$ of sampling frequency $f_s = 15,0588 \, kHz$.

**Fig. 4.** Excitation (4 kHz and 64 kHz) signal (green) and sampling instants ('x' - blue)

![Graph](image)

Simulation results are shown for all real and imaginary components for kHz and 64 kHz components on Fig.6 – Fig.9. For Fig.6 and Fig.7, the first order IIR-filter with time-constant $T_0 = 1$ second was used for filtering of multiplication of the input samples and reference cosine and sinewaves. For Fig.8 and Fig.9, averaging over 68 samples is used. So the averaging time window is $T_0 = 68 * T_s = 4,515 \, ms$. Fig.6 and Fig.8 are for DUT $C = 1.75 \, nF$ and Fig.7 and Fig.9 for $C = 7 \, nF$.

**Fig. 5.** Results for $C = 1,75 \, nF$, averaging-IIR $T_0 = 1s$, 4-kHz- real (green) and imaginary (cyan), and 64 kHz- real (blue) and imaginary (red) components

![Graph](image)

**Fig. 6.** Results for $C = 7 nF$, averaging-IIR $T_0 = 1s$, 4-kHz- real (green) and imaginary (cyan), and 64 kHz- real (blue) and imaginary (red) components

![Graph](image)
3. CONCLUSIONS

The undersampling methods have probably a strong potential for implementation in the digital multifrequency measurement of electrical bio-impedance variations. This method allows significantly exceed the Nyquist frequency limit, valid for the traditional uniform sampling, and enables significantly expand the frequency range of the excitation signals, and also relax requirements to the real-time operating circuits and computational power of the DSP.

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