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Multi-Weight Multi-Length Strict Optical Orthogonal Codes

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ABSTRACT

In this paper, we present the construction of multi-weight, multi-length, strict optical orthogonal codes. This code set satisfies the conditions of the strictly orthogonal codes, namely, the maximum non-zero shift autocorrelation and the maximum cross-correlation constraints of one. Multi-length codes are designed to support multi-rate services, while the multi-weight codes are designed to support differentiated quality of service (QoS). A code set with two user classes is constructed, and its bit error rate performance is investigated.

1. INTRODUCTION

Optical fibers offer a potential of THz bandwidths, that is larger, the shorter the cable length is. This makes optical fiber especially applicable for very high-speed local area network (LAN) applications. Typical LAN traffic is bursty by nature. Therefore, static channelization (or multiplexing) schemes, realized by permanently allocated time- or wavelength division multiplexing (TDM, WDM) slots waste resources for idle users [1]. Users can share the communication bandwidth more effectively by using dynamic medium access techniques (eg statistical multiplexing). This can be realized for instance by code division, or carrier sense multiple access techniques (CDMA, CSMA).

Let us now inspect the realization of CDMA by employing spreading and dispreading in optical domain. In intensity modulation, direct detection (IM-DD) non-coherent optical communication systems, intensity of the optical signal is modulated and detected. Therefore, only unipolar codes can be applied for optical, non-coherent multiple access. In [2], Optical Orthogonal Codes (OOC) are proposed for non-coherent optical CDMA. The OOCs can be generated simply by using optical passive components, such as optical delay lines, optical splitters and optical combiners [3]. The OOCs are highly sparse codes, and the number of supported users is quite limited. However, the number of users can be increased by using longer codes with lower weights. On the other hand, the usage of longer codes requires faster modulation of lasers, while lower weights degrade the bit error rate performance significantly.

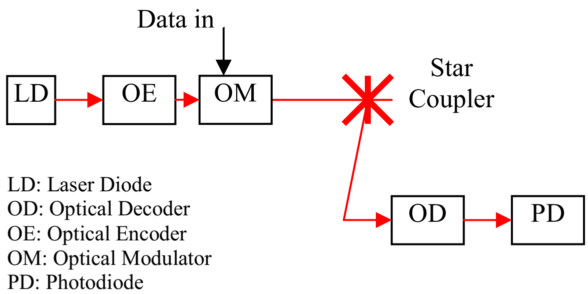


Fig. 1. Optical CDMA network

Fig. 1. shows a schematic diagram of a non-coherent fiber optic communication system employing optical code division encoding and decoding. The laser source produces very short optical pulses, which are encoded by using the optical encoder. The number of taps in the encoder determines the code weight, which can be controlled by the optical switches in the taps of the encoder. The optical decoder at the receiving end is matched to the desired code with the optical delays equal to the complements of that at the encoder (eg. it is the optical matched filter for the applied code).
When suggested for the first time, OOC was proposed for certain equal code weights and lengths to satisfy the code correlation properties planned to guarantee a high number of simultaneous, equal data rate and equal error rate users. This work extends the earlier results in [2] to the case of different weight and different rate users while preserving the original correlation properties of the OOCs. In section 2, the original OOC system is reviewed and the multi-weight, multi-length OOC is introduced. In order to illustrate our ideas, we describe an OOC based system with two user classes. Finally, in section 3 we summarize the results of the performance analysis for the proposed system.

2. CODE CONSTRUCTION

The generated OOC code set C is characterized by the quadruple $(N,W,\lambda_a,\lambda_c)$ where N, W, λ_a and λ_c are the code length, code weight, maximum non-zero shift autocorrelation and the maximum cross-correlation, respectively [4]. OOC codes satisfy the property

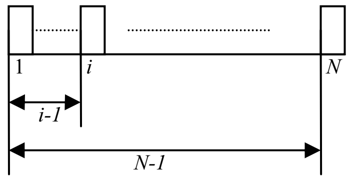


Fig. 2. Mark position differences

$$R_{xx}(m) = \sum_{n=0}^{N-1} x_n x_{n+m} \le \lambda_a \qquad (1)$$

for any $x \in C$ and any integer $[\![m]\!]_N \neq 0$, where $[\![m]\!]_N$ is taken to be m modulo N, and,

$$R_{xy}(m) = \sum_{n=0}^{N-1} x_n y_{n+m} \le \lambda_c \qquad (2)$$

for any $(x \neq y) \in C$, and any integer m.

The property in (1) makes the synchronization at the intended receiver simpler by constraining the autocorrelation side-lobes to λ_a, while (2) allows for distinguishing the different users accessing the LAN simultaneously meaning that the cross-correlations of the users are limited by λ_c.

The codes with the maximum non-zero shift autocorrelation and the maximum cross-correlation bounded by one are called the strict OOCs [5]. These are characterized by $\lambda_a = \lambda_c = 1$ and denoted by $(N,W,1)$. The number of codes in the strict OOC code set is upper bounded by [1],

$$K \le \left\lfloor \frac{N-1}{W(W-1)} \right\rfloor \qquad (3)$$

where $\lfloor x \rfloor$ is the integer part of x.

An OOC code set with fixed weight and length can be generated by using the set of mark position differences which follows from the principle of incoherent optical delay line decoder operation. The difference between chip positions containing marks in any OOC code can be of any integer number in *{1,2,...,N-1}* (see Fig. 2). The difference between mark positions d_n within the code can be constructed by using,

$$d_n = N - [\![N + P_i - P_j]\!]_N \qquad (4)$$
$$i \neq j\,;\; i,j = 1,2,...,W$$

where P_i is the i^{th} mark position.

To satisfy (1) and (2), it is required that there are no repeated elements in their code difference vectors, and no common elements between any of the two difference vectors of all the codes. For example, consider the two (150,3,1) OOC code vectors with $c_1 = (1,61,104)^T$ and $c_2 = (1,105,114)^T$, and the corresponding difference sets (4), $d_1 = \{60,103,43,90,47,107\}$ and $\{104,113,9,46,37,141\}$. It can be verified that the two codes are part of a strict OOCs. In order to support multiple rate, and QoS differentiation, the construction of strict OOC can be generalized based on the concept of mark position difference set. The code set construction is flexible in such a way that the code length and weight can be selected arbitrarily as long as the number of desired codes in each class of users can be supported.

Let us now consider the construction of a strict OOC code set with two user classes. W denote the higher data rate class with the code length of N_{HR}, and weight W_{HR}, and the lower data rate class with the code length of N_{LR}, and weight of W_{LR} ($N_{HR} < N_{LR}$). For each of the code vectors, we then construct a difference set by following (4). Then the length of the difference set is $W_{HR}(W_{HR}-1)$ for the high rate users, and $W_{LR}(W_{LR}-1)$ for the low rate users. In order to construct a code set with two user classes we then suggest proceeding as follows.

1. Construct the initial allowed differences set for the high rate class as integers from 1 to $N_{HR} - 1$.
2. Construct the codes (for instance, by using a random search [4]) for the high rate class with the mark positions in the range of 2 to $N_{HR} - 1$.
3. Construct the allowed differences for the low rate class as integers from 1 to $N_{LR} - 1$.
4. Remove the difference set elements of the high rate class codes from the allowed differences for the low rate class.
5. Construct the codes for the low rate class with the mark positions in the range of N_{HR} to $N_{LR} - 1$.

By using this procedure, a number of high rate and low rate user codes with $N_{HR} = 150$, $N_{LR} = 500$, $W_{HR} = 3$ and $W_{LR} = 5$ were generated (Table 1). Note that the mark positions for the high rate codes are all less than 150 and the mark positions for the low rate codes are more than 150 except the first mark at the first chip position (as indicated by the underline in the Table I). A sample of code auto- and cross-correlation magnitude is shown in Fig. 3. In part (a) of the figure, the autocorrelation peak of the low rate codes is 5 and the non-zero shift peak is 1 as required by (1). Also, the cross-correlation between any two users from the two classes is bounded by one as shown in Fig. 3(b).

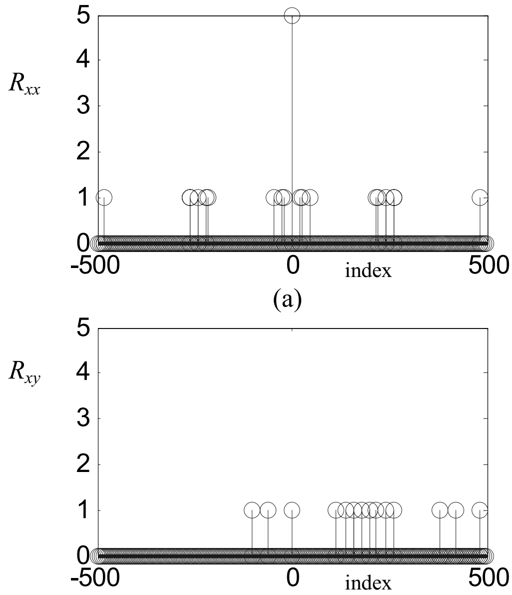
Table 1. (a) Low rate (500,5,1) and (b) high rate (150,3,1) strict OOC code set. (Note that, columns of the sub-tables indicate the positions of the marks in the code)

(a)

1	1	1	1	1	1	1	1	1	1
218	162	195	248	173	247	204	285	175	205
242	184	200	412	187	334	395	365	193	231
262	194	212	446	229	372	418	401	257	289
482	413	274	453	322	449	426	466	325	362

(b)

1	1	1	1
61	105	112	46
104	114	118	97



(b)

Fig. 3 Correlation amplitude (a) autocorrelation of a low rate code (b) cross-correlation between a low rate and a high rate codes.

3. PERFORMANCE ANALYSIS AND SIMULATION RESULTS

In asynchronous *K*-user CDMA systems, the received signal is the sum of the desired user signal and the signals of the other *K*-1 interfering users, with the chip time delay offsets that are multiples of chip period. Let us assume the condition of perfect power balance (e.g. the received power of all users is equal), and let us neglect other noise sources, such as the thermal noise from receiver's electronics, or shot noise from the photo-detector. For the system of two user classes, accumulated user interference is the sum of two components: Interference from the users in the same class, and the interference from the users in the other class. These two components are binomially distributed random variables with different parameters. Thus the total interference will be the convolution of these two random variables. The signal to interference ratio (SIR) for the high rate and low rate user classes for the strict OOC can be approximated as [6],

$$SIR_{HR} = \frac{W_{HR}^2}{(K_{HR}-1)\sigma_{HR}^2 + K_{LR}\sigma_{LR,HR}^2} \tag{6}$$

and

$$SIR_{LR} = \frac{W_{LR}^2}{(K_{LR}-1)\sigma_{LR}^2 + K_{HR}\sigma_{HR,LR}^2} \tag{7}$$

where, σ_{HR}^2 is the average variance of the cross-correlation magnitude the high-rate users, σ_{LR}^2 is the average variance of the cross-correlation magnitude the low-rate users, $\sigma_{HR,LR}^2$ is the average variance of the cross-correlation magnitude of the high-rate user class over the low-rate class, and $\sigma_{LR,HR}^2$ is the average variance of the cross-correlation magnitude of the low-rate user class over the high-rate user class. Then, by using the Gaussian approximation, the bit error rate for the high- and low-rate users classes of the OOC system can be written as,

$$P_e^{HR} \simeq Q\left(\frac{\sqrt{SIR_{HR}}}{2}\right) \tag{8}$$

and

$$P_e^{LR} \simeq Q\left(\frac{\sqrt{SIR_{LR}}}{2}\right), \tag{9}$$

where *Q(x)* is the well know *Q* function given by,

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-u^2/2)du \tag{10}$$

The BER is plotted in Fig. 4 for both user classes. In Fig. 4(a) we use the same weight for the two classes. The BER for the high rate users is smaller because the interference from the low rate users is smaller. Fig. 4(b) shows the BER for the strict OOC code set with the high-rate user class (500,5,1), and the low-rate user class of (1500,7,1), while part (c) with the high-rate user class of (500,7,1), and the low-rate user class of (1500,5,1) in part (c). In all cases it is assumed that the number of high-rate users is fixed to 4, and the number of low-rate users is varied from 2 to 20. We note that the effect of changing the weight of the high rate users on the BER is smaller as compared to that of the low rate users due to the lower value of $\sigma_{LR,HR}^2$.

4. CONCLUSIONS

In this paper we proposed and presented the construction of multi-weight, multi-length strict optical orthogonal code set. The code set is flexible and can be designed for any code lengths and weights limited only by the possible number of codes. The code set contains codes with different code lengths and code weights while satisfying the required correlation properties of the strict OOCs. The different code lengths support data rate differentiation, and the different code weights support QoS differentiation. The presented technique is interesting from a practical point of view, since the hardware requirement is the same as that of the fixed length fixed weight OOC. We have considered a system with two user classes, with two code lengths and two code weights, and demonstrated how the multi-length, multi-weight strict OOCs can provide the proposed differentiation. Simulations showed that the BER performance can be controlled by changing the code weight of the high rate and low rate users.

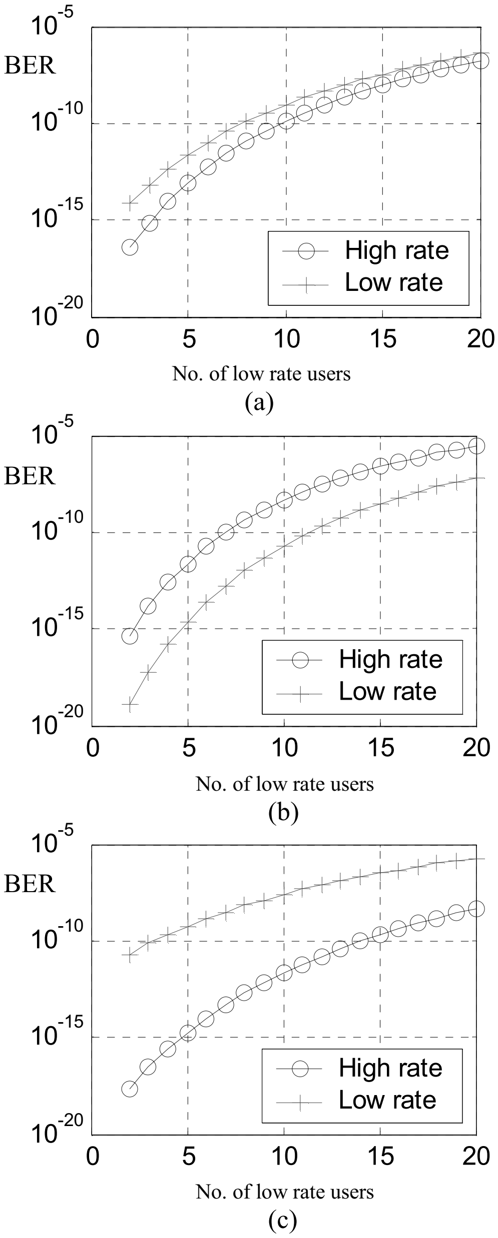


Fig. 4 BER for the class OOC high rate and low rate classes with $K_{HR} = 4$ and (a) $W_{HR} = 5$, $W_{LR} = 5$ (b) $W_{HR} = 5$, $W_{LR} = 7$ (c) $W_{HR} = 7$, $W_{LR} = 5$.

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