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Soft Output Detection using Path Detector for Multiple Antennas

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ABSTRACT

This paper presents a detection method based on path search providing soft decision outputs for MIMO systems. The number of paths searched is reduced in comparison to tree search in List Sequential Methods. As an example, we consider 16QAM signaling with 2 transmitting and 2 receiving antennas. The proposed method, Soft Output Path Detector (SOPD), shows BER performance similar to Sphere Decoder (SD) [2], but the SOPD has lower complexity than the SD at low SNR. Soft decisions provided by the SOPD noticeably improve performance in MIMO systems with channel coding.

1. INTRODUCTION

Soft detection methods are widely used in modern communications systems to minimize the information loss, by making a "hard" decision as late as possible in the receiver. This paper presents a detection method for Multiple-Input Multiple-Output (MIMO) systems providing soft decision outputs, called in this paper as Soft Output Path Detector (SOPD).

The SOPD searches sequentially for the closest symbols to a received vector $\mathbf{r}$ from an M-QAM constellation and creates Log-Likelihood Ratios (LLR) for each decision bit. The SOPD algorithm combines concepts from the Sphere Decoder (SD) [1] – [3], the Tree Search Detector (TSD) [4] and the List Sequential Detector (LISS) [5]. However, in some cases the proposed SOPD has lower complexity. As an example, the paper describes the SOPD algorithm for 16-QAM signaling. Detection for other M-QAM modulation schemes with the SOPD can be derived in a similar way.

Section 2 introduces the SOPD algorithm for 16-QAM systems. In Section 3 the complexity and performance results of the SOPD and the SD are shown and compared, with conclusions followed in Section 4.

1.1. System Model

We consider a multiantenna system with N_t transmitters, N_r receivers and the symmetry constrain $N_t = N_r$. A vector transmitted during each time period is denoted as $\mathbf{x} = [\, \mathrm{x}_1, \mathrm{x}_2, ..., \mathrm{x}_{N_t}]^T$, where each component is an independent choice from a complex M-QAM constellation. The received vector $\mathbf{r} = \mathbf{Hx} + \mathbf{n}$ is the result of the transmitted symbols contained in $\mathbf{x}$ over the flat fading channel $\mathbf{H}$ and corrupted by AWGN $\mathbf{n}$.

The algorithm of the SOPD is suitable only for real constellation, therefore, we need to define an equivalent real-valued (RV) model with dimensions $2N_t$ by $2N_r$. We redefine then the received vector as $\boldsymbol{r} = \boldsymbol{Hx} + \boldsymbol{n}$, where

$$\boldsymbol{r} = [Real(\mathbf{r})\ Imag(\mathbf{r})]^T \tag{1}$$

$$\boldsymbol{x} = [Real(\mathbf{x})\ Imag(\mathbf{x})]^T \tag{2}$$

$$\boldsymbol{n} = [Real(\mathbf{n})\ Imag(\mathbf{n})]^T \tag{3}$$

$$\boldsymbol{H} = \begin{bmatrix} Real(\mathbf{H})\ Imag(\mathbf{H}) \\ -Imag(\mathbf{H})\ Real(\mathbf{H}) \end{bmatrix} \tag{4}$$

1.2. Soft Output Detection

In order to provide soft output information per bit each received symbol has to be compared with the closest constellation point in which the bit in consideration takes the logical values 1 and 0. For this purpose, and to comply with the real and imaginary part decomposition of the received vector, the constellation is also divided into RV matrices, each containing the axis values in which each bit represents a logical 1 or 0 in the constellation. Considering the 16-QAM constellation of Figure 1 in a system with $N_t = 2$, these matrices are defined as

$$\mathbf{L1^A} \quad = \quad \begin{bmatrix} c_2^1\, is\, 1 \\ c_2^2\, is\, 1 \\ c_1^3\, is\, 1 \\ c_1^4\, is\, 1 \end{bmatrix} \quad = \quad \begin{bmatrix} 1 & 3 \\ 1 & 3 \\ -3 & -1 \\ -3 & -1 \end{bmatrix} \tag{5}$$

$$\mathbf{L1^B} \quad = \quad \begin{bmatrix} c_3^1\, is\, 1 \\ c_3^2\, is\, 1 \\ c_4^3\, is\, 1 \\ c_4^4\, is\, 1 \end{bmatrix} \quad = \quad \begin{bmatrix} -3 & 3 \\ -3 & 3 \\ -3 & 3 \\ -3 & 3 \end{bmatrix} \tag{6}$$

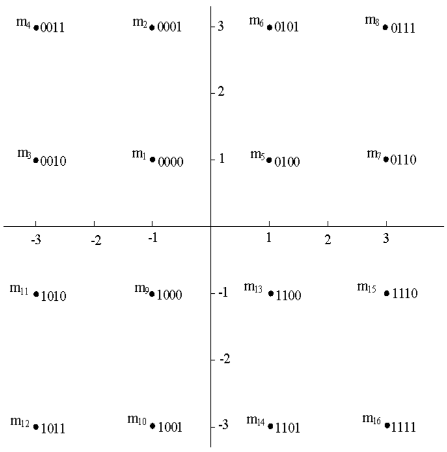

Fig. 1. 16-QAM constellation with Gray Mapping

$$\mathbf{L0^A} = \begin{bmatrix} c^1_2\, is\, 0 \\ c^2_2\, is\, 0 \\ c^3_1\, is\, 0 \\ c^4_1\, is\, 0 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ -3 & -1 \\ 1 & 3 \\ 1 & 3 \end{bmatrix} \tag{7}$$

$$\mathbf{L0^B} = \begin{bmatrix} c^1_3\, is\, 0 \\ c^2_3\, is\, 0 \\ c^3_4\, is\, 0 \\ c^4_4\, is\, 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} \tag{8}$$

where c^i_j denotes the j^{th} bit of the RV received symbol r_i (each complex constellation symbol m_k is formed by the bits $[c^k_1\, c^k_2\, c^k_3\, c^k_4]$ in which c^k_1 is the most significant bit (MSB) and c^k_4 is the least significant bit (LSB)). We should notice that $c^i{}_2$ and $c^i{}_3$ represent the real axis of the constellation, while $c^i{}_1$ and $c^i{}_4$ the imaginary one.

The procedure used to find bit likelihoods can be summarized in five steps:

1. Find the closest point to the received symbol, from the constellation axis that does not belong to the bit in consideration, save this distance as d_p.

2. Find the closest point to the received symbol, from the pair of constellation axis in which the bit in consideration takes the logical value 0, save this distance as d'_0.

3. Find the closest point to the received symbol, from the pair of constellation axis in which the bit in consideration takes the logical value 1, save this distance as d'_1.

4. Calculate the Euclidean distances d_0 and d_1 as

$$\begin{aligned} d_0 &= d_p + d'_0 \\ d_1 &= d_p + d'_1 \end{aligned} \tag{9}$$

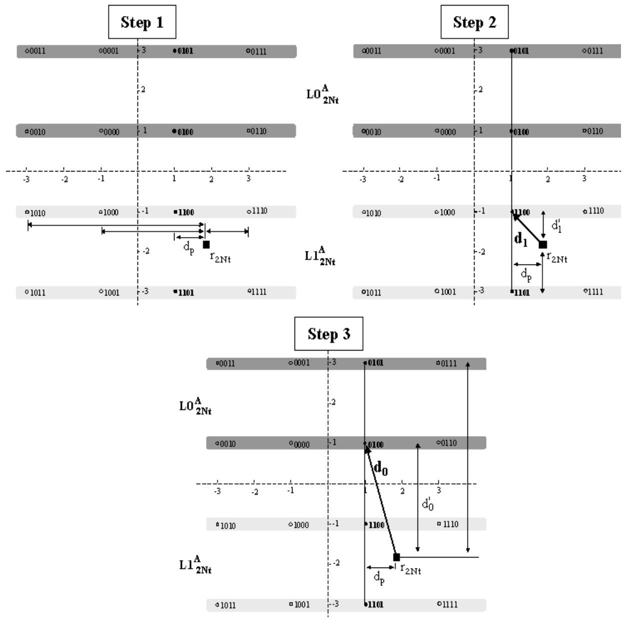

Fig. 2. Steps to obtain d_0 and d_1 for the first bit (MSB) of the received symbol r_{2N_t} in a 16-QAM system.

5. With the use of these distances the bit likelihoods are obtained as

$$LLR(c^i_j \mid r_i) = log\frac{P\left[c^i_j = 1 \mid r_i\right]}{P\left[c^i_j = 0 \mid r_i\right]} \tag{10}$$

where

$$P\left[c^i_j = 1 \mid r_i\right] = \frac{d_0}{d_0 + d_1} \tag{11}$$

and

$$P\left[c^i_j = 0 \mid r_i\right] = \frac{d_1}{d_0 + d_1} \tag{12}$$

The necessary steps to obtain the distances d_0 and d_1 from the MSB of the received symbol r_{2Nt} are graphically depicted in Figure 2.

2. SOFT OUTPUT PATH DETECTOR ALGORITHM

The SOPD searches sequentially for the vector $\hat{\boldsymbol{x}}$ with the minimum Euclidean distance to the received vector $\boldsymbol{r}$ in a reduced number of vector candidates. The sequential search starts with RV symbol r_{2N_t}, moves backwards to r_{2N_t-1} and so on. The candidates of each symbol r_i are used to find likelihood ratios per bit, simultaneously, the candidates are defined as those points in which each bit can take the logical value 1 or 0. Thus, the possible number of paths to visit is limited by the number of bits used in the modulation scheme.

The search starts from the first bit $c^{2N_t}_1$ (MSB) of the RV symbol r_{2N_t}. The closest constellation symbols from $L1^A_{2N_t}$ and $L0^A_{2N_t}$ to r_{2N_t} are chosen and saved as $Y^A_{2N_t,1}$ and

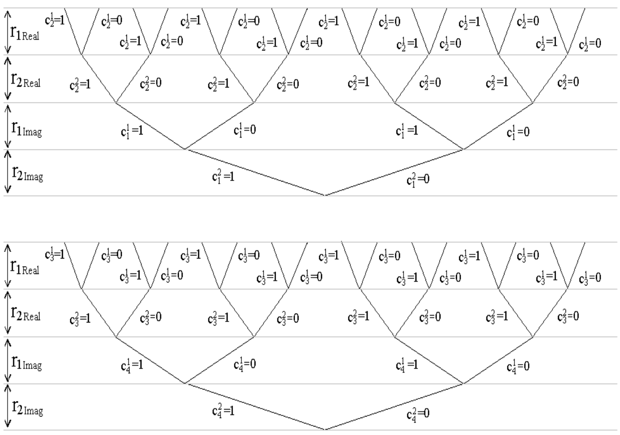

Fig. 3. Possible paths for a 16-QAM system with $N_t = 2$.

$Y^B_{2N_t,2}$, representing the root of the first set of paths. These points are found as

$$Y^A_{2N_t,1} = \min_{L1^A_{2N_t}} \left| S_{2N_t} - L1^A_{2N_t} \right|^2 \tag{13}$$

and

$$Y^A_{2N_t,2} = \min_{L0^A_{2N_t}} \left| S_{2N_t} - L0^A_{2N_t} \right|^2 \tag{14}$$

where the expression min refers to one of the two elements in $L1^A_{2N_t}$ or $L0^A_{2N_t}$ that computed the minimum distance, $S_{2N_t} = \rho_{2N_t}$ and $\boldsymbol{r} = \boldsymbol{\rho H}$ with $\boldsymbol{\rho} = (\rho_1, ..., \rho_{N_t})$. At this point, we save also the distances $d_{1,c_1^{2N_t}}$ and $d_{0,c_1^{2N_t}}$ as

$$d_{1,c_1^{2N_t}} = q_{ii} \left| S_{2N_t} - Y^A_{2N_t,1} \right|^2 \tag{15}$$

and

$$d_{0,c_1^{2N_t}} = q_{ii} \left| S_{2N_t} - Y^A_{2N_t,2} \right|^2 \tag{16}$$

as well as the accumulated distance per path, defined as

$$d_{tot,p} = d_{tot,p} + q_{ii} \left| S_i - Y^A_{i,k} \right|^2 \tag{17}$$

for all paths p affected by the symbol $Y^A_{i,k}$ ($1 \leq p \leq 8$ considering $Y^A_{2N_t,1}$, which is the root for the first 8 paths when $N_t = 2$). The terms q_{ii} in the previous equations, are placed to consider the linear transformation suffered by the constellation over the channel. The channel correlation matrix $\boldsymbol{G} = \boldsymbol{H}\boldsymbol{H}^{\mathbf{T}}$ is factorized using the Cholesky decomposition to find an upper triangular matrix $\boldsymbol{U}$ such that $\boldsymbol{G} = \boldsymbol{U}^{\mathbf{T}}\boldsymbol{U}$. The elements of the matrix $\boldsymbol{U}$ are then substituted as $q_{ii} = u^2_{ii}$ and $q_{ij} = u_{ij}/u_{ii}$.

To move to the next level, the first path elements $Y^A_{2N_t,1}$ and $Y^A_{2N_t,2}$ will be used to update two versions of **S**

$$S_{i,k} = \rho_i + \sum_{j=i+1}^{N_t} q_{ij} \left| \rho_j - Y^A_{j,k} \right|^2 \tag{18}$$

Now, the bit $c_1^{2N_t-1}$ of r_{2N_t-1} is considered, and the closest symbols from $L1^A_{2N_t-1}$ and $L0^A_{2N_t-1}$ to the two points $S_{2N_t-1,1}$ and $S_{2N_t-1,2}$ are saved. This time four different results will be obtained and the number of paths is incremented, saving all path elements and their distances. Next, **S** is updated four times with Equation (18) obtaining $S_{2N_t-2,1}$, $S_{2N_t-2,2}$, $S_{2N_t-2,3}$ and $S_{2N_t-2,4}$. The recursions continue for r_{2N_t-2}, and so on. It should be noticed, from matrices $\mathbf{L1^A}$ and $\mathbf{L0^A}$ that when we reach point r_{N_t} the bit searched is switched to $c_2^{N_t}$ (the second most significant bit), which is considered until reaching r_1.

In a similar way the matrices $\mathbf{L1^B}$ and $\mathbf{L0^B}$ are used to find a second set of paths $Y^B_{i,k}$, $d^B_{1/0\ c^i_j}$ and $d_{tot,p}$. At the end of the recursions the information stored in the paths consists in all the points $Y^A_{i,k}$ and $Y^B_{i,k}$ found during the iterations, the updated versions of vector **S** and also the distances of each path and point. The number of paths necessary to complete the search is $2^{(2N_t+1)}$ for the case of 16-QAM. Figure 3 shows the paths to be searched by a 16-QAM system with $N_t = 2$ and helps also to clarify the "Path" concept.

Once all the $2^{(2N_t+1)}$ paths have been created the soft values can be calculated with the formulas

$$LLR(c^i_j \mid r_i) = log \left[\frac{d_{1,c^i_j} + min(d_{0,c_j^{i-N_t/2}}, d_{1,c_j^{i-N_t/2}}) + \alpha}{d_{0,c^i_j} + min(d_{0,c_j^{i-N_t/2}}, d_{1,c_j^{i-N_t/2}}) + \beta} \right] \tag{19}$$

for $N_t/2 < i \leq N_t$, and

$$LLR(c^i_j \mid r_i) = log \left[\frac{d_{1,c^i_j} + min(d_{0,c_j^{i+N_t/2}}, d_{1,c_j^{i+N_t/2}}) + \alpha}{d_{0,c^i_j} + min(d_{0,c_j^{i+N_t/2}}, d_{1,c_j^{i+N_t/2}}) + \beta} \right] \tag{20}$$

for $1 \leq i \leq N_t/2$. Where $\alpha = min(d_{tot,p})$ refers to the minimum accumulated distance of all paths p in which the j^{th} bit is 1, and $\beta = min(d_{tot,v})$ to the minimum accumulated distance of all paths v in which the j^{th} bit is 0.

3. SIMULATION RESULTS

Comparison of the complexity of SOPD and Reduced Complexity SD (RCSD) [3], in terms of number of recursions done by each detector for different levels of signal to noise ratios (SNR), is shown at Figure 4. As one can see, the SOPD complexity is lower at low SNR, while the SD is a preferable solution at high SNR.

Comparing to the LISS detector [5], the SOPD reduces the number of searched paths. The LISS limits the number of paths by a constant L_{max}, while the SOPD has a fixed number of paths to search of 2^{2N_t+1}.

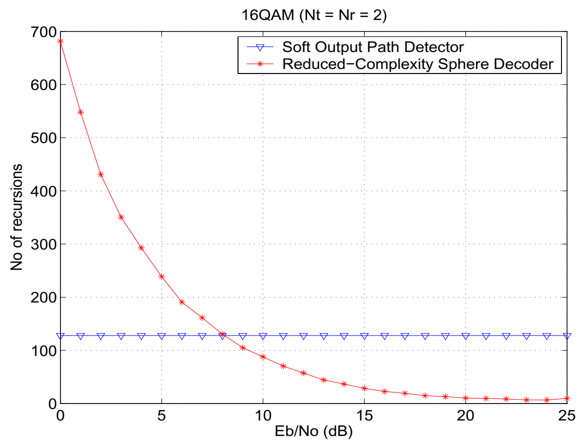

Fig. 4. Soft Output Path Detector and Reduced Complexity Sphere Decoder, number of recursions computed in a 16-QAM system with $N_t = N_r = 2$.

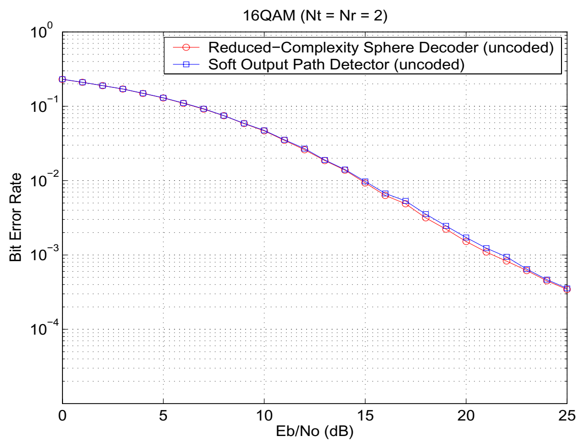

Fig. 5. Soft Output Path Detector and Reduced Complexity Sphere Decoder, Error Performance for Hard Symbols Detection in a 16-QAM system with $N_t = N_r = 2$

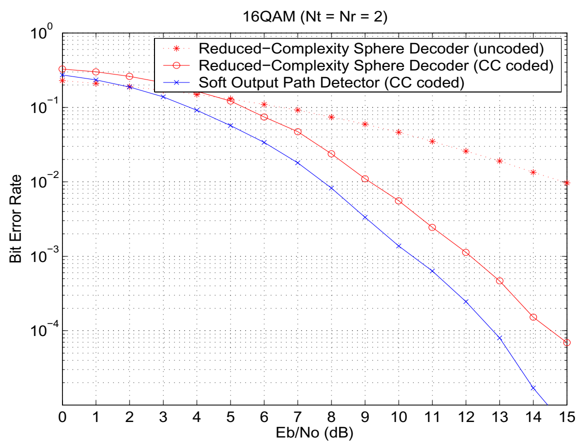

Fig. 6. Soft Output Path Detector and Reduced Complexity Sphere Decoder, Error Performance for a 16-QAM system with $N_t = N_r = 2$ and Convolutional Coding/Decoding.

To evaluate SOPD detector performance, first we compare the SOPD with hard decision outputs to the RCSD. It is reported that the RCSD provides the ML detection for hard decisions. Simulation results presented at Figure 5 confirm that the SOPD and the RCSD detectors have the same performance.

The quality of soft values generated by SOPD is evaluated with the use of a rate 1/2 convolutional code with constrain length 7 and generator polynomials $G_8^{(1)} = 133$, $G_8^{(2)} = 171$. Figure 6 shows that performance gain provided by soft outputs with respect to hard decisions is about 1.8dB. Recall that the theoretical gain due to soft decisions in AWGN channel is about 2dB, confirming that soft outputs from the SOPD are close to optimal.

4. CONCLUSIONS

In this paper the SOPD, the detection method for MIMO systems, is proposed and evaluated. This method is based on principles behind tree search detectors, the List Sequential Detector and the Reduced-Complexity Sphere Decoder. It is shown that the proposed SOPD method provides the same performance as the LISS and RCSD detectors, but has lower complexity at low SNR. Besides, the SOPD creates soft decision outputs of acceptable quality that noticeably improve performance in MIMO systems with channel coding.

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