Proceedings of the 6th Nordic Signal Processing Symposium - NORSIG 2004
June 9 - 11, 2004
Espoo, Finland

Application of the Generalized Predictive Control Method in Closed-Loop Power Control of CDMA Cellular Communication Systems

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ABSTRACT

In this paper we investigate adaptive closed-loop power control algorithms based on the Generalized Predictive Control (GPC) method that are able to alleviate the effect of the loop delay inherent in the closed-loop power control of DS-CDMA systems, even if the loop delay is unknown. The numerical results indicate that the GPC-based algorithms can significantly improve the radio network capacity without any increase in power control signaling, even with unknown loop delay.

1. INTRODUCTION

Transmitter power control (PC) is needed in wireless cellular communication systems for managing the co-channel interference powers. In particular, in direct sequence code division multiple access (DS-CDMA) systems, power control is an essential tool for coping with the near-far problem, that is, a strong signal overpowering weaker signals at a receiver. The power control problem has been investigated by many researchers from many perspectives during recent decades (see [1] and the references therein for an overview of power control in CDMA cellular communication systems). Practical implementations of power control in CDMA systems utilize closed-loop control, where the transmitter adjusts its power based on commands received from the receiver in a feedback channel. To minimize signaling overhead, typically one bit is used for the power control command. In practice the command must be derived based on measurements made at the receiver, transmitted over the feedback channel to the transmitter, and finally processed and applied at the transmitter. All these operations constitute a loop delay, which can cause problems if it is not properly taken care of in the design of the power control algorithm [2]. In many cases the loop delay is known due to a specific frame structure inherent in the system. For this case, the adaptive closed-loop algorithms we proposed in [3] and the loop delay cancellation scheme proposed in [2] can be very effective. However, if the loop delay is unknown or incorrectly estimated, the performance advantages of those algorithms are lost. In [4] we introduced new adaptive closed-loop power control algorithms that are based on generalized predictive controller (GPC) [5]. Those algorithms are more robust to incorrect loop delay estimation.

In this paper we give a detailed overview of the GPC method and evaluate the adaptive GPC-based PC algorithms in [4] by computer simulations. The paper is organized as follows: Section 2 gives a detailed derivation of the GPC method starting from a specific linear system model. Section 3 introduces the closed-loop power control model and the evaluated PC algorithms. In Section 4 we present some numerical results from a cellular network simulator. Finally, Section 5 concludes the paper.

2. GENERALIZED PREDICTIVE CONTROL [5][6]

2.1 System model

In the GPC approach, the controlled process, or *system*, is modeled as an autoregressive integrated moving average process with exogenous input (ARIMAX):

$$Ay(t) = Bu(t-k) + \frac{C}{\Delta}\xi(t) , \qquad (1)$$

where $y(t)$ is the output and $u(t)$ is the input of the process, k is the delay of the process, $\{\xi(t)\}$ is a white Gaussian sequence with variance σ_ξ^2. The backward shift operator q^{-1} is defined by $q^{-1}x(t) \equiv x(t-1)$, and $\Delta = 1 - q^{-1}$ is the differencing operator. A, B and C are polynomials in q^{-1}, given by

$$\begin{aligned} A &= 1 + a_1 q^{-1} + \ldots + a_{n_a} q^{-n_a} \\ B &= b_0 + b_1 q^{-1} + \ldots + b_{n_b} q^{-n_b} \\ C &= 1 + c_1 q^{-1} + \ldots + c_{n_c} q^{-n_c} \end{aligned} \qquad (2)$$

We refer to the model by ARIMAX(n_a, n_b, n_c, k). It was proposed in [7] that relatively low-order models ($n_a, n_b, n_c \leq 2$) are sufficient for the power control process.

2.2 Generalized predictive control derivation

The generalized predictive control method was proposed in [5] to overcome the robustness problems encountered with the standard minimum variance controllers. It has since then gained popularity both in industry and academia. It has been successfully applied in many industrial processes [6][8], and has shown good performance and robustness with respect to overparametrization or poorly known delays.

The basic idea of GPC is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a prediction horizon. Thus, instead of calculating one j-step-ahead prediction and a single control signal as in the minimum variance methods, a set of future predictions and a sequence of control signals are calculated at each iteration. The GPC cost function is

$$J_{GPC}(N_1, N_2, N_u) = E\left\{ \sum_{j=N_1}^{N_2} \delta(j)[\hat{y}(t+j\,|\,t) - w(t+j)]^2 + \sum_{j=1}^{N_u} \lambda(j)[\Delta u(t+j-1)]^2 \right\} \quad (3)$$

where $\hat{y}(t+j\,|\,t)$ is an optimum j-step ahead prediction of the system output based on data up to time t, N_1 and N_2 are the minimum and maximum output (or costing) horizons, N_u is the control horizon, $\delta(j)$ and $\lambda(j)$ are weighting sequences and $w(t+j)$ is the future reference trajectory, which can be calculated as

$$w(t) = y(t); \quad w(t+j) = \alpha w(t+j-1) + (1-\alpha) r(t+j), \quad j = 1 \ldots N \quad (4)$$

where $\alpha \in [0,1]$ is a selectable parameter that controls the smoothness of the approximation from the actual system output towards the known reference $r(t)$, and thus influences the dynamic response of the system. In the original formulation [5] $\delta(j)$ is considered to be 1 and $\lambda(j)$ is considered to be constant. The idea of GPC is to calculate a sequence of future control signals $u(t)$, $u(t+1)$, ... in such a way that the future output of the system $y(t+j)$ is driven close to $w(t+j)$. This is accomplished by minimizing $J_{GPC}(N_1, N_2, N_u)$.

2.2.1 Control law

In the following it is assumed that $C = 1$. The colored noise case can be found in [6]. To minimize the GPC cost function $J_{GPC}(N_1, N_2, N_u)$, one needs to calculate the optimal predictions of $y(t+j)$ for $N_1 \leq j \leq N_2$. To do this, the following Diophantine equation is employed:

$$1 = E_j A \Delta + q^{-j} F_j \quad (5)$$

where E_j and F_j are polynomials with degrees $j-1$ and n_a, respectively. They are uniquely defined given A and the prediction interval j. Multiplying (1) by $E_j \Delta q^j$ and substituting for $E_j A \Delta$ from (5) one gets

$$y(t+j) = E_j B \Delta u(t+j-k) + F_j y(t) + E_j \xi(t+j) \quad (6)$$

As E_j is of degree $j-1$ the noise components are all in the future. Thus the optimal predictor of $y(t+j)$ is

$$\hat{y}(t+j\,|\,t) = G_j \Delta u(t+j-k) + F_j y(t) \quad (7)$$

where $G_j = E_j B$. The polynomials

$$E_j = e_{j,0} + e_{j,1} q^{-1} + \cdots + e_{j,j-1} q^{-(j-1)}$$
$$F_j = f_{j,0} + f_{j,1} q^{-1} + \cdots + f_{j,n_a} q^{-n_a}$$

can be calculated recursively as follows:

$$\begin{aligned} E_{j+1} &= E_j + f_{j,0} q^{-j} \\ f_{j+1,i} &= f_{j,i+1} - f_{j,0} \tilde{a}_{i+1} \quad ; \quad i = 0 \ldots n_a - 1, \end{aligned} \quad (8)$$

where $\tilde{a}_i$ denotes the coefficient of the ith term in the polynomial ΔA. From this, the polynomial G_j can be obtained recursively as

$$G_{j+1} = E_{j+1} B = (E_j + f_{j,0} q^{-j}) B = G_j + f_{j,0} q^{-j} B \quad (9)$$

so the first j coefficients of G_{j+1} are identical to those of G_j and the remaining coefficients are given by $g_{j+1,j+i} = g_{j,j+i} + f_{j,0} b_i$; $i = 0 \ldots n_b$.

Since the control signal $u(t)$ influences the system output after k sampling periods, the values N_1, N_2 and N_u defining the horizons in $J_{GPC}(N_1, N_2, N_u)$ can be defined as

$$N_1 = k; \quad N_2 = k + N - 1; \quad N_u = N \quad (10)$$

where N is the length of the output horizon. Casting equation (7) in matrix form, one gets

$$\mathbf{y} = \mathbf{G}\mathbf{u} + \mathbf{f} \quad (11)$$

where

$$\mathbf{y} = [\hat{y}(t+k \mid t) \quad \hat{y}(t+k+1 \mid t) \quad \cdots \quad \hat{y}(t+k+1 \mid t)]^T \tag{12}$$

$$\mathbf{u} = [\Delta u(t) \quad \Delta u(t+1) \quad \cdots \quad \Delta u(t+N-1)]^T \tag{13}$$

$$\mathbf{G} = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ g_{N-1} & g_{N-2} & \cdots & g_0 \end{bmatrix} \tag{14}$$

$$\mathbf{f} = \begin{bmatrix} F_k y(t) + (G_k - g_0) q \Delta u(t-1) \\ F_{k+1} y(t) + \left(G_{k+1} - g_0 - g_1 q^{-1}\right) q^2 \Delta u(t-1) \\ \vdots \\ F_{k+N-1} y(t) + \left(G_{k+N-1} - \sum_{i=0}^{N-1} g_i q^{-i} \right) q^N \Delta u(t-1) \end{bmatrix} \tag{15}$$

The control law is designed to minimize $J_{GPC}(N_1, N_2, N_u)$, which can be written in matrix form as (assuming $\delta(j) = 1$ and $\lambda(j) = \lambda$)

$$J_{GPC} = (\mathbf{Gu} + \mathbf{f} - \mathbf{w})^T (\mathbf{Gu} + \mathbf{f} - \mathbf{w}) + \lambda \mathbf{u}^T \mathbf{u} \tag{16}$$

where

$$\mathbf{w} = [w(t+k),\, w(t+k+1), \ldots, w(t+k+N-1)]^T \tag{17}$$

The minimization of (16) with respect to **u** yields

$$\mathbf{u} = \left(\mathbf{G}^T \mathbf{G} + \lambda \mathbf{I}\right)^{-1} \mathbf{G}^T (\mathbf{w} - \mathbf{f}) \tag{18}$$

and from (13)

$$\Delta u(t) = [1,0,...,0]\mathbf{u} \tag{19}$$

Thus only the first element of the vector **u** is actually used at each iteration.

To reduce the computational burden involved in the N by N matrix inversion in (18), the control horizon N_u can be selected so that $N_u < N$, where it is assumed that the projected control signals are going to be zero after $N_u < N$ samples. In this case **G** is replaced with a matrix **G'** formed by taking the first N_u columns of **G**. Then **G'** is used instead of **G** in (18), i.e.,

$$\mathbf{u} = \left(\mathbf{G}'^T \mathbf{G}' + \lambda \mathbf{I}\right)^{-1} \mathbf{G}'^T (\mathbf{w} - \mathbf{f}) \tag{20}$$

Thus the calculation of the control command involves the inversion of only an N_u by N_u matrix. Note that in the case $N_u = 1$ the matrix inversion in (20) reduces to scalar computation. This can still give acceptable control for simple systems [6].20

3. POWER CONTROL ALGORITHMS

Fig. 1 shows the flowchart of the conventional fixed-step power control (FSPC) algorithm [9]. In the Figure all values are in decibels, $\gamma(t)$ and $\gamma^t(t)$ are the signal-to-interference ratio (SIR) at the receiver and SIR target, $u_{tx}(t)$ and $u_{rx}(t)$ are the PC transmitted and received PC commands, $p(t)$ is the transmitter power, $g(t)$ is the link gain, $I(t)$ is the interference power at the receiver, $e(t)$ is the PC misadjustment and k is the total loop delay.

The FSPC algorithm belongs to the class of *decision feedback* (DF) algorithms, i.e., only two states are used for signaling. This is of great practical value, since CDMA requires relatively high PC update rate, and all signaling consumes valuable radio resources. If the relay block is removed and $k = 1$, the block diagram is equivalent to the distributed power control (DPC) algorithm [10]:

$$p(t+1) = p(t) + \beta\left(\gamma^t(t) - \gamma(t)\right), \quad \beta \in \left]0,1\right] \tag{21}$$

with $\beta = 1$. The DPC algorithm belongs to the class of *information feedback* (IF) algorithms, since analog values are signaled back in the loop. This naturally consumes a lot of feedback bandwidth. The GPC method can be used in both IF and DF forms. By defining $u(t) = u_{tx}(t)$, $y(t) = \gamma(t)$ and $r(t) = \gamma^t(t)$, the application of the GPC method to the closed-loop power control is straightforward. In the DF case the control output signal $u(t)$ is passed through a relay before transmission, and this quantized signal is also used in the internal feedback in the GPC controller. The model parameters are estimated with the recursive least squares (RLS) algorithm. The GPC-based PC algorithms are called the GPC-PC and GPCD-PC in the IF and DF cases, respectively. Details are omitted here, and they can be found from [4].

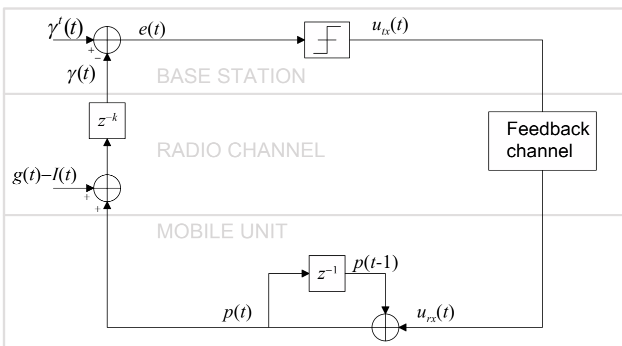

Fig. 1. Flowchart of closed-loop power control.

4. SIMULATION RESULTS

In this section we give results of uplink simulations of a cellular network. In the simulations, 80 users are uniformly distributed over seven hexagonal cells. Each user transmits at a constant data rate of 45 kb/s. The chip rate is 3.84 Mchip/s. The target bit-energy-to-interference-spectral-density ratio (E_b / I_o) is 6 dB for every user. The velocities of the users are randomly selected from 0 km/h to 30 km/h. Each user is connected to the base station with the largest link gain at all times. The radio link gain is modeled as a product of three variables: the distance-dependent propagation loss, log-normal shadowing with 0 dB mean and 8 dB standard deviation, and motion-induced Rayleigh-distributed multipath fading generated by Jakes' model [11]. It is assumed that the receivers are able to combine two paths with equal mean strength but independent fading. The log-normal shadowing component is correlated according to the correlation model proposed in [12].

The results of the simulations are shown in Fig. 2 and Fig. 3 for the IF and DF cases, respectively. Also the minimum variance (MV) and generalized MV (GMV) based algorithms from [3] are simulated. The total loop delay is $k = 2$ power control periods (PC update rate is 1.5 kHz), and all the algorithms incorrectly assume that $k = 1$. It can be seen that the GPC-based algorithms perform well despite of the incorrect loop delay assumption, in contrast to the MV/GMV-based algorithms. The performance is better with longer horizons and higher model orders, but this also increases the complexity of the algorithms.

5. CONCLUSIONS

The generalized predictive control can be used to improve the performance of closed-loop power control in CDMA cellular communication systems even when the loop delay

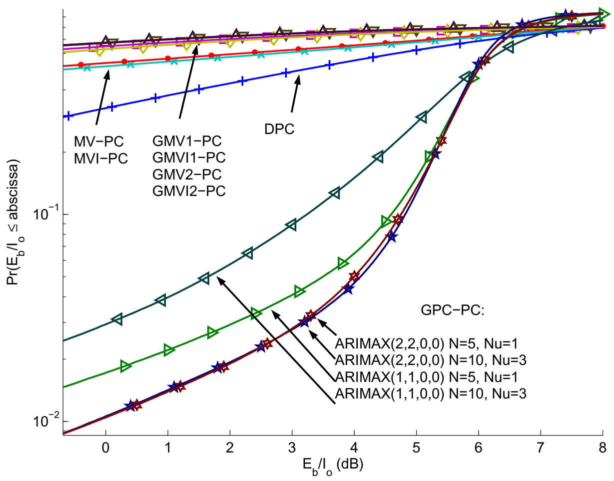

Fig. 2. Empirical cumulative distribution function (CDF) of E_b/I_o, information feedback.

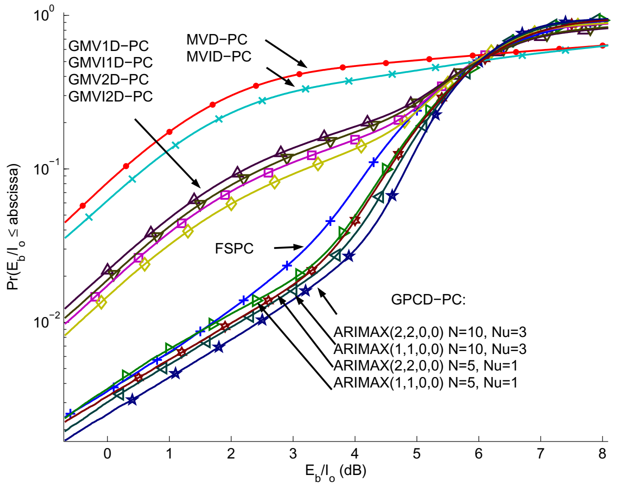

Fig. 3. Empirical cumulative distribution function (CDF) of E_b/I_o, decision feedback.

is unknown, if the relatively high computational complexity can be tolerated. However, if the loop delay is known with certainty, the less complex minimum variance based algorithms in [3] are a better choice. Further improvements may be achievable by the inclusion of the power limit and PC command quantization constraints in the GPC cost function. This is left for future work.

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