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Quantization and Dynamic Range Effects on the Energy Detection

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ABSTRACT

The signal and threshold quantization effects on the performance of the total-power radiometer detector are studied. The goal is to define the sufficient number of quantization bits to maintain acceptable performance when the noise level estimate is assumed to be perfect. In addition, we study the effects of the dynamic range related to noise level. The simulations illustrate that the selection of the dynamic range affects the performance enormously. The noise should not be allowed to be saturated. If the dynamic range is set correctly, 4 bits is a sufficient number for signal quantization.

1. INTRODUCTION

In signal detection, the goal is to decide whether only noise or noise and signal(s) are present. The total-power radiometer is a detector which calculates the total energy of the received signal [1]. In practice, the input signal is sampled and quantized before detection which causes performance degradation. The main principles of quantization are presented in [2]. Therein, various quantization methods, such as companding, robust quantization, and optimum quantization has been introduced. Additionally, the quantization with memory was briefly considered. The optimum (non-uniform) quantization has been more precisely studied in [3]. The optimum and nearly optimum quantization in signal detection applications have been presented in [4], [5], and [6]. The goal of the optimum quantization is to reduce the distortion caused by quantization to as small level as possible. In the unknown signal detection, the optimal quantizing levels cannot be calculated. The quantization effects within the constant false alarm rate (CFAR) detection have been studied in [7]. In the CFAR detection, the noise level estimation could be done before decisions. In this paper, we concentrate on the impact of signal quantization errors on the total-power radiometer. Additionally, the effect of dynamic range on the performance are studied. The quantization is assumed to be uniform, i.e., non-optimum and memoryless. The goal is to define sufficient number of signal quantization bits that the performance of the radiometer does not degrade substantially. Additionally, the adequate dynamic range size will be examined.

The structure of this paper is as follows. In Section 2 the main theory of signal detection and quantization in energy detection is presented. Section 3 includes the simulated performance examples and discussion. Conclusions are drawn in Section 4.

2. SYSTEM DESCRIPTION

2.1. Signal Detection

Signal detection is based on the statistical hypothesis testing. We assume that the received sequence $\{x_k\}_{k=1}^K$ consists of K statistically independent samples x_k. There are two possible hypotheses: the noise only hypothesis is denoted as

$$H_0 : x_k = n_k, \quad k = 1, 2, \dots, K \tag{1}$$

and the signal-plus-noise hypothesis is denoted as

$$H_1 : x_k = \theta s_k + n_k, \quad k = 1, 2, \dots, K. \tag{2}$$

Herein, $\{n_k\}_{k=1}^K$ is a zero mean white Gaussian process with variance σ^2, $\{s_k\}_{k=1}^K$ is the transmitted signal sequence, and θ is a parameter which determines the power level of the signal. The total-power radiometer calculates the energy of the samples

$$E = \sum_{k=1}^{K} |x_k|^2, \tag{3}$$

which is compared to a threshold T. The threshold is set by using the assumed sample statistics under the hypothesis H_0. If E exceeds the threshold, signal is declared to be present. The optimum threshold depends on the noise level, which has to be estimated. A usual way to do this is to initially switch the antenna off and to measure the noise level. Then, the antenna is switched on and the actual process can begin. Unfortunately, the one-shot estimate is not perfect and the noise level is easily estimated to be too low which causes the increase in the false alarm probability P_{FA} [8]. This is due to the fact that the noise level is slightly lower when the antenna is switched off than when it is switched on. However, we assume that the noise level is known accurately and concentrate on the signal quantization effects.

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2.2. Quantized Energy Detection

The block diagram of the digital total-power radiometer is presented in Fig. 1. The received noisy signal (or noise only) is discretized by an analog-to-digital converter (ADC). The ADC consists of sampling, quantization, and coding operations but we concentrate on the quantization. Since the sampling rate is designed according to the radiometer, the noise can be assumed to be white. In the quantization process, the sample values x_k are converted to quantized values from the finite set. In unknown signal detection, the signal level can not be predicted and the signal values can be arbitrarily low. To solve this problem, we use the uniform midriser quantizer, i.e., the one where a decision level is located at zero. The switch between the antenna and the ADC selects whether the noise estimation mode or receiving mode is used. The discretized samples are fed to the radiometer which calculates the total energy of the signal. Finally, the decision is done based on the threshold test.

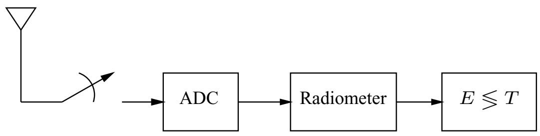

Fig. 1. The block diagram of the digital total-power radiometer.

The quantization process induces errors which are known as granular and saturation noise. The granular noise results from the conversion of the values from a infinite set to a finite set. Consequently, the amount of granular noise depends on the distance between the decision levels known also as a quantile. When the number of decision levels is large, the granular noise is assumed to be a sequence of zero mean, independent random variables. The saturation noise is caused when the input signal amplitude exceeds the dynamic range of the quantizer. Saturation causes more serious distortion to the signal than the granular error.

3. SIMULATION RESULTS

The desired false alarm probability $P_{\text{FA,des}}$ is set to be either 10^{-1} or 10^{-3}. The false alarm probability of 10^{-1} is far too high for a single detector but it might be reasonable if the detection is performed sequentially [9]. The transmitted signal is a random binary phase shift keying (BPSK) sequence. A single bit is sampled 12 times. This refers to the fact that the bandwidth of the total-power radiometer differs from the bandwidth of the sampled signal. The threshold is quantized by b_{thr} bits and signal by b_{sig} bits. The adequate b_{thr} is examined by quantizing only the threshold, but not the input sequence. The selected threshold quantization level is used to investigate signal quantization effects. The false alarm probability P_{FA} vs. signal-to-noise ratio (SNR) for different b_{thr} when $P_{\text{FA,des}} = 10^{-3}$ is presented in Fig. 2. It can be seen that 10 bit threshold quantization is adequate and 8 bit quantization almost adequate. The corresponding false alarm probabilities for $P_{\text{FA,des}} = 10^{-1}$ are depicted in Fig. 3. Equal conclusions can be drawn, although the relative differences between the curves are slightly smaller than in the previous case. Accordingly, $b_{\text{thr}} = 10$ from now on.

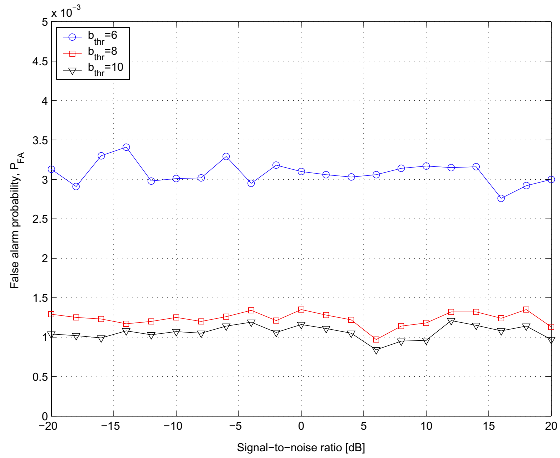

Fig. 2. False alarm probabilities for radiometer when only threshold quantized, $P_{\text{FA,des}} = 10^{-3}$.

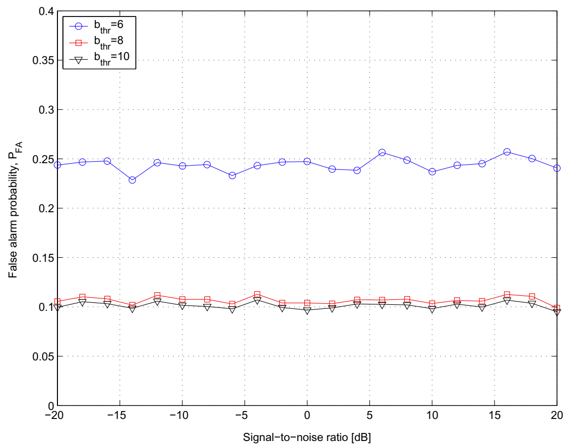

Fig. 3. False alarm probabilities for radiometer when only threshold quantized, $P_{\text{FA,des}} = 10^{-1}$.

The dynamic range of the radiometer can be set such that amplitude of the noise fills a part of it, fills it all, or such that noise is saturated. Next, we investigate this effect together with signal quantization. If we set the dynamic range $R = 6\sigma$, then, given our assumptions, 1% of the noise samples are saturated. If the signal is present, it starts to saturate significantly when SNR exceeds 12 dB. The miss probability for the quantized radiometer vs. SNR is presented in Fig. 4. In this case, the miss probability P_{miss} (and hence also probability of detection since $P_{\text{d}} = 1 - P_{\text{miss}}$) performance of the radiometer with only 2 signal quantization bits is quite close to the theoretical one. When 4 or more bits are used the performance is almost identical to the theoretical performance. It should be noted that above SNR $= 0$ dB detector performs ideally.

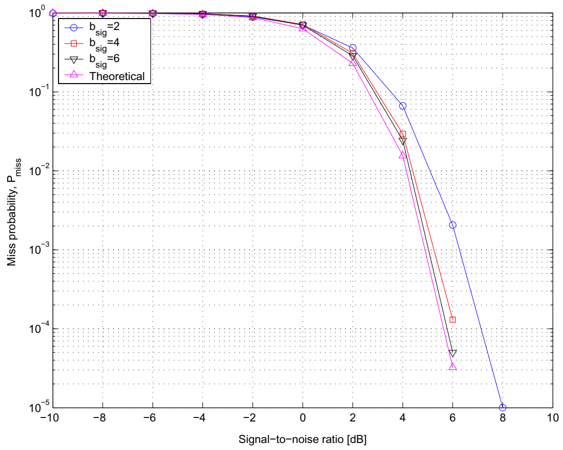

Fig. 4. Miss probabilities for radiometer with 2, 4, and 6 quantization bits compared to the theoretical performance, $P_{\text{FA,des}} = 10^{-3}$, $b_{\text{thr}} = 10$, and $R = 6\sigma$.

Then, we investigate the situation that the dynamic range is $R = 12\sigma$ and $R = 3\sigma$. In the first case, the signal starts to saturate substantially when SNR exceeds 20 dB. In the latter case, if SNR $=$ 3 dB. The miss probability for the first case is depicted in Fig. 5 and compared to the miss probability of the quantizer with $R = 6\sigma$. The corresponding false alarm probabilities vs. SNR are presented in Fig. 6. The miss probability decreases as the dynamic range increases but the false alarm probability increases drastically. This increase can be compensated by increasing b_{sig}. The false alarm values for the case $P_{\text{FA,des}} = 10^{-1}$ can be seen in Fig. 7. In this case, the relative increase in the false alarm probability is considerably smaller. For example, when $b_{\text{sig}} = 3$ and $R = 12\sigma$, the false alarm probability is approximately 2.5 times higher than the desired one, whereas in the case of Fig. 6, it is approximately 8 times higher.

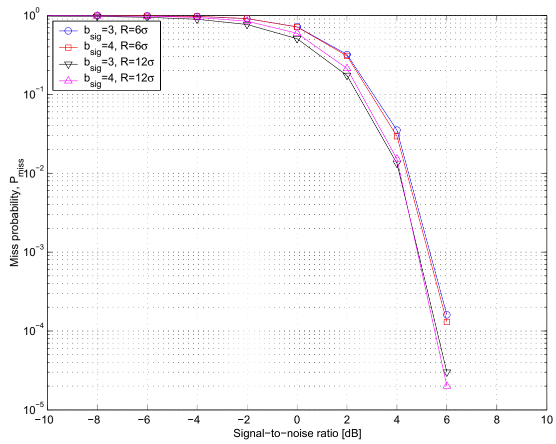

Fig. 5. Comparison of miss probabilities for radiometer with $R = 6\sigma$ and $R = 12\sigma$, $P_{\text{FA,des}} = 10^{-3}$, $b_{\text{thr}} = 10$.

The miss probability of the radiometer when $R = 3\sigma$ is presented in Fig. 8 (note the scale of the vertical axis). It can be observed that the miss probability increases substantially when the noise saturation occurs. As can be seen, increasing b_{sig} does not offer notable performance gain as the number of bits exceeds 8. The analogous comparison for $P_{\text{FA,des}} = 10^{-1}$ is done in Fig. 9. The gap between the simulated and the theoretical values is narrower but still very large. The false alarm probability is close to zero in this case.

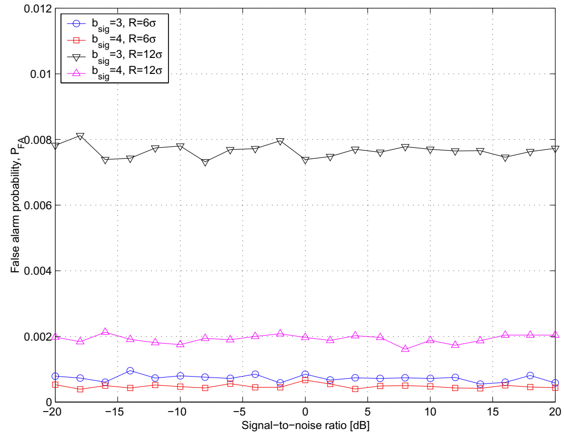

Fig. 6. Comparison of false alarm probabilities for radiometer with $R = 6\sigma$ and $R = 12\sigma$, $P_{\text{FA,des}} = 10^{-3}$, $b_{\text{thr}} = 10$.

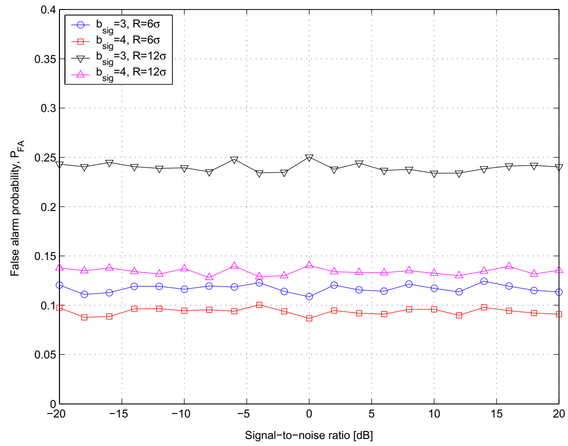

Fig. 7. Comparison of false alarm probabilities for radiometer with $R = 6\sigma$ and $R = 12\sigma$, $P_{\text{FA,des}} = 10^{-1}$, $b_{\text{thr}} = 10$.

4. CONCLUSIONS

The effect of signal and threshold quantization on the performance of the total-power radiometer was studied. The results show that the dynamic range of the quantizer related to the noise level is the most critical parameter when considering the performance. If it is set correctly the acceptable miss probability can be obtained by using only 4 quantizing bits. The saturation of noise can cause drastic performance degradation which can not be compensated by increasing the number of quantizing bits. This is due to changed statistic, i.e., the noise is non-Gaussian. Contrary, the signal can be allowed to be clipped. The possible fur-

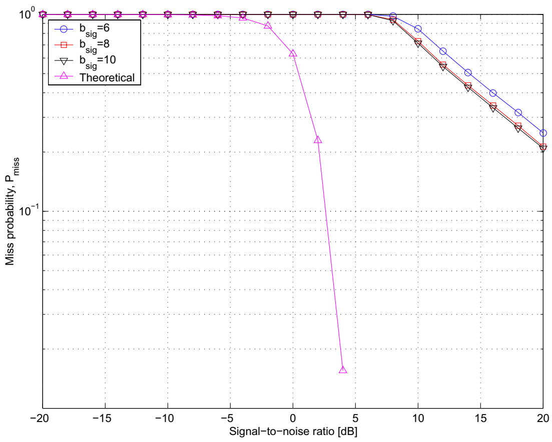

Fig. 8. Miss probabilities for radiometer with 6, 8, and 10 quantization bits compared to the theoretical performance, $P_{\text{FA,des}} = 10^{-3}$, $b_{\text{thr}} = 10$, and $R = 3\sigma$.

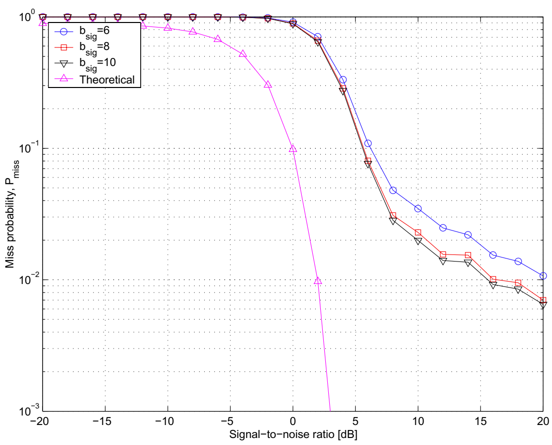

Fig. 9. Miss probabilities for radiometer with 6, 8, and 10 quantization bits compared to the theoretical performance, $P_{\text{FA,des}} = 10^{-1}$, $b_{\text{thr}} = 10$, and $R = 3\sigma$.

ther research topics are the impacts of the noise level estimation and fixed-point implementation on the performance of the radiometer. These will provide us a full understanding of the practical radiometers.

REFERENCES

[1] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*, Prentice-Hall, Inc., Upper Saddle River, NJ, 1st edition, 1998.

[2] A. Gersho, "Principles of quantization," *IEEE Transactions on Circuits and Systems*, vol. CAS-25, no. 7, pp. 427–436, July 1978.

[3] J. Max, "Quantizing for minimum distortion," *IRE Transactions on Information Theory*, vol. IT-6, pp. 7–12, Mar. 1960.

[4] S. A. Kassam, "Optimum quantization for signal detection," *IEEE Transactions on Communications*, vol. COM-25, no. 5, pp. 479–484, May 1977.

[5] B. Aazhang and H. V. Poor, "On optimum and nearly optimum data quantization for signal detection," *IEEE Transactions on Communications*, vol. COM-32, no. 7, pp. 745–751, July 1984.

[6] H. V. Poor, "Fine quantization in signal detection and estimation," *IEEE Transactions on Information Theory*, vol. 34, no. 5, pp. 960–972, Sept. 1988.

[7] P. P. Gandhi, "Data quantization effects in CFAR signal detection," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 32, no. 4, pp. 1277–1289, Oct. 1996.

[8] A. Sonnenschein and P. M. Fishman, "Radiometric detection of spread-spectrum signals in noise of uncertain power," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 28, no. 3, pp. 654–660, July 1992.

[9] J. Tsui, *Digital Techiques for Wideband Receivers*, Artech House, Inc., Norwood, MA, 1st edition, 1995.