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Performance Analysis of CCK Modulation under Multipath Fading Channel

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ABSTRACT

The complementary codes have been applied in the high speed wireless local area network due to their good correlation properties. However, the performance analysis formulae of CCK modulation under multipath fading channel have not been found in published papers so far. In this paper, the closed-form BER and CWER (codeword error rate) performance expressions are derived according to complementary code properties, CCK demodulation regulation and procedure. Simulation link of IEEE 802.11b DSSS baseband is constructed over COSSAP platform. The BER and CWER versus Eb/N0 curves are obtained through numerical calculations and simulations under multipath fading channels. The validity of the formulae has been tested by comparing the numerical and simulation results. The difference between numerical and simulation results is discussed and interpreted.

1. INTRODUCTION

Complementary code keying (CCK) has been adopted in IEEE 802.11b Standard for 11Mbps DSSS baseband. The CCK modulation performance is described approximately in [1] using the performance analysis formulae derived for M-ary Bi-Orthogonal keying (MBOK), the numerical results and simulation results of CCK performance under AWGN channel condition are given. However, the performance analysis formulae of CCK modulation under multipath fading channel condition have not been found in the published papers so far.

In this paper, the closed-form analytical formulae are derived for BER (bit error rate) and CWER (codeword error rate) performance of the CCK modulation, according to the CCK modulation regulation, multipath model and demodulation procedure. Numerical results and simulation results under single-path and multipath fading channel conditions are presented. Validity of the derived formulae is proved by comparing the analytical curves with the simulation curves.

2. CCK MODULATION AT TRANSMITTER

Eight successive information bits B=(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7) can be denoted as B=(B_1, B_2), where B_1= (b_0, b_1) and B_2=(b_2, b_3, b_4, b_5, b_6, b_7). B is encoded to an 8-chip codeword C, and each chip is of a 4-phase complex value,

$$C=\{e^{i(\varphi_1+\varphi_2+\varphi_3+\varphi_4)},e^{i(\varphi_1+\varphi_3+\varphi_4)},e^{i(\varphi_1+\varphi_2+\varphi_4)},$$

$$-e^{i(\varphi_1+\varphi_4)},e^{i(\varphi_1+\varphi_2+\varphi_3)},e^{i(\varphi_1+\varphi_3)},-e^{i(\varphi_1+\varphi_2)},e^{i\varphi_1}\}$$

$$=e^{i\varphi_1}\{e^{i\phi_0},e^{i\phi_1},e^{i\phi_2},e^{i\phi_3},e^{i\phi_4},e^{i\phi_5},e^{i\phi_6},e^{i\phi_7}\} \quad (1)$$

where φ_1 is obtained by DQPSK from the first two data bits B_1=(b_0, b_1) (Table 1); φ_2, φ_3 and φ_4 are determined from the bit pairs in B_2 respectively (Table 2).

Table 1 DQPSK encoding [2]

B_1=(b_0,b_1)	φ_1 of even chip	φ_1 of odd chip
00	0	π
01	$\pi/2$	$3\pi/2$
10	π	0
11	$3\pi/2$	$\pi/2$

Table 2 QPSK encoding

(b_{2n-2}, b_{2n-1}) (n =2,3,4)	φ_n
00	0
01	$\pi/2$
10	π
11	$3\pi/2$

There are 256 codewords since they are mapped from 8 binary data bits. It can be observed in (1) that every codeword has a common factor $e^{i\varphi_1}$ which has 4 possible values. Given $e^{i\varphi_1}$, there is a set of 64 different codewords determined by B_2. These 64 codewords have ideal cross-correlation properties due to orthogonality [1]. The m-th (m=1,2,…,64) codeword in the set is[3]

$$C_m(t)=\sum_{k=0}^{7}rect(t-kT_c)e^{i\phi'_{k,m}}=e^{i\varphi_1}\Phi_m(t)$$

where, $\Phi_m(t)=\sum_{k=0}^{7}rect(t-kT_c)e^{i\phi_{k,m}}$, T_c denotes chip duration, $rect(t)$ a rectangular function spanning over $0\le t<T_c$, $\phi'_{k,m}=\phi_{k,m}+\varphi_1$. There is one to one correspondence between function $\Phi_m(t)$ and data bits B_2.

Let P_m denote transmission power and T_s (=8 T_c) presents the time duration of codeword C, the transmitted signal can be expressed as

$$s_m(t)=\sqrt{P_m}C_m(t),\qquad 0\le t<T_s \quad (2)$$

3. MULTIPATH RAYLEIGH FADING CHANNEL

The baseband model of a multipath channel is

$$h(t)=\sum_{l=0}^{L-1}h_l\delta(t-\tau_l)$$

where h_l and τ_l are the l-th path complex gain and time delay, respectively. The real and imaginary parts of h_l are both zero-mean Gaussian random variables, and | h_l | is Rayleigh distributed.

4. MULTIPATH DIVERSITY USING CMF

Multipath diversity is an effective way to combat fading. Typical RAKE receiver extracts multipath replicas with several correlators, and performs maximum ratio

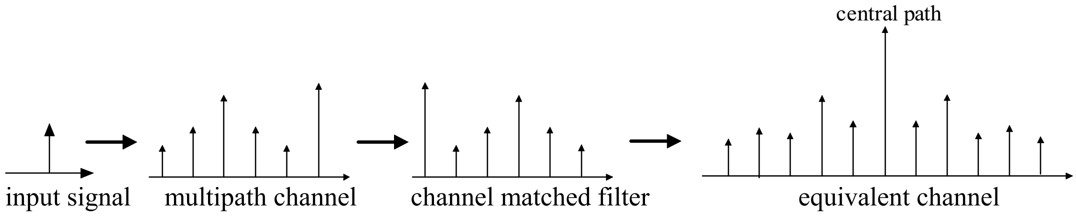

Fig.1. Impulse responses of multipath channel, CMF and the equivalent channel.

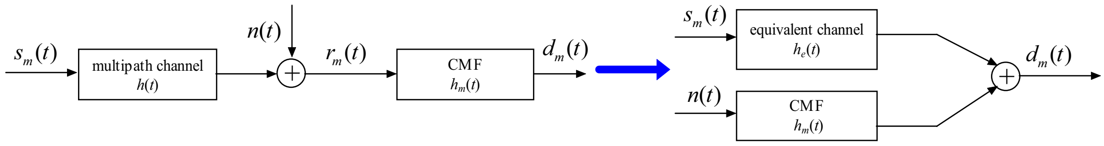

Fig.2. The equivalent expression of the output signal of CMF

combining (MRC) using delay and weighting summation (i.e., linear filtering). The weighting coefficients are the conjugates of the channel estimates $\hat{h}_l$. Another structure of multipath diversity was proposed by Intersil Corporation[1]. The linear filtering is performed before correlation, resulting in simpler implementation.

4.1 Channel Matched Filter

This linear filter is referred to channel matched filter (CMF)[1]. Its impulse response $h_m(t)$ is the conjugate and time reversal of the multipath channel impulse response in (2).

$$h_m(t) = h^*(\tau_{L-1} - t) = \sum_{l=0}^{L-1} h_l^* \delta(\tau_{L-1} - \tau_l - t)$$

where the subscript "m" denotes match, and the superscript "*" presents complex conjugation.

4.2 Equivalent Channel

CMF is located at the front-end of the receiver, and performed before CCK demodulation. The series interconnection of multipath channel and CMF can be seen as an equivalent channel (EC) with $2L-1$ paths, as shown in Fig. 1. Its impulse response is

$$h_e(t) = h(t) * h_m(t) = \sum_{l=0}^{L-1} |h_l|^2 \delta(t - \tau_{L-1}) + \sum_{l=0}^{L-1} \sum_{k=0,k\neq l}^{L-1} h_l h_k^* \delta(t - \tau_l - \tau_{L-1} + \tau_k) \quad (3)$$

where the subscript "e" denotes equivalent, and the mark "$*$" expresses convolution. The first term in (3) is the central path of EC in Fig. 1, with a delay τ_{L-1}. The second term corresponds to the other paths of EC.

4.3 Output of CMF

As shown in Fig. 2, the signal $s_m(t)$ is transmitted through multipath channel $h(t)$ and corrupted by noise $n(t)$, where $n(t)$ is a zero-mean Gaussian noise with dual-sided power spectral density $N_0/2$. Let $r_m(t)$ denote the received signal which is the input of CMF, $d_m(t)$ is the output of CMF. $d_m(t)$ is the sum of two components: the output of EC with input $s_m(t)$, and the output of CMF with input $n(t)$,

$$\begin{aligned} d_m(t) &= s_m(t) * h_e(t) + n(t) * h_m(t) \\ &= \sum_{l=0}^{L-1} |h_l|^2 \sqrt{P_m} C_m(t - \tau_{L-1}) + \sum_{l=0}^{L-1} h_l^* n(t + \tau_l - \tau_{L-1}) \\ &+ \sum_{l=0}^{L-1} \sum_{k=0,k\neq l}^{L-1} h_l h_k^* \sqrt{P_m} C_m(t - \tau_l - \tau_{L-1} + \tau_k) \end{aligned} \quad (4)$$

The first term in (4) is the signal component. It is the maximum ratio combination of multipath replicas, and has a delay τ_{L-1} as compared with the transmit signal in (2). The last term reflects multipath interference, and the second term is noise.

5. CCK DEMODULATION

CCK demodulation is performed after channel matched filtering, and the demodulation procedure is described in Fig. 3. There are 64 correlators. Correlation is performed between $d_m(t)$ and $\Phi_j(t)$ (j=1,2,…,64) respectively. 64 cross-correlation values at time instance τ_{L-1}, $R_{m,1}(\tau_{L-1})$ to $R_{m,64}(\tau_{L-1})$, are compared in their absolute values. Let $\hat{m}$ denote the sequence number of the correlator whose output cross-correlation at time instance τ_{L-1} has the maximal absolute value, and $\hat{m}$ is thus the estimate of the sequence number of the transmitted codeword in a 64 codeword set. $\hat{B}_2$, the estimate of the transmitted 6-bit B_2, is then obtained according to $\hat{m}$. $\hat{B}_1$ can be obtained by DQPSK demodulation. The decoded codeword $\hat{B}$ is generated by multiplexing $\hat{B}_1$ and $\hat{B}_2$ in order.

There is a delay τ_{L-1} between $d_m(t)$ and $s_m(t)$ because of the EC in Fig.1. Therefore, the output cross-correlation value of the j-th correlator at time instance τ_{L-1} is

$$\begin{aligned} R_{m,j}(\tau_{L-1}) &= \frac{1}{T_s} \int_{\tau_{L-1}}^{\tau_{L-1}+T_s} d_m(t) \Phi_j^*(t - \tau_{L-1}) dt \\ &= D_{m,j} + S_{m,j}(\tau_k - \tau_l) + \upsilon_j \end{aligned} \quad (5)$$

where

$$D_{m,j} = \left(\sum_{l=0}^{L-1} |h_l|^2 \right) \sqrt{P_m} e^{j\varphi_1} \delta(m - j)$$

$$S_{m,j}(\tau_k - \tau_l) = \sqrt{P_m} \sum_{l=0}^{L-1} \sum_{k=0,k\neq l}^{L-1} h_l h_k^* e^{j\varphi_1} \frac{1}{T_s} \int_0^{T_s} \Phi_m(t - \tau_l + \tau_k) \Phi_j^*(t) dt$$

$$\upsilon_j = \sum_{l=0}^{L-1} h_l^* \frac{1}{T_s} \int_0^{T_s} n(t + \tau_l) \Phi_j^*(t) dt$$

In (5), $D_{m,j}$ is the desired signal term, $S_{m,j}(\tau_k - \tau_l)$ is multipath interference term. Noise term υ_j can be proved to be subject to zero-mean Gaussian distribution with variance $(N_0/2T_s) \times \sum_{l=0}^{L-1} |h_l|^2$.

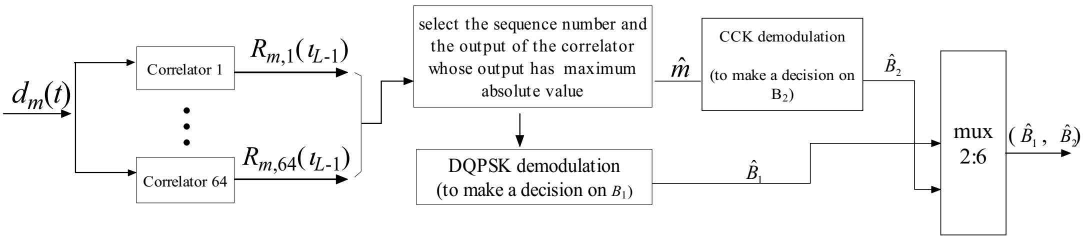

Fig.3. CCK demodulation procedure

6. PERFORMANCE UNDER AWGN CHANNEL

In order to provide useful formulae for multipath channel case, CCK modulation performance is studied first under additive white Gaussian noise (AWGN) channel. AWGN channel is a special case of multipath channel with L=1, h_0=1 and delay spread τ_0=0. Equ. (5) is thus simplified to

$$R_{m,j}(0)=\begin{cases}\sqrt{P_m}e^{i\varphi_1}+\upsilon_m, & j=m\\ \upsilon_j, & j\neq m\end{cases} \tag{6}$$

where υ_j is subject to Gaussian distribution $N(0, N_0/2T_s)$. Without loss of generality, assume that the transmitted codeword and corresponding signal are $C_1(t)$ and $s_1(t)$ respectively.

6.1 Output of the First Correlator

It is known from (6), the output of the first correlator can be expressed as $R_{1,1}(0)=Y_1+iY_2$, where the real part $Y_1=\sqrt{P_m}\cos\varphi_1+\upsilon_x$, and the imaginary part $Y_2=\sqrt{P_m}\sin\varphi_1+\upsilon_y$. Y_1 and Y_2 are subject to Gaussian distributions $N(\sqrt{P_m}\cos\varphi_1, \sigma^2)$ and $N(\sqrt{P_m}\sin\varphi_1, \sigma^2)$ respectively. Where $\sigma^2=var(\upsilon_j)/2$. Let $R_1=|R_{1,1}(0)|$, then R_1 is of Rice distribution with a probability density function (pdf) [4]

$$p_{R_1}(r_1)=\frac{r_1}{\sigma^2}e^{-(r_1^2+P_m)/2\sigma^2}I_0\left(\frac{r_1}{\sigma^2}\sqrt{P_m}\right),\quad r_1\geq 0$$

where $I_0(x)$ is the 0th-order modified Bessel function of the first kind.

6.2 Output of the Other Correlators

The output of the j-th correlator is $R_{1,j}=\upsilon_j$ (j=2,3, … ,M), and its real part and imaginary part are both subject to Gaussian distribution $N(0, \sigma^2)$. Let $R_j=|R_{1,j}(0)|$, then all the variables $R_j=|\upsilon_j|$ (j=2,3, … ,M) are statistically independent and subject to an identical Rayleigh distribution[4] with a pdf expression

$$p_{R_j}(r_j)=\frac{r_j}{\sigma^2}e^{-r_j^2/2\sigma^2},\quad r_j\geq 0,\quad j=2,3,\ldots,M$$

6.3 Probability of Deciding Data Bits B_2 Correctly

In order to decide data bits B_2 correctly, R_1 has to be larger than each of R_2, R_3, … , R_M . The probability of making correct decision can thus be expressed as

$$P_c(B_2)=\int_0^{+\infty}P(R_1>R_2,R_1>R_3,\ldots,R_1>R_M\mid C_1)p(R_1)dR_1 \tag{7}$$

where subscript "c" means correct. $P(R_1>R_2, R_1>R_3, \ldots, R_1>R_M|C_1)$ denotes the joint probability that R_1 is larger than each of R_2, R_3, … , R_M, given the transmitted codeword C_1. Then this joint probability is averaged over all values of R_1, and the probability of deciding B_2 correctly can be obtained. Since the $\{R_j\}$(j=2,3, … ,M) are statistically independent with identical distribution, the joint probability can be expressed as a product of M-1 marginal probabilities

$$P(R_1>R_2,R_1>R_3,\ldots,R_1>R_M\mid C_1)=\left(P(R_1>R_2\mid C_1)\right)^{M-1} \tag{8}$$

The marginal probability $P(R_1>R_2|C_1)$ can be expressed as

$$P(R_1>R_2\mid C_1)=\int_0^{R_1}p_{R_2}(r_2)dr_2=1-e^{-R_1^2/2\sigma^2}$$

Let $t=R_1^2/(2\sigma^2)$ and substitute into (8) and then (7), Equation (7) can be further derived as

$$P_c(B_2)=e^{-\gamma_s}\int_0^{+\infty}\left(1-e^{-t}\right)^{M-1}e^{-t}I_0\left(\sqrt{4t\gamma_s}\right)dt \tag{9}$$

where the signal-to-noise ratio per symbol γ_s is

$$\gamma_s=\frac{|D_{m,m}|^2}{\mathrm{var}(\upsilon_m)}=\frac{P_m}{N_0/(2T_s)}=\frac{2T_sP_m}{N_0}=\frac{2E_s}{N_0}=\frac{16E_b}{N_0}=8\gamma_b$$

where $E_S=T_SP_m=8E_b$ indicates the energy per transmitted symbol, E_b denotes the energy per bit, γ_b is SNR per bit.

6.4 Probability of Making a Correct Decision on B_1

Given that B_2 has been decided correctly, data bits $B_1=(b_0, b_1)$ are DQPSK demodulated from the output of the m-th correlator. The probability of symbol error of DQPSK modulation is [4]

$$P_b(B_1\mid B_2)=Q_1(a,b)-0.5I_0(ab)\exp\left[-2\gamma_b\right]$$

where, $a=\left[2\gamma_b\left(1-1/\sqrt{2}\right)\right]^{1/2}$, $b=\left[2\gamma_b\left(1+1/\sqrt{2}\right)\right]^{1/2}$, and $Q_1(a,b)$ is Markum Q function.

B_1 consists of two data bits, hence the relation between the conditional probabilities of symbol error and bit error can be expressed as[4]

$$P_b(B_1\mid B_2)=\frac{2^{2-1}}{2^2-1}P_s(B_1\mid B_2)$$

The conditional probability of making correct decision on B_1 is thus

$$P_c(B_1\mid B_2)=1-P_s(B_1\mid B_2)=1-1.5P_b(B_1\mid B_2) \tag{10}$$

6.5 Codeword Error Rate and Bit Error Rate

Codeword C can be decided correctly when both B_1 and B_2 have been decided correctly. Probability of making correct decision on C is

$$P_C(C)=P_C(B_1B_2)=P_C(B_2)\,P_C(B_1|B_2).$$

CWER is thus

$$P_e(C)=1-P_c(C)=1-P_c(B_1B_2)=1-P_c(B_2)P_c(B_1\mid B_2)$$

Bit error rate (BER) is then

$$P_b = \frac{2^{8-1}}{2^8 - 1} P_e(C) \approx \frac{1}{2} P_e(C) = 0.5 - 0.5 P_c(B_2) P_c(B_1 | B_2) \quad (11)$$

7. CCK MODULATION PERFORMANCE UNDER MULTIPATH CHANNEL

Multipath fading channel is considered now. According to (5), the m-th transmitted signal is demodulated correctly when the absolute value of the output of the m-th correlator is larger than those of all other correlators. And SNR of codeword is

$$\gamma_s = \frac{|D_{m,m}|^2}{|S_{m,m}(\tau_k - \tau_l)|^2 + \mathrm{var}(\upsilon_j)} = \frac{2\left(\sum_{l=0}^{L-1} |h_l|^2\right)^2 E_s}{2T_s |S_{m,m}(\tau_k - \tau_l)|^2 + N_0 \sum_{l=0}^{L-1} |h_l|^2} = 8\gamma_b \quad (12)$$

Adopting anti-multipath techniques like decision feedback equalization at the receiver, the multipath interference term $S_{m,m}(\tau_k - \tau_l)$ can be ignored, then (12) becomes $\gamma_b = (N_0/2T_s) \times \sum_{l=0}^{L-1} |h_l|^2$. Assume that every path of the multipath channel has identical average SNR per bit $\bar{\gamma}_c$, then

$$\gamma_b = \frac{2E_b}{N_0} \sum_{l=0}^{L-1} |h_l|^2, \qquad l = 0,1,\ldots,L-1$$

Because all $|h_l|$ are subject to identical Rayleigh distribution, γ_b is subject to χ^2 distribution with degree of freedom of $2L$, and its pdf is [4]

$$p(\gamma_b) = \frac{1}{(L-1)! \bar{\gamma}_c^{\,L}} \gamma_b^{L-1} e^{-\gamma_b/\bar{\gamma}_c}, \qquad \gamma_b \geq 0$$

The bit error rate is then

$$P_b = \int_0^\infty P_b(\gamma_b) p(\gamma_b) d\gamma_b \quad (13)$$

Using Equations (11), (10) and (9), $P_b(\gamma_b)$ can be written as

$$P_b(\gamma_b) = 0.5 - 0.5 e^{-8\gamma_b} \int_0^{+\infty} \left(1 - e^{-t}\right)^{M-1} e^{-t} I_0\left(\sqrt{32 t \gamma_b}\right) dt$$
$$\times \left\{1 - 1.5\left[Q_1(a,b) - 0.5 I_0(ab) \exp(-2\gamma_b)\right]\right\}$$

In practice, the codeword error probability is more concerned. CWER can be derived from (11) as

$$P_e(C) = 2P_b \quad (14)$$

8. NUMERICAL AND SIMULATION RESULTS

The BER and CWER versus E_b/N_0 results are obtained using Equations (13) and (14), under single path fading channel and 2-path identical strength fading channel conditions respectively. Simulation link of IEEE 802.11b DSSS baseband are constructed over COSSAP platform. Under the above channel conditions and assuming various mobile speeds, simulations are carried out; BER and CWER results are obtained. Part of the numerical and simulation results are plotted in Fig. 4.

It can be observed that under single path fading channel condition and for the same E_b/N_0, CWER is about 2 times of BER, this verifies the relation in (14). Same conclusion also stands under 2-path fading channel condition.

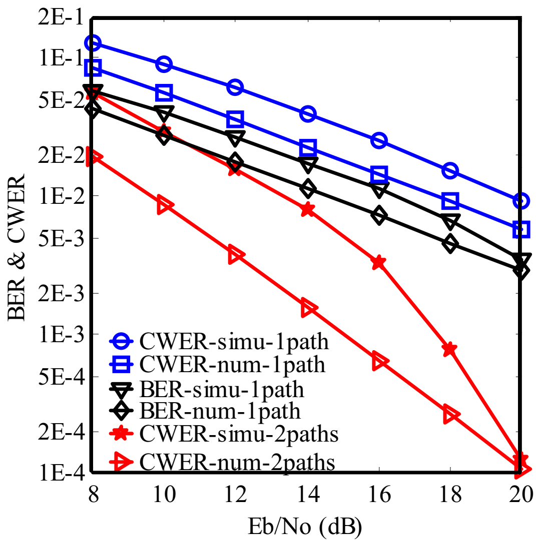

Fig. 4. BER and CWER (fading channel, pedestrian speed)

For both single path and 2-path identical strength fading channels, to achieve the same CWER or the same BER, the difference between the numerical and simulation values of the required E_b/N_0 is around 2 dB.

Under 2-path fading channel, the difference between numerical and simulation values is relatively large at low E_b/N_0. The reason is that the multipath interference in (12) is ignored in the derivation. In contrast, at high E_b/N_0, the difference between numerical and simulation values is small, indicating that larger value of $N_0 \sum |h_l|^2$ in the denominator of (12) allows ignoring the multipath interference term.

9. CONCLUSIONS

In this paper, closed-form formulae for BER and CWER performances of CCK modulation have been obtained through mathematical derivations. Numerical results have been compared with simulation results, and the accuracy of the presented formulae has been validated. The main reason causing the difference between numerical results and simulation results has been interpreted.

ACKNOWLEDGMENT

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