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Performance Analysis of Parallel Interference Cancellation Detector in Downlink MC-CDMA Systems

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ABSTRACT

This paper proposes two types of low-complexity iterative multiuser detectors for downlink MC-CDMA systems, based on parallel interference cancellation (PIC). It will be shown that per-carrier MMSE-PIC detector is equivalent to per-user MMSE-PIC detector in the fully loaded case, and per-carrier MMSE-PIC obtains the same BER performance as per-user MMSE-PIC with one tenth of computational complexity for lower loaded cases. In addition, EGC based PIC detector is 1 dB worse than per-user MMSE-PIC, however the complexity is about one half of per-user MMSE-PIC detector's complexity for the fully loaded case and about one twentieth for lower loaded cases.

1. INTRODUCTION

Multi-Carrier Code Division Multiple Access (MC-CDMA) combines the advantages of orthogonal frequency division multiplexing (OFDM) and CDMA [1]. It is an attractive candidate for future mobile broadband communications. However, in a multiuser setting, the orthogonality of the spreading codes is severely distorted under fading channels, causing MC-CDMA systems to suffer from multiple access interference (MAI). Thus, the overall BER (bit error rate) performance decreases abruptly with increasing number of users. Multiuser detection (MUD) techniques have been introduced to improve the system performance [2]. MUD jointly combines all the detected signals to reduce the total amount of MAI in detection. Among the MUD techniques, PIC is a non-linear MUD technique wherein the transmitted signal of each user is detected in parallel over a number of iterations.

Various PIC detectors have been proposed to improve the performance by mitigating MAI in MC-CDMA systems. However, the gap between single user bound and the PIC detector is not discussed in [3-5]. The iterated-decision multiuser detector presented in [6] is originally used in the uplink CDMA systems. In [7], the detector is analyzed in the linear precoded (LP-) OFDM context. It has been shown that the iterative PIC detector can achieve the single user bound with only a few iterations. For the first iteration, the detector reduces to the per-user Minimum Mean Square Error (MMSE) multiuser detector, thus, we refer to it as per-user MMSE-PIC detector. The per-user MMSE-PIC detector has been extended to MC-CDMA systems, and it also obtains the single user bound when the number of subchannels is large enough [8].

Based on [7-8], we propose per-carrier MMSE-PIC and Equal Gain Combining (EGC) based PIC detectors for downlink MC-CDMA systems reducing dramatically the complexity of the detector.

The paper is organized as follows: Section 2 describes the MC-CDMA system model. Section 3 presents the different PIC detector schemes and compares their complexities. Section 4 provides simulation results. Finally, Section 5 summarizes the conclusions.

2. SYSTEM DESCRIPTION

A single-cell downlink MC-CDMA system model is adopted in this paper, where the data of K users are transmitted simultaneously on N subchannels. IFFT and FFT are used for modulation and demodulation. The guard interval (GI) implemented as a cyclic prefix (CP) is longer than the channel delay spread, such that the transmission channel is effectively flat within each subchannel. Moreover, all the user channels are identical.

With these assumptions, intersymbol interference is avoided, and then only one symbol from each user is contributing to the observed data within one detection interval. After GI removal and FFT, a direct frequency domain signal model can be used. The signal within one detection interval appears as (time index omitted)

$$\mathbf{r} = \begin{bmatrix} H_1 c_{1,1} & H_1 c_{2,1} & \cdots & H_1 c_{K,1} \\ H_2 c_{1,2} & H_2 c_{2,2} & \cdots & H_2 c_{K,2} \\ \vdots & \vdots & \ddots & \vdots \\ H_N c_{1,N} & H_N c_{2,N} & \cdots & H_N c_{K,N} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_K \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix} \quad (1)$$
$$= \mathbf{Sa} + \mathbf{n}$$

where $\mathbf{r} = [r(1), r(2), ..., r(N)]^T$ with $r(i)$ being the i-th subchannel observation. A_k is the transmitted symbol for user k and **n** denotes the noise vector. The code chips of user k are denoted as $c_{k,1}, c_{k,2},..., c_{k,N}$ and H_i denotes the i-th subchannel response. Further, **S** can be written as **S** = **ΛC**, where

$$\mathbf{C}=\begin{bmatrix} c_{1,1} & c_{2,1} & \cdots & c_{K,1} \\ c_{1,2} & c_{2,2} & \cdots & c_{K,2} \\ \vdots & \vdots & \ddots & \vdots \\ c_{1,N} & c_{2,N} & \cdots & c_{K,N} \end{bmatrix} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_K \end{bmatrix} \quad (2)$$

and $$\mathbf{\Lambda} = diag\{H_1, H_2, \cdots, H_N\} \quad (3)$$

3. PARALLEL INTERFERENCE CANCELLATION DETECTORS

3.1 Per-user MMSE-PIC Detector

The per-user MMSE-PIC detector is depicted in Fig. 1. The basic idea underlying the detection process is to retrieve simultaneously after equalization all the interfering symbols based on previous estimations. The MAI is regenerated and cancelled at the next iteration. This reduces the multiuser interference for all the symbols and increases the SINR.

At iteration l, the received signal **r** is multiplied by a $K\times N$ feed forward matrix $(\mathbf{B}^l)^H$ producing the equalized $K\times 1$ vector $\tilde{\mathbf{r}}^l = (\mathbf{B}^l)^H \mathbf{r}$. Then, based on the symbol estimates of the previous iteration, an estimate of the MAI is subtracted from $\tilde{\mathbf{r}}^l$ to produce $\tilde{\mathbf{a}}^l$, i.e. $\tilde{\mathbf{a}}^l = \tilde{\mathbf{r}}^l - (\mathbf{D}^l)^H \hat{\mathbf{a}}^{l-1}$. The diagonal elements of the backward matrix $\mathbf{D}^l$ are restricted to be zero in order to estimate MAI. A bank of slicers applied to $\tilde{\mathbf{a}}^l$ generates the current estimate $\hat{\mathbf{a}}^l$ of the transmitted symbols.

Let **a** and $\hat{\mathbf{a}}^l$ be vectors of zero-mean uncorrelated symbols with energy ε (equal energy symbols). Their correlation can be written in the form [6]

$$E[\mathbf{a}(\hat{\mathbf{a}}^l)^H] \approx \varepsilon\boldsymbol{\rho}^l = \varepsilon\, diag[\rho_1^l, \rho_2^l, \cdots, \rho_K^l] \quad (4)$$

where ρ_k^l is taken as a measure of the reliability of $\hat{a}_k^l$ and is called confidence coefficient. The matrices $\mathbf{B}^l$ and $\mathbf{D}^l$ which maximize the SINR are given by [7]:

$$\mathbf{D}^l = (\boldsymbol{\rho}^{l-1})^H[(\mathbf{B}^l)^H\mathbf{S} - diag\{((\mathbf{B}^l)^H\mathbf{S})_{1,1}, \cdots, ((\mathbf{B}^l)^H\mathbf{S})_{K,K}\}]^H \quad (5)$$

$$\mathbf{B}^l \propto \varepsilon[\sigma^2\mathbf{I} + \varepsilon\mathbf{S}(\mathbf{I} - \boldsymbol{\rho}^{l-1}(\boldsymbol{\rho}^{l-1})^H)\mathbf{S}^H]^{-1}\mathbf{S} \quad (6)$$

The expression of the SINR β_k^l for the k[th] user is

$$\beta_k^l = \frac{\eta_k^l\varepsilon}{1-(1-{\rho_k^{l-1}}^2)\eta_k^l\varepsilon} \quad (7)$$

where $\eta_k^l = \mathbf{S}[k]^H[\sigma^2\mathbf{I} + \varepsilon\mathbf{S}(\mathbf{I} - \boldsymbol{\rho}^{l-1}(\boldsymbol{\rho}^{l-1})^H)\mathbf{S}^H]^{-1}\mathbf{S}[k]$, and $\mathbf{S}[k]$ is the k^{th} column of **S**. For QPSK modulation,

$$\rho_k^l = 1 - 2Q\left(\sqrt{\beta_k^{l-1}}\right) \quad (8)$$

with $Q(x) = \frac{1}{2\pi}\int_x^\infty e^{-t^2/2}dt,\ x \ge 0$. In the downlink case, the confidence coefficients for all the users are the same, we denote it as ρ^l for the lth iteration. For QPSK modulation, the per-user MMSE-PIC obtains the single user bound with 5 iterations when having 256 subchannels or more [7-8].

3.2 Per-carrier MMSE-PIC Receiver

In the fully loaded case (K=N), the per-user MMSE estimation principle coincides in the downlink with the per-carrier MMSE detection [9]. Similarly, the per-carrier MMSE-PIC detector can be deduced from the per-user MMSE-PIC in fully loaded case. Due to code orthogonality, for any N and K, $\mathbf{C}^T\mathbf{C} = \mathbf{I}$, but with K=N, $\mathbf{C}\mathbf{C}^T = \mathbf{I}$ is also satisfied. Thus, the feed forward matrix is given by:

$$\mathbf{B}^l \propto diag\{\frac{H_1}{\sigma^2+\varepsilon\left(1-{\rho^{l-1}}^2\right)|H_1|^2}, \frac{H_2}{\sigma^2+\varepsilon\left(1-{\rho^{l-1}}^2\right)|H_2|^2}, \cdots, \frac{H_N}{\sigma^2+\varepsilon\left(1-{\rho^{l-1}}^2\right)|H_N|^2}\}\mathbf{C} \quad (9)$$

The backward matrix is the same as equation (5), whereas, the expression of the SINR β_k^l for the k[th] user is given by:

$$\beta_k^l = \frac{\mathbf{B}^l[k]^H\mathbf{S}[k]\mathbf{S}[k]^H\mathbf{B}^l[k]\varepsilon}{\{\mathbf{B}^l[k]^H[\sigma^2\mathbf{I} + \varepsilon(1-{\rho^{l-1}}^2)\mathbf{S}\mathbf{S}^H - \varepsilon(1-{\rho^{l-1}}^2)\mathbf{S}[k]\mathbf{S}[k]^H]\mathbf{B}^l[k]\}} \quad (10)$$

The SINR β_k^l expressed in (10) reduces to (7) with the matrix $\mathbf{B}^l$ defined in (6) [7]. As equations (6) and (9) are the same for fully loaded case, it can be concluded that per-carrier MMSE-PIC is equivalent to per-user MMSE-PIC in fully loaded case.

3.3 EGC based PIC Detector

Unlike in per-user MMSE-PIC and per-carrier MMSE-PIC detectors, the feed forward matrix is fixed for EGC based PIC detector. The feed forward matrix is a pure EGC equalizer.

$$\mathbf{B}^l = diag\{\frac{H_1}{|H_1|}, \frac{H_2}{|H_2|}, \cdots, \frac{H_N}{|H_N|}\}\mathbf{C} \quad (11)$$

The backward matrix is also chosen in the way that SINR is maximized for each user at the input of the slicer and is again as written in (5). Hence, we only need to get the equalized vector $\tilde{\mathbf{r}}^l$ once, and the complexity of the detector is reduced. The SINR β_k^l for the kth user is given by (10).

3.4 Complexity Comparison

Based on the equations for the three PIC detectors, the number of real multiplications divided by the number of detected symbols in the MC-CDMA multiplex is used as the complexity measure. Table 1 provides the corresponding results. It can be observed that per-carrier MMSE-PIC detector and EGC based PIC detector have less multiplications than per-user MMSE-PIC detector for the lower loaded cases. In the downlink channel, per-carrier MMSE-PIC is equivalent to per-user MMSE-PIC for the fully loaded case. If we have less users, per-carrier MMSE-PIC detector has about one tenth of the multiplications of per-user MMSE-PIC for the same number of iterations. The complexity of EGC based PIC is comparable with per-carrier MMSE-PIC at the MMSE/EGC stage and the first iteration of PIC. For more iterations, the complexity of EGC based PIC is about one half of that of per-carrier MMSE-PIC.

4. NUMERICAL AND SIMULATION RESULTS

Some simulation results are presented in this section. The MC-CDMA system model described in Section 2 is used. The spreading is done by Walsh-Hadamard codes of the same length as the number of subchannels. Subchannels are assumed to be independently Rayleigh fading and non-frequency-selective. QPSK modulation is used for all users in the simulation.

Fig. 2, Fig. 3 and Fig. 4 present the simulation and theoretical results of the three detectors for half loaded case when the number of subcarriers is 256. Fig. 5 and Fig. 6 are for the fully loaded case.

Both the simulation and theoretical results show that per-carrier-MMSE-PIC detector can achieve the same performance as per-user MMSE-PIC with enough iterations in the half loaded case. It should also be noted that EGC based PIC is about 1dB worse in performance than per-user MMSE-PIC in downlink channel theoretically. Moreover, the simulated and theoretical curves match when having 256 subchannels for EGC based PIC.

5. CONCLUSIONS

In this paper, per-carrier MMSE-PIC and EGC based PIC detectors are proposed for the downlink scenario of MC-CDMA systems. The gap between EGC based PIC detector and per-user MMSE detector is 1dB with 5 iterations, however, the complexity is reduced to about one half for the fully loaded case and about 1/20 for other cases. Per-carrier MMSE PIC detector is equivalent to per-user MMSE PIC receiver in the fully loaded case. For lower loaded cases, per-carrier MMSE PIC detector is as good as linear MMSE-PIC receiver with the complexity of about 1/10. Anyway, the computational complexity of iterated PIC detector is rather high with high number of subcarriers. In the future work, it is important to try to find ways to reduce the observed gap between theoretical and actual performance of iterated PIC detector while using a considerably lower number of subcarriers in the MC-CDMA multiplex.

ACKNOWLEDGMENT

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Table 1. Multiplications per detected symbol (downlink), N=256.

K		per-user MMSE-PIC	per-carrier MMSE-PIC	EGC based PIC
128	MMSE /EGC	1.051×10^6	0.016×10^5	0.010×10^5
	1st PIC	1.970×10^6	1.349×10^5	1.329×10^5
	2nd PIC	2.889×10^6	2.683×10^5	1.336×10^5
	3rd PIC	3.809×10^6	4.017×10^5	1.344×10^5
256	MMSE /EGC	0.015×10^5	0.015×10^5	0.010×10^5
	1st PIC	2.657×10^5	2.657×10^5	2.637×10^5
	2nd PIC	5.299×10^5	5.299×10^5	2.652×10^5
	3rd PIC	7.941×10^5	7.941×10^5	2.668×10^5

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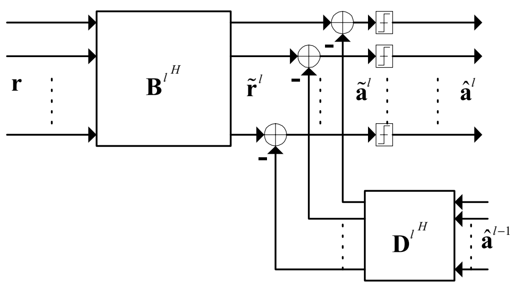

Fig. 1. Iterated-decision multiuser PIC detector.

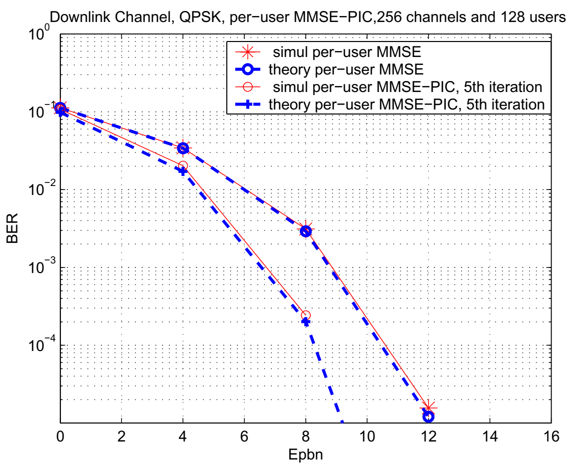

Fig. 2. Performance for per-user MMSE-PIC detector (half loaded case).

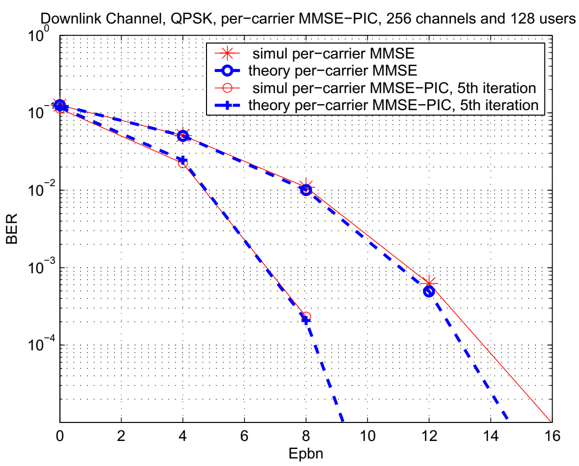

Fig. 3. Performance for per-carrier MMSE-PIC detector (half loaded case).

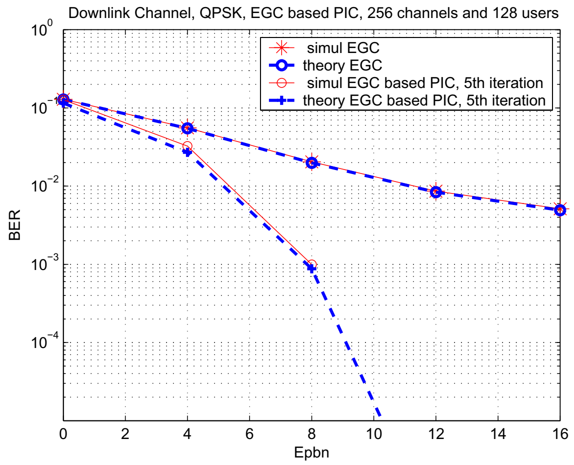

Fig. 4. Performance for EGC based PIC detector (half loaded case).

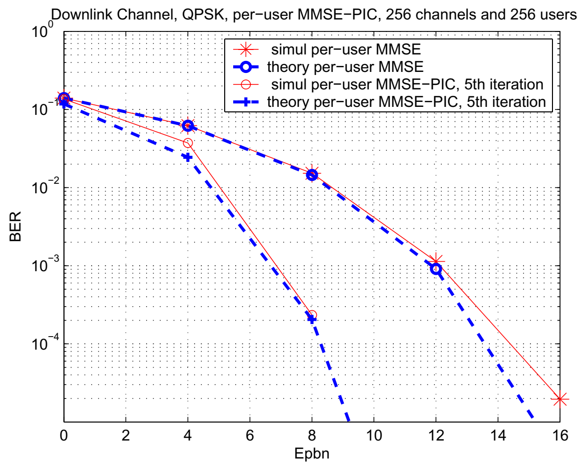

Fig. 5. Performance for per-user MMSE-PIC detector (fully loaded case).

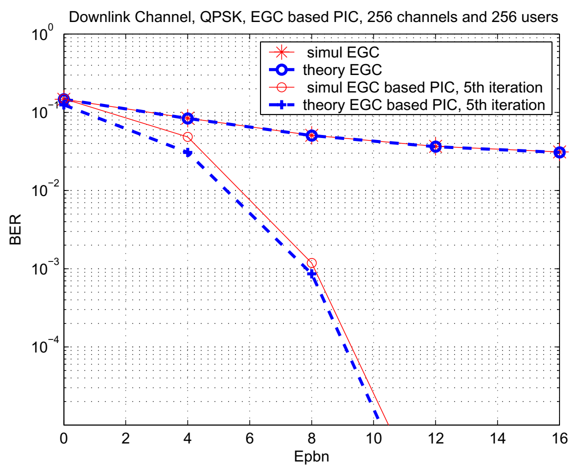

Fig. 6. Performance for EGC based PIC detector (fully loaded case).