Adaptive Steiglitz-McBride Notch Filter Design for Radio Interference Suppression in VDSL Systems

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Abstract— **In this paper, Recursive least square (RLS) based complex Adaptive notch filter (ANF) using Steiglitz-McBride (SM) method is proposed to suppress the Radio frequency interference (RFI) in very-high-speed digital subscriber line (VDSL) systems. The proposed RLS-SM ANF converges fast and requires less computational complexity than the existing direct form constrained ANF using recursive prediction error (RPE) algorithm. The proposed algorithm is specially advantageous when dealing with multiple RFI's.**

I. Introduction

VDSL is an emerging broadband access technology to provide a fast (up to 52 Mbps) data connection by using the existing twisted-pair telephone cables. As the VDSL transmission spectrum can occupy a bandwidth of up to 12 MHz [1], a VDSL receiver has to face radio frequency interference (RFI) from the amateur radio transmissions. Such an RFI is a single side band (SSB) modulated narrowband signal and it can be extremely damaging to VDSL transmission because the ingressed interfering noise level may be as high as 0 dBm peak envelope power (PEP) at the receiver input and the average power can be -3 dBm for the worst-case [1]. To alleviate the RFI problem, an analog RFI canceller can be implemented to prevent the receiver analog-to-digital convertor (ADC) from saturation. However, the interference level after the ADC can still be several orders of magnitude higher than the desired signal. Thus, it is important to combat the RFI also in the digital domain.

In this paper, we focus on digital RFI suppression in the baseband. Since the amateur radio interference band is very narrow compared to the sampling frequency of the VDSL signal, we can model the RFI as a sum of complex sinusoids embedded in a white noise like VDSL signal in the baseband. We propose a new complex ANF algorithm using Steiglitz-McBride method to suppress RFI.

The paper is organized as follows. In Section 2, the system model is defined. Complex ANF algorithms using the SM method are reviewed in Section 3. Section 4 analyzes the ANF convergence. In Section 5, simulation results show the improved performance of ANF using SM method. Finally, Section 6 concludes the paper.

II. System model

The simplest time-domain model for an RFI corrupted baseband VDSL signal is a sum of complex sinusoids embedded in complex white Gaussian noise. Note that there may exist multiple RFI sources simultaneously, the model can be expressed as,

$$y(k) = \sum_{i=1}^{M} R_i exp(j\omega_i k + \phi) + \epsilon(k) \tag{1}$$

where R_i are the amplitudes of the RFI signals. They depend on the distances between the amateur radio transmitters and the VDSL cables. R_i also depend on the electromagnetic characteristics of the cables. We consider the worst case that the power spectra density (PSD) of the RFI can be as high as -40 dBm/Hz. $\epsilon(k)$ is a sequence of i.i.d. complex random variable with zero mean and variance denoted by σ^2. It models the wideband VDSL signal which is standardized to have -60 dBm/Hz PSD mask. ω_i is a pseudo-carrier frequency where has the highest power. Because of the SSB-SC modulation, the frequency where most energy are distributed is not around the carrier frequency, but shifted 1-2 kHz due to the PSD of the voice signal. It is known that (1) can be represented by an ARMA model [2],

$$A(q^{-1})y(k) = A(\rho q^{-1})\epsilon(k) \tag{2}$$

where $A(q^{-1})$ is a monic polynomial of order M and its roots are on the unit circle with arguments equal to $\{\omega_i\}$. The parameter $\rho \in (0, 1)$ is a pole radius which keeps the filter $A(q^{-1})/A(\rho q^{-1})$ stable. Such filter is also known as constrained form notch filter. In our application, $\epsilon(k)$ in (2) approximates the wideband VDSL signal to an order $o(\sqrt{1-\rho})$, where $o(x)$ is defined such that $|o(x)/x|$ is bounded as $x \to 0$ [3]. If the notch filter can be identified online, filter output $\epsilon(k)$ is the clean VDSL signal. To guarantee reliable VDSL transmission, at least 20 dB signal to noise ratio (SNR) is needed in the RFI band. Therefore, the digital RFI suppressor should reduce the RFI PSD by 40 dB at the incident frequencies. In addition, the adaptive RFI suppressor should converge fast to the incident frequency and be able to deal with multiple RFI frequencies.

III. Complex ANF algorithms using SM method

In an early contribution to ANF algorithms by Nehorai [4], the real-valued Gauss-Newton type recursive prediction error (RPE) algorithm was derived for constrained notch filter. The

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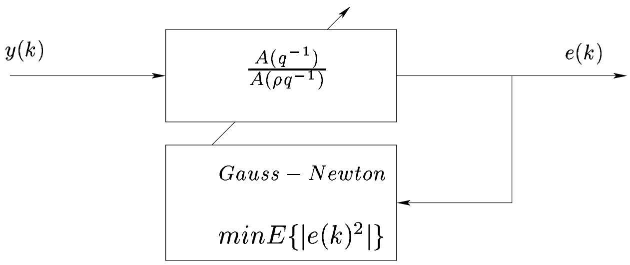

Fig. 1. Adaptive notch filter with recursive prediction error algorithm for coefficient adjustment

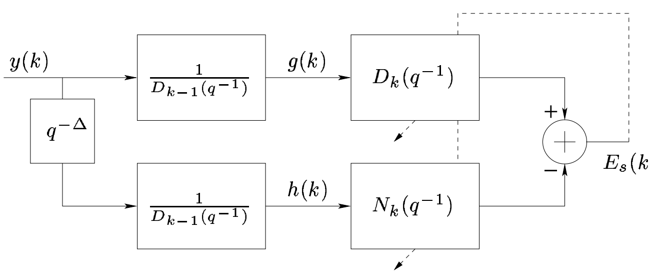

Fig. 2. Adaptive notch filter based on Steiglitz-McBride method

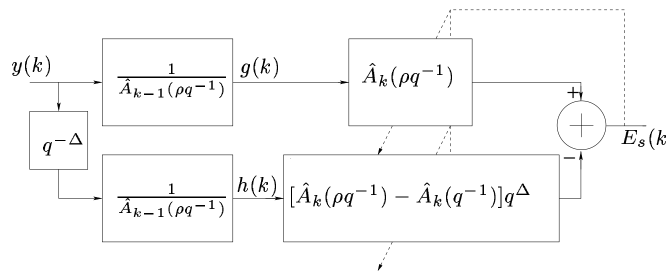

Fig. 3. Block diagram of ANF using Steiglitz-McBride method in adaptive line enhancer structure

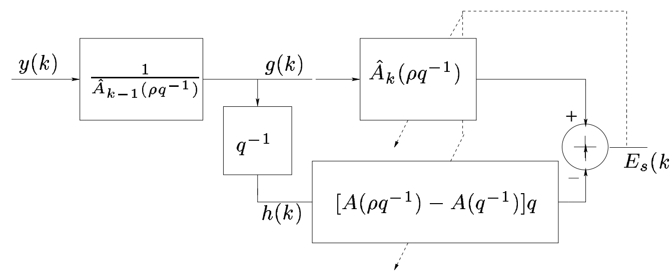

Fig. 4. Block diagram of modified ANF using Steiglitz-McBride method

RPE algorithm has the structure shown in Fig. 1. The algorithm adjusts the filter coefficients to minimize the cost function $E\{|e(k)^2|\}$ by calculating the gradient recursively. Based on the same objective function, Pei [5] extended the RPE algorithm to complex coefficient ANF, which converges to a small biased solution. Cheng [3] derived a new real-valued ANF algorithm using the well-known SM method. Cheng's idea comes from the system identification application by using delayed signal as the reference signal [6]. The resulting block diagram is depicted in Fig. 2. The function of the delay factor Δ in the figure is to decorrelate the prefilter outputs $g(k)$ and $h(k)$ in the upper and lower paths. By letting the structure to approximate a notch filter, the structure shown in Fig. 3 is obtained. As we notice that there is an advance operation in the lower branch of the filter, such a structure is unrealizable except when $\Delta = 1$. Cheng [3] modified the structure by introducing delays also at the upper branch to eliminate the advance operation. It is shown that the algorithm converges to an unbiased solution. In this paper, we extend the idea of [3] and derive a complex coefficient ANF algorithm using the SM method with a simplified structure.

A. Simplified ANF structure

Rather than introducing delays before the prefilter, we move the delay operation at the lower branch after the prefilter shown in Fig. 2, such arrangement can save one prefilter block. Because our input VDSL signal is modelled as white noise, we can define $\Delta = 1$. Larger Δ can be chosen in other applications when the noise is colored. Since the resulting transfer function after the convergence is desired to be a notch filter, the following equation should be satisfied:

$$\lim_{k\to\infty}\left(1 - q^{-1}\frac{N_k(q^{-1})}{D_k(q^{-1})}\right) = \frac{A(q^{-1})}{A(\rho q^{-1})} \tag{3}$$

This yields the block diagram shown in Fig 4. Therefore the polynomials $D_k(\rho q^{-1})$ and $N_k(q^{-1})$ in modified Fig. 2 can be defined as

$$\begin{aligned} D_k(q^{-1}) &= A(\rho q^{-1}) \\ N_k(q^{-1}) &= [A(\rho q^{-1}) - A(q^{-1})]q \end{aligned} \tag{4}$$

B. Algorithm derivation

The adaptive algorithm can be derived directly from Fig 4. Let the estimated coefficient vector $\Theta_{k-1} = [a_1, a_2, \cdots, a_M]^T_{k-1}$ where the superscript T denotes the transpose operation. Using the recursive least square (RLS) procedure, we derive the detailed algorithm as below.

Step1: Prefilter

$$g(k) = \frac{1}{\hat{A}_{k-1}(\rho q^{-1})}y(k) \tag{5}$$

where $\hat{A}_{k-1}(\rho q^{-1}) = 1 + a^*_{1,k-1}\rho q^{-1} + a^*_{2,k-1}\rho^2 q^{-2} + \cdots + a^*_{M,k-1}\rho^M q^{-M}$.

Rearranging the input-output, we obtain,

$$g(k) = y(k) - \Theta_{k-1}^H \mathbf{G}_k \tag{6}$$

where the superscript H denotes conjugate transpose, and

$$\mathbf{G}_k = [\rho g(k-1), \rho^2 g(k-2), \cdots, \rho^M g(k-M)]^T$$

Since the prefilter output for lower branch $h(k) = g(k-1)$ is a delayed version of $g(k)$, one prefilter can be saved.

Step 2: Output expression
The output can also be arranged in vector form,

$$\begin{aligned}\epsilon(k) &= g(k)\hat{A}_k(q^{-1}) - h(k+1)[\hat{A}_k(\rho q^{-1}) - \hat{A}_k(q^{-1})] \\ &= g(k) - \Theta_{k-1}^H \Phi(k) \end{aligned} \tag{7}$$

where

$$\Phi(k) = [\phi_1(k), \phi_2(k), \cdots, \phi_M(k)]^T$$

and

$$\phi_i(k) = -\rho^i g(k-i) + (\rho^i - 1)h(k-i+1)$$

Step 3: Covariance matrix update

$$\mathbf{P}(k+1) = \frac{1}{\lambda(k)}\left[\mathbf{P}(k) - \frac{\mathbf{P}(k)\Phi(k)\Phi^H(k)\mathbf{P}(k)}{\frac{\lambda(k)}{\alpha(k)} + \Phi^H(k)\mathbf{P}(k)\Phi(k)}\right] \tag{8}$$

where $\lambda(k) = 1 - \alpha(k)$ is the forgetting factor in the RLS algorithm.

Step 4: Estimation parameter update

$$\Theta(k+1) = \Theta(k) + \alpha(k)\mathbf{P}_{k+1}\Phi(k)\epsilon(k)^* \tag{9}$$

The algorithm is in the RLS form. The differences between the RPE algorithm and our RLS algorithm using the SM methods are on the choice of regression vector $\Phi(\mathbf{k})$ and error $\epsilon(k)$. A better choice of these parameters can lead to faster convergence and less excess mean square error (MSE) at the output.

IV. Convergence Considerations

In both RPE and RLS-SM algorithms, the convergence speed and the excess MSE depends on two parameters: the pole radius ρ and the stepsize α.

A. Time-varying ρ

In most of the ANF algorithms the pole radius is a time-varying function [5], [3]. The reason is that ρ determines the bandwidth of the notches. Practically, if no *a priori* information is available on the input sinusoid, when the notches are too narrow, the algorithm may not converge. On the other hand, a larger pole radius ($\rho \to 1$) will lead to less excess MSE after convergence. Therefore an exponential function is often used for $\rho \to 1$ by letting ρ grow from an initial value $\rho(1)$ to the desired value $\rho(\infty)$ according to

$$\rho(k+1) = \rho_0 \rho(k) + (1-\rho_0)\rho(\infty) \tag{10}$$

where ρ_0 determines the rate of change in $\rho(k)$.

B. Optimal α for SM method

In the algorithms derived by Pei [5] and Cheng [3], the stepsize is treated in the same way as ρ, which approaches exponentially the predefined value. Without*a priori* knowledge of the input, the choice of α is usually a difficult task. In this paper, we apply the optimal stepsize derived in [7] for IIR filters using the SM method. An optimal stepsize puts a proper weight on the new incoming data at each updating step, which will lead to the maximal reduction of MSE, thus speeding up the convergence. The optimal α has the form

$$\alpha(k) = \frac{\kappa}{1+\tau(k)} \tag{11}$$

where $0 < \kappa < 1$ is a reduction factor which is related to the filter order and $\tau(k) = \Phi^H(k)\mathbf{P}(k)\Phi(k)$. Note that $\tau(k)$ is an intermediate result of (8), so that finding the optimal convergence factor does not increase the complexity of the algorithm.

V. Simulations

We apply the proposed complex RLS-SM ANF and the direct form ANF using RPE algorithm to suppress RFI respectively in the first downstream channel of a single carrier modulated (SCM) VDSL systems standardized by [1]. The channel occupies band from 0.138 MHz to 3 MHz. The data rate varies according to the constellation size, ranging from 8.1 to 12.96 Mbps. The amateur radio signal that interferes with this channel is expected to appear between 1.81 MHz and 2.0 MHz and is bandlimited to 4 kHz. If we downsample the baseband signal according to the RFI band, VDSL signal is white noise, and RFI will appear between $(-0.5, 0.5)$ in the normalized frequency range. By cascading first order ANF, we can estimate the corresponding RFI frequencies. The reason for downsampling is that the RFI only appears between $(-0.03, 0.03)$ in the normalized frequency range if we use the VDSL sampling rate. ANF cannot discriminate so closely located multiple RFI sources. The ANF will converge to local minimum and RFI suppression will distort the VDSL signal. In all the following experiments, The pole radius is time-varying according to (10), where $\rho_0 = 0.99$, $\rho(1) = 0.7$ and $\rho(\infty) = 0.995$. We use optimal stepsize derived in (11) for RLS-SM algorithms, assuming no *a priori* knowledge of the input signal, whereas the stepsize in RPE algorithm is optimized according to the input signal to interference ratio (SIR). The input signal is modelled as in (1). The RFI and VDSL power are chosen such that the interference level from one RFI sources is 20 dB higher than the VDSL signal. In multiple RFI case, the SIR is calculated by the sum of RFI powers and VDSL. The output signal is normalized with respect to the VDSL power, in other words, 0 dB output is the best suppression result that we can achieve. The simulation results are averaged over 100 independent runs. The SIR and normalized output power are shown in Table I.

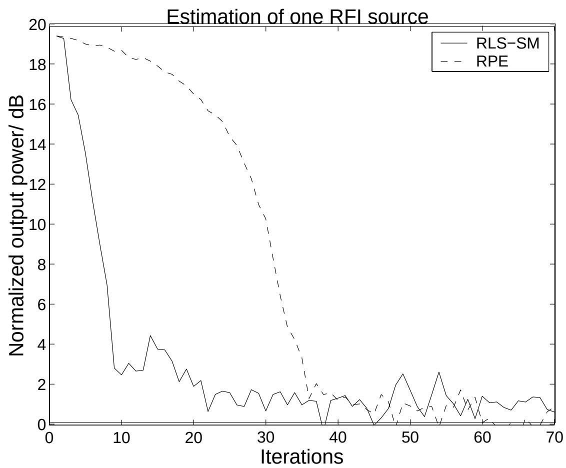

Fig. 5. The output MSE when estimating 1 RFI source

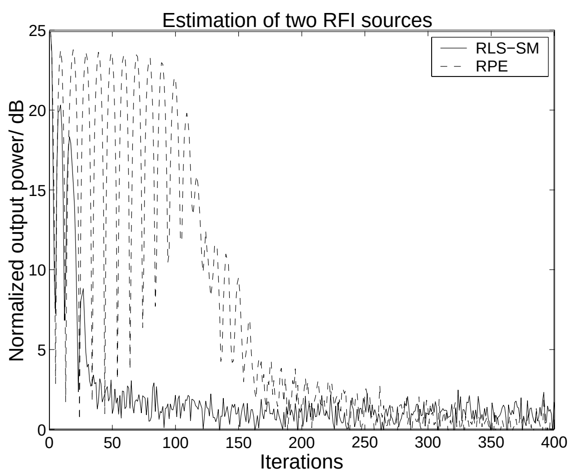

Fig. 6. The output MSE when estimating 2 RFI sources

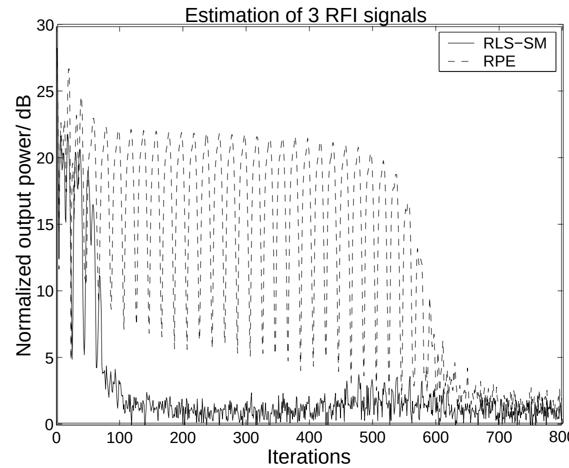

Fig. 7. the output MSE when estimating 3 RFI sources (Fixed α and ρ for RPE)

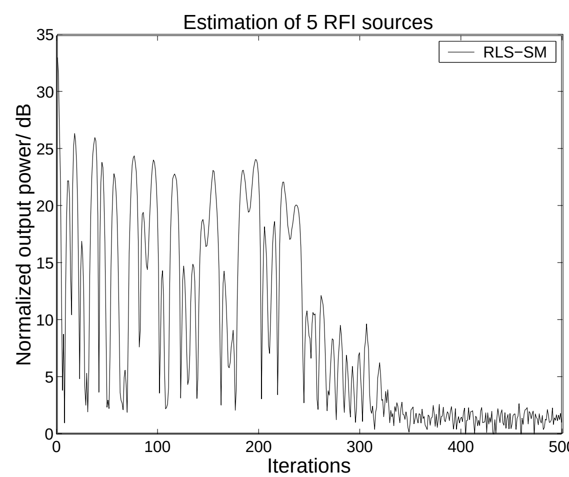

Fig. 8. The output MSE when estimating 5 RFI sources using RLS-SM algorithm

A. *The first-order notch filter*

There is one complex sinusoid signal embedded in white noise, whose frequency is $\omega_1 = 0.015$. The filter output MSE is shown in Fig. 5. It can be seen that using RLS-SM algorithm leads to the optimal solutions in ca. 20 iterations, whereas the RPE algorithm requires ca. 40 iterations to converge.

B. *The second-order notch filter*

Now there are two RFI sources whose frequencies are $\omega_1 = 0.1$ and $\omega_2 = 0.3$. The slower convergence can be seen in Fig. 6. As we can also notice, RPE algorithm leads to lower excess MSE in the stationary model because of the choice of the stepsize. However, since RFI frequency are slowly time-varying in practice, RLS-SM algorithm has better tracking ability and converges fast at the same time.

C. *The third-order notch filter*

The third experiment is for the case of three RFI signal where their frequencies are $\omega_1 = 0.1$, $\omega_2 = 0.2$ and $\omega_3 = 0.4$. In this case, RPE algorithm with time-varying pole radius does not converge, therefore ρ is set fixed at 0.8. As can be seen in Fig. 7, compare with RLS-SM algorithm, the RPE algorithm converges slower and generates higher excess MSE.

D. *Estimation of 5 RFI sources using RLS-SM algorithm*

This is an extreme case that 5 RFI sources exist with frequencies $\omega_1 = 0.1$, $\omega_2 = 0.2$, $\omega_3 = 0.25$, $\omega_4 = 0.3$ and $\omega_5 = 0.4$. Since this is a difficult situation for ANF to converge, we loose the criteria on excess MSE and let $\rho(\infty) = 0.95$. As can be seen in Fig. 8, the algorithm converges in ca. 400 iterations,

TABLE I

NORMALIZED OUTPUT MSE AFTER CONVERGENCE

	SIR(dB)	RLS-SM	RPE
1^{st} order	-20	0.6	0.09
2^{nd} order	-25	1.18	0.97
3^{rd} order	-29	1.2	1.7
5^{th} order	-34	1.32	–

whereas RPE algorithm fails to converge within 1024 iterations.

VI. CONCLUSIONS

In this paper, the ANF using SM method proposed in [3] is extended to the complex-coefficient case. We propose a simplified structure by relocating the delay elements on one branch of the filter. Simulations show RLS-SM algorithm converges faster than the RPE algorithm when suppressing sinusoid embedded in white Gaussian noises. It is also more robust since it can deal with up to 5 RFI sources. Furthermore, optimized stepsize developed in [7] for RLS-SM algorithms is also employed to speed up the convergence. By cascading first order notch filters, we can efficiently suppress multiple RFI signals in VDSL systems.

REFERENCES

[1] "Very high-speed digital subscriber line; part 2: Transceiver specification," Tech. Rep. TS 101 270-2, V0.1.1, ETSI, Sophia, France, March 2000.

[2] J. A. C. Bingham, *ADSL, VDSL, and Multicarrier Modulation*, Wiley, 2000.

[3] M. H. Cheng and J. L. Tsai, "A new adaptive notch filter with constrained poles and zeros using Steiglitz-McBride method," in *Proceedings of ICASSP98*, Seattle, Washington, USA, May 12-15 1998, pp. 1469–1472.

[4] A. Nehorai, "A minimal parameter adaptive notch filter with constrained poles and zeros," *IEEE Trans.on ASSP*, vol. ASSP-33, no. 4, pp. 983–996, 1985.

[5] S. C. Pei and C. Tseng, "Complex adaptive IIR notch filter algorithm and its applications," *IEEE Trans. on Circuits and Systems-II: Analog and Digital Signal Processing*, vol. 41, no. 2, pp. 158–163, 1994.

[6] P. S. R. Diniz, *Adaptive filtering: Algorithms and Practical Implementation*, Kluwer, 1997.

[7] J. E. Cousseau and P. S. R. Diniz, "On optimal convergence factor for IIR adaptive filters," in *Proceedings of ISCAS '94*, Tokyo, Japan, October 1994, pp. 137–140.