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Measurements and Modelling of Nonlinear Power Amplifiers

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ABSTRACT

In this paper, nonlinear power amplifier modelling for fourth-generation mobile communications is studied. We carried out measurements in the 2 to 8 GHz range using a nonlinear power amplifier and developed a nonlinear model with memory based on the Hammerstein structure (static nonlinearity + linear filter). A polynomial model for the nonlinearity and an FIR filter with a bulk delay for the linear part were employed. The parameters were obtained with simple least-squares (LS) estimation. The obtained model agrees well with the measurement results.

1. INTRODUCTION

The major driver for future wireless broadband multimedia communication systems is the increasing demand for personal mobile communications that require increased data rates, capacity, flexibility and reliability. The new radio interfaces are predicted to support data rates up to 100 Mbit/s for mobile access and up to 1 Gbit/s for wireless local area access. This means that the limited bandwidth, transmit power and other resources must be used as efficiently as possible, close to the optimal theoretical limits.

Orthogonal Frequency Division Multiplexing (OFDM) is one of the most promising modulation technologies for these future systems. OFDM is a multicarrier modulation technique where a single data stream is transmitted over a number of lower rate subcarriers. Some of the key advantages of OFDM are efficient handling of multipath environments, channel capacity enhancement in slow time-varying channels, robustness against narrowband interference and high spectral efficiency. The most notable drawbacks are the high Peak-to-Average Power Ratio (PAPR) and sensitivity to both frequency offset and phase noise.

High PAPR indicates that a highly linear power amplifier is required at the transmitter. The linearity requirement can be met by driving the power amplifier well below its saturation point. This causes poor power efficiency, which is especially bad in a mobile transmitter. Driving the power amplifier closer to its saturation point is appealing, since it would increase power efficiency and prolong battery life of a mobile transmitter. However, driving the power amplifier above the linear region results in nonlinear distortion effects. The nonlinear distortion makes it more difficult to receive the signal and generates harmonic and intermodulation signals outside of the intended frequency band. These unwanted distortion products are potential interfering sources to other users of the radio interface, especially on adjacent frequency bands. The interference must therefore be reduced to a level where both systems can operate satisfactorily.

In this paper, we want to develop a simple model for a nonlinear amplifier based on new measurement results. The paper is organized as follows. In Section 2, the measurement system and methods are described. Section 3 describes the chosen Hammerstein-type model structure and the proposed parameter estimation method. Section 4 presents the modelling results. Conclusions and further work is discussed in Section 5.

2. MEASUREMENTS

Measurements of a nonlinear power amplifier must be done in order to obtain realistic gain and phase distortion data for modelling purposes. The measurements used in this research were done in cooperation with the Communications Laboratory and the Radio Communications Laboratory of Helsinki University of Technology.

Amplitude Modulation to Amplitude Modulation (AM-AM) and Amplitude Modulation to Phase Modulation (AM-PM) conversion, as well as frequency sweep of gain and phase measurements were performed [1]. The measured amplifier is a Mini-Circuits ZVE-8G with a wide frequency band of 2 to 8 GHz and nominal gain of 30 dB. An Agilent Technologies PNA Series Microwave Network Analyzer model E8363A was the main measuring device used, together with two Midwest microwave attenuators with 3 dB and 20 dB nominal value. The setup system for these measurements is shown in Figure 1.

The amplifier's amplitude and phase responses for a single-tone input were measured using a frequency sweep from 1.5 GHz to 8.5 GHz with a 5 MHz sampling step. The AM-AM and AM-PM conversion characteristics of the amplifier were measured varying the input level from -27 dBm to 5 dBm with a 0.5 dBm sampling step.

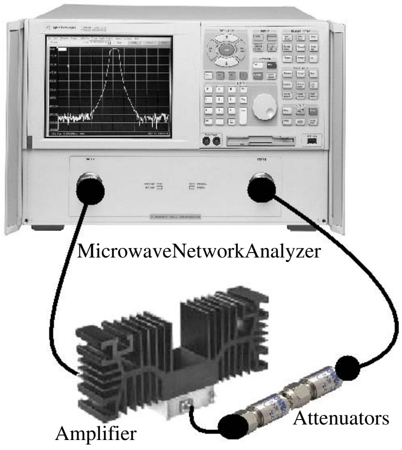

Fig. 1: *Setup system for amplifier measurements.*

3. EFFICIENT POWER AMPLIFIER MODEL

The objective is to model the measured frequency-dependent nonlinear behavior of the amplifier using a Hammerstein-type model where the nonlinearity and the dynamics of the system are assumed separable [2]. The nonlinearity is implemented by a polynomial model and the dynamics by an FIR filter [3, 4]. The estimation problem is to minimize the difference between the output of the power amplifier and the Hammerstein-type model as illustrated in Figure 2. Mathematically this can be formulated as

$$E = \left|H_{d,p}(A_l, \omega_k) - H_p(A_l, \omega_k)\right|^2 \tag{1}$$

where $H_{d,p}$ is the desired power-dependent frequency response, H_p is the power-dependent frequency response of the model, A_l and ω_k are the discrete amplitude and frequency points at which both $H_{d,p}$ and H_p are evaluated. Direct minimization of the error function defined in Equation (1) is very difficult [5] and hence we want to further simplify it.

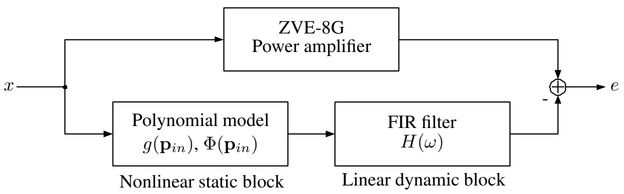

Fig. 2: *Illustration of the estimation problem. The nonlinear static block is implemented using polynomials and the linear dynamic block is implemented as an FIR filter.*

The problem can be simplified by assuming that the identification of the two blocks can be done separately. The polynomial model is given by the AM-AM and AM-PM measurements at the center frequency of the desired operating band. The FIR filter is given by the small-signal frequency response of the modelled system. The simplified estimation problem is to minimize the error functions

$$E_g = \left|A_{d,l} - g(A_l)\right|^2 \tag{2a}$$
$$E_\Phi = \left|\phi_{d_s,l} - \Phi(A_l)\right|^2 \tag{2b}$$
$$E_H = \left|H_{d,s_0}(\omega_k) - H(\omega_k)\right|^2 \tag{2c}$$

where $A_{d,l}$ are the desired discrete output amplitude values, $\phi_{d_s,l}$ are the desired scaled discrete output phase values and $H_{d,s_0}(\omega_k)$ is the desired scaled discrete small-signal response.

The desired discrete output phase values must be scaled to zero degree at the small-signal input power level A_s at which $H(\omega_k)$ is fitted to or else the phase delay of the amplifier is included twice in the estimation. Similarly, the small-signal response $H_{d,s}(\omega_k)$ must be scaled to have unit gain at the center frequency ω_c at which $g(A)$ and $\Phi(A)$ are fitted to. Otherwise the gain of the amplifier is included twice in the estimation. Mathematically the required scaling operations can be written as

$$\phi_{d_s,l} = \phi_{d,l} - \phi_{d,s} \tag{3}$$
$$\left|H_{d,s_0}(\omega_k)\right| = \frac{\left|H_{d,s}(\omega_k)\right|}{\left|H_{d,s}(\omega_0)\right|}. \tag{4}$$

3.1. Static nonlinearity

The AM-AM and AM-PM measurement results at a given frequency can be represented by three vectors

$$\mathbf{p}_{in} = \begin{bmatrix}p_{in}(0) & p_{in}(1) & \cdots & p_{in}(L-1)\end{bmatrix}^{\mathrm{T}} \tag{5a}$$
$$\mathbf{p}_{out} = \begin{bmatrix}p_{out}(0) & p_{out}(1) & \cdots & p_{out}(L-1)\end{bmatrix}^{\mathrm{T}} \tag{5b}$$
$$\boldsymbol{\phi}_{out} = \begin{bmatrix}\phi_{out}(0) & \phi_{out}(1) & \cdots & \phi_{out}(L-1)\end{bmatrix}^{\mathrm{T}} \tag{5c}$$

where $\mathbf{p}_{in}$ contains the input power values, $\mathbf{p}_{out}$ contains the measured output power values, $\boldsymbol{\phi}_{out}$ contains the measured output phase shift values and L is the number of measured frequency points. Defining the coefficient vectors $\mathbf{a}$ and $\mathbf{b}$

$$\mathbf{a} = \begin{bmatrix}a_0 & a_1 & \cdots & a_N\end{bmatrix}^{\mathrm{T}} \tag{6a}$$
$$\mathbf{b} = \begin{bmatrix}b_0 & b_1 & \cdots & b_N\end{bmatrix}^{\mathrm{T}} \tag{6b}$$

and the observation matrix $\mathbf{U}$

$$\mathbf{U} = \begin{bmatrix}1 & p_{in}(0) & \cdots & p_{in}^N(0) \\ 1 & p_{in}(1) & \cdots & p_{in}^N(1) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & p_{in}(L-1) & \cdots & p_{in}^N(L-1)\end{bmatrix}, \tag{7}$$

which has the special form of a Vandermonde matrix, the polynomial model can be compactly written in vector notation as

$$g(\mathbf{p}_{in}) = \mathbf{U}\mathbf{a} \tag{8a}$$
$$\Phi(\mathbf{p}_{in}) = \mathbf{U}\mathbf{b}. \tag{8b}$$

As can be seen from Equation (8), the polynomial model is linear in coefficients and hence linear least-squares estimation can be directly applied to solve it. The least-squares estimators (LSE) can be written as [6]

$$\hat{\mathbf{a}} = \left(\mathbf{U}^{\mathrm{T}}\mathbf{U}\right)^{-1}\mathbf{U}^{\mathrm{T}}\mathbf{p}_{out} \tag{9a}$$

$$\hat{\mathbf{b}} = \left(\mathbf{U}^{\mathrm{T}}\mathbf{U}\right)^{-1}\mathbf{U}^{\mathrm{T}}\boldsymbol{\phi}_{out}. \tag{9b}$$

3.2. Linear dynamic block

The frequency response of a Mth-order ($M+1$ coefficients) causal FIR filter is defined as

$$H\left(e^{j\omega_k}\right) = \sum_{m=0}^{M} h_m e^{-j\omega_k m} \tag{10}$$

where the filter coefficients $h_0, h_1, \ldots, h_m$ are assumed to be real-valued and ω_k is a set of discrete angular frequencies where the response is evaluated. The filter order can be reduced by adding a bulk delay of D samples before the filter.

The filter design problem is to choose the filter coefficients $h_0, h_1, \ldots, h_M$ so that the weighted least-squares error is minimized:

$$E = \sum_{k=0}^{K-1} W(\omega_k) \left| H_d\left(e^{j\omega_k}\right) - \sum_{m=0}^{M} h_m e^{-j\omega_k(m+D)} \right|^2 \tag{11}$$

where ω_k is a set of K discrete angular frequencies in the range $[-\pi, \pi]$, $H_d\left(e^{j\omega_k}\right)$ is the desired complex-valued frequency response, $W(\omega_k)$ is a real-valued nonnegative weight function, M is the desired order of the filter and D is the chosen length of the bulk delay.

Equation (11) can be elaborated into the form [7]

$$E = \mathbf{h}^{\mathrm{T}}\mathbf{P}\mathbf{h} - 2\mathbf{h}^{\mathrm{T}}\mathbf{p}_1 + p_0 \tag{12}$$

where

$$\mathbf{h} = \begin{bmatrix} h_0 & h_1 & \cdots & h_M \end{bmatrix}^{\mathrm{T}} \tag{13}$$

$$\mathbf{P} = \sum_{k=0}^{K-1} W(\omega_k)\mathbf{C}\left(\omega_k\right) \tag{14}$$

$$\mathbf{p}_1 = \sum_{k=0}^{K-1} W(\omega_k)\Big[\mathrm{Re}\Big\{H_d\left(e^{j\omega_k}\right)\Big\}\mathbf{c}\left(\omega_k\right) - \mathrm{Im}\Big\{H_d\left(e^{j\omega_k}\right)\Big\}\mathbf{s}\left(\omega_k\right)\Big] \tag{15}$$

$$p_0 = \sum_{k=0}^{K-1} W(\omega_k)\left|H_d\left(e^{j\omega_k}\right)\right|^2 \tag{16}$$

and

$$\mathbf{c}\left(\omega_k\right) = \begin{bmatrix} \cos D\omega_k & \cdots & \cos(D+M)\omega_k \end{bmatrix}^{\mathrm{T}} \tag{17a}$$

$$\mathbf{s}\left(\omega_k\right) = \begin{bmatrix} \sin D\omega_k & \cdots & \sin(D+M)\omega_k \end{bmatrix}^{\mathrm{T}} \tag{17b}$$

$$\mathbf{C}\left(\omega_k\right) = \begin{bmatrix} 1 & \cdots & \cos\left(M\omega_k\right) \\ \cos\left(\omega_k\right) & \cdots & \cos\left[(M-1)\,\omega_k\right] \\ \vdots & \ddots & \vdots \\ \cos\left(M\omega_k\right) & \cdots & 1 \end{bmatrix} \tag{17c}$$

and the optimal least-squares solution is obtained in the normal form as

$$\mathbf{h} = \mathbf{P}^{-1}\mathbf{p}_1. \tag{18}$$

4. MODELLING RESULTS

Figure 3 shows the detailed structure of the employed Hammerstein-type model. The static nonlinearity was implemented with a 5th-order polynomial, and the linear part with a 13th-order FIR filter preceded by a bulk delay of 17 samples. The bulk delay reduces the required filter order from 30 to 13.

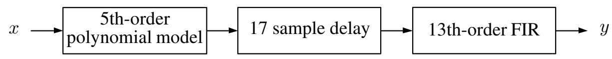

Fig. 3: *Implementation of the Hammerstein-type model.*

Figure 4 shows the nonlinearity approximation fitted to single-tone data measured at 6 GHz, for the amplitude dynamics between $-27\ldots+5$ dBm. The approximation is seen to be very good for both the amplitude and phase.

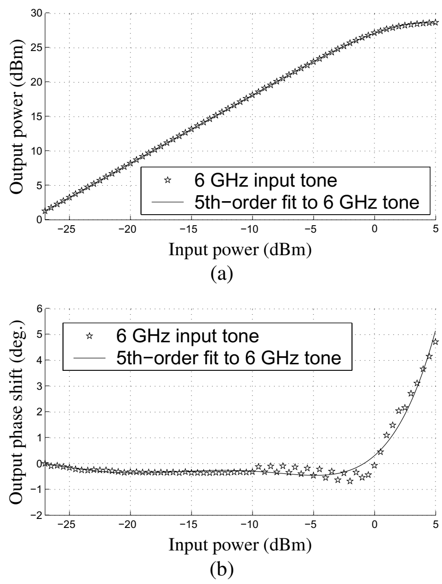

Fig. 4: *A 5th-order least-squares polynomial fit of the measured AM-AM and the AM-PM characteristics at 6 GHz.*

Figure 5 illustrates the measured and the estimated frequency response at -27 and 0 dBm input signal power levels. The frequency-dependent gain variations are clearly observed, as well as the saturation (reduced gain) for the 0 dBm signal. The dotted curves show frequency response approximations with the 13th-order FIR filter, with 10^3 higher weight in the frequency band of special interest (5 to 7 GHz). The small-signal (-27 dBm) response was chosen for the final model.

Figure 6 shows a set of amplitude and phase curves in the range 5 to 7 GHz obtained with the complete model of

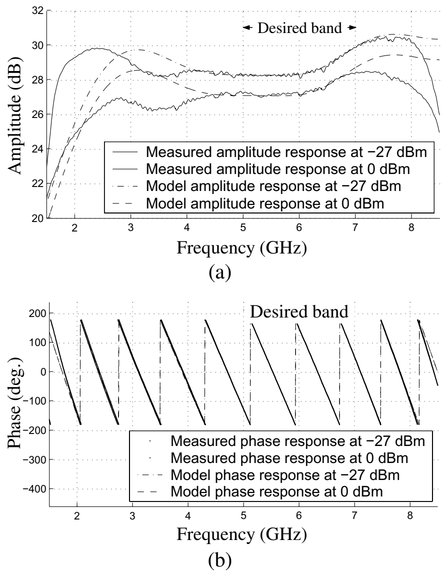

Fig. 5: *A comparison between the measured and estimated frequency response at -27 dBm and 0 dBm input power levels.*

Fig. 3, together with the measurement data. The match is seen to be excellent. This is also confirmed by Fig. 7 which shows the total LS error averaged over different input levels, which is seen to be below 60 dB in the band of interest.

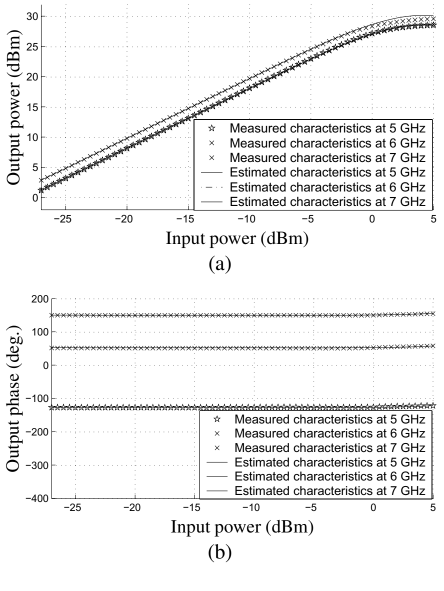

Fig. 6: *A comparison between the measured and estimated AM-AM & AM-PM characteristics at selected frequencies.*

5. CONCLUSIONS

A wideband nonlinear model with memory was developed for power amplifiers for mobile communications. Based on new single-tone measurement data, a Hammerstein-type model structure was introduced with practical LS-type method for model parameter estimation. The results show that accurate model can be developed with low-order structures.

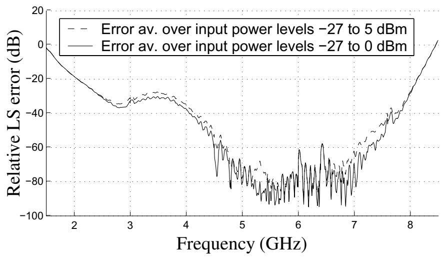

Fig. 7: *The average estimation error as a function of the frequency.*

Future research includes testing the models with wideband OFDM signals and designing efficient predistortion structures for practical amplifiers, as discussed in [8].

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