PREDICTION-BASED ADAPTIVE BLIND EQUALIZATION: A PERFORMANCE STUDY

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ABSTRACT

Blind equalization of a communication channel using a prediction-based Lattice Blind Equalizer (LBE) is considered. Second order cyclostationary statistics and a single-input multiple-output model arising from fractional sampling of the received data are used. The performance of the LBE algorithm is studied in extensive simulations where commonly used example channels are employed. Convergence in the Mean Square Error (MSE) and Symbol Error Rate (SER) as well as the number of symbols required to open the eye are studied at different SNRs. Robustness in the face of channel order mismatch and channels with common subchannel zeros is considered. The simulation results are compared to the results obtained by the fractionally spaced Constant Modulus Algorithm, the Cyclic-RLS algorithm and the subspace method by Moulines et al.

1. INTRODUCTION

Second-order cyclostationary (CS) statistics are the basis of most recent blind channel equalization/identification algorithms. Since [1] and [2], research has focused on methods based on the CS-statistics for practical purposes; shorter data sequences and less computations are required than with higher-order-statistics (HOS). Despite the quest for practicality, few CS-based algorithms have been designed to be adaptive in time and to have a feasible computational complexity for a real-time communication environment. Adaptivity is needed, because the equalizer must account for the time-varying channel. Also, the computational burden of block-type algorithms can be decreased, if the equalizer parameters are computed in a recursive manner.

The prediction-error approach, originally published in [3], has great potential for an adaptive implementation. Many algorithms have been developed to recursively obtain the prediction error, which is a part of the solution to the blind equalization problem. In our earlier work [4], a Least Squares Lattice (LSL) algorithm was used to construct the Lattice Blind Equalizer (LBE). Adaptive prediction was also used in [5] in conjunction with mutually referenced signals. The correlation matrix based blind equalizer of [6] can be implemented adaptively by the Cyclic-RLS (CRLS) algorithm. The CS-based equalizers of [6] were shown to equivalent to the prediction based equalizers in [4].

In this paper, the performance of the LBE is studied with simulations and compared to the CRLS and the fractionally spaced version of the Constant Modulus Algorithm (FS-CMA) (c.f. [7]). The FS-CMA uses HOS implicitly and it has become popular due to its adaptivity and simplicity of implementation. The performance of the block-type subspace-based algorithm [8] is also given for reference purposes.

This paper is organized as follows: The system model is described in section 2. The considered algorithms are introduced briefly in section 3. The simulation results are presented in section 4. SER and MSE curves are given and the number of received symbols required to open the eye are measured. Robustness to inaccurate channel order estimation and to channels with common subchannel zeros is also studied. Finally, section 5 concludes the paper.

2. SYSTEM MODEL

The output of a linear communication channel at time t is

$$y(t) = \sum_{k=-\infty}^{\infty} x(k)h(t-kT) + v(t) \tag{1}$$

where $h(t)$ is the complex baseband channel impulse response, $x(k)$ the sequence of complex information symbols and $v(t)$ additive noise. When P samples per symbol are available at the receiver, we can consider a multichannel model, where at time nT a vector $\mathbf{y}(n) = [y_0(n), \ldots, y_{P-1}(n)]^T$ is received, $()^T$ denoting transposition. The output $y_i(n)$ of each subchannel $h_i(n), n = 0, \ldots, P-1$ can be written as

$$y_i(n) = \sum_{k=0}^{L_h-1} x(n-k)h_i(k) + v_i(n) \tag{2}$$

where $v_i(n)$ are samples of $v(t)$ corresponding to $y_i(n)$. The maximum length of $h_i(n)$ is L_h. The channel impulse response can also be represented as a vector $\mathbf{h}(n) = [h_0(n), \ldots, h_{P-1}(n)]^T$.

The multichannel communication system can be equalized with a fractionally spaced equalizer (FSE) that is organized into P parallel filter banks, as shown in Fig. 1 for $P = 2$. The equalization is possible if the subchannels do not have any common zero [9]. Equalizer $g_i(n)$ is used in cascade with the subchannel $h_i(n)$. The equalizer coefficients are collected in a vector $\mathbf{g}_d = [\mathbf{g}^T(0), \ldots, \mathbf{g}^T(M)]$, where $\mathbf{g}(n) = [g_0(n), \ldots, g_{P-1}(n)]^T$ and d denotes the equalizer delay. We consider a finite symbol sequence of length M and define an $(MP \times 1)$-vector $\mathbf{y}_M(n) = [\mathbf{y}^T(n), \ldots, \mathbf{y}^T(n-M+1)]^T$. The output of the d-delay equalizer is then

$$\hat{x}(n-d) = \mathbf{g}_d \mathbf{y}_{M+1} \tag{3}$$

Because the system output is sampled at symbol rate, the overall impulse response seen at the equalizer output is

$$\Sigma_{i=0}^{P-1} h_i(n) * g_i(n) \tag{4}$$

where $*$ denotes convolution.

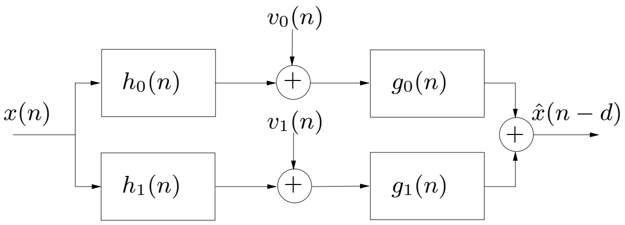

Figure 1: A multichannel communication system.

3. ALGORITHMS

In this section, the adaptive LBE-algorithm is briefly presented. The adaptive CRLS and FS-CMA as well as the block-type subspace algorithm of Moulines et al. [8] are also considered.

The LBE was introduced in [4]. The linear one-step prediction error vector of $\mathbf{y}(n)$ is denoted by $\mathbf{f}_M(n)$ and its covariance matrix by $\mathbf{P}_{f,M}$, where M is the prediction order. The zero-delay MMSE-equalized symbol estimate is now [10]

$$\hat{x}(n) = \sigma_x^2 \mathbf{h}^H(0)\mathbf{P}_{f,M}^{-1}\mathbf{f}_M \quad (5)$$

where σ_x^2 is the variance of the transmitted symbol. The LBE uses the adaptive LSL-algorithm [11] to recursively compute the prediction error and its covariance matrix. The parameters of the lattice filter are the reflection coefficient matrices $\kappa_{f,m}(n)$ and $\kappa_{b,m}(n)$ (Fig. 2). The forward and backward prediction errors of order m are obtained as $\mathbf{f}_m(n) = \mathbf{f}_{m-1}(n) + \kappa_{f,m}(n)\mathbf{b}_{m-1}(n-1)$ and $\mathbf{b}_m(n) = \mathbf{b}_{m-1}(n-1) + \kappa_{b,m}(n)\mathbf{f}_{m-1}(n)$, respectively. The covariance matrix $\mathbf{P}_{f,m}$ is also obtained recursively. In the LBE, the required first channel coefficient vector $\mathbf{h}(0)$ is obtained adaptively by eigen-vector and eigen-value tracking of $\mathbf{P}_{f,m}$ [12]. The computations are recursive both in the prediction order m and time n, which provides several benefits in a real-time communication environment. In addition, lattice filters have advantages over transversal filters. No block-type initialization nor exact knowledge of the channel order is needed.

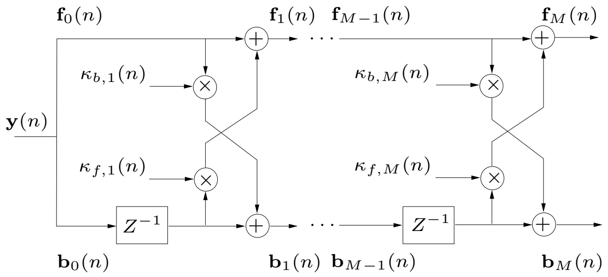

Figure 2: A lattice predictor.

In the simulations, the LBE is compared to two adaptive equalizers and one block-type algorithm. The first adaptive equalizer was introduced in [6]. It includes the inversion of the block-Toeplitz correlation matrix $\mathbf{R}_y = \mathrm{E}[\mathbf{y}_{M+1}\mathbf{y}_{M+1}^H]$, where $()^H$ denotes Hermitian transposition. The equalizer was shown in [4] to be equivalent to (5). The matrix inversion can be performed recursively resulting in the adaptive CRLS-algorithm. A short block type matrix inversion is used for initialization. As opposed to the LBE, the first channel coefficient $\mathbf{h}(0)$ is assumed known in this approach. The second adaptive algorithm that the LBE is compared to is the FS-CMA, c.f. [7]. Due to the fractional sampling, the local minima inherent in the error surface of the symbol-spaced CMA-algorithm can be avoided. Similarly to the algorithms using second-order CS-statistics, the subchannels are not allowed to have any common zero. Simplicity of implementation is the main advantage of the FS-CMA. Finally, the subspace-based equalizer of [8] — denoted here by SBE — is also used in the simulations. The algorithm exploits the orthogonality property between the signal and noise subspaces of $\mathbf{R}_y$. Eigen-decomposition provides vectors spanning the subspaces. A matrix containing equalizers of various delays can be constructed. The disadvantages of the method are high computational complexity and block type estimation.

4. SIMULATION RESULTS

In this section, the performance of the LBE is studied in simulations. All channels that are used are $T/2$ fractionally spaced. The channels are described in the following:

- Channel 1 is a minimum-phase channel of length $4T$. See [6] and [4] for details.
- Channel 2 is an empirically measured mixed-phase microwave channel. The magnitude of its impulse response is shown at the top of Fig. 3. The leftmost zero-plot of Fig. 3 shows the zeros of the $T/2$-spaced channel. In the center, the zeros of the individual subchannels $h_0(n)$ and $h_1(n)$ are shown with a circle and a cross, respectively. The rightmost plot shows the zeros of the SIMO channel as seen by the receiver, i.e. $h_0(n) + h_1(n)$.
- Channel 3 is a minimum-phase channel that has subchannels with exactly common zeros. The zero-locations are $[-0.2736-0.3917i, 0.0074-0.4999i, -0.4560+0.2706i, -0.2472+0.3870i, 0.5213-0.0539i, 0.4149+0.1422i, 0.1961+0.7765i]$.
- Channel 4 is modified from channel 3 by changing the zero $-0.2736 - 0.3719i$ of both subchannels to $-0.9500 - 0.8380i$. The channel has exactly common zeros and is mixed-phase.

Channel 2 was obtained from the web-site *http://spib.rice.edu/spib /microwave.html* and was further time-decimated to length 16 as explained in [13]. Channels 3 and 4 were modified from the near-common zero channel used in [13] and [6].

The equalizer performance is evaluated after every 100 received symbols. The performance criteria are the sample estimate of the SER and the MSE. A test set of 1000 symbols is used at each point and the results are averaged over 100 Monte-Carlo runs. Nevertheless, for high SNRs the number of symbol errors can remain too low for the SER to be statistically reliable. In these cases the number of runs is increased as needed. The channel input is a white 16-QAM sequence with unit variance. The additive noise is white and Gaussian. The SNR is defined as in [6] as SNR $:= \mathrm{E}[\Sigma_{j=0}^{P-1}|y_j'(n)|^2/\Sigma_{k=0}^{P-1}|v_k(n)|^2]$, where $y_j'(n)$ is the signal component of $y_j(n)$.

The LBE produces zero- or maximum-delay equalizers only. The other algorithms can be applied to arbitrary-delay equalization. All equalizers were forced to zero-delay equalization in order to make the results comparable. The CRLS has a block-type initialization period of 100 symbols and the first channel coefficient $\mathbf{h}(0)$ is assumed to be known. The channel order is required in the

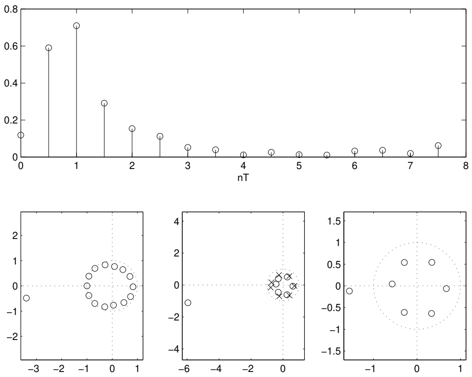

Figure 3: Channel 2 impulse response magnitude and zero locations.

SBE. None of the above assumptions are needed in the LBE nor in the FS-CMA. At first, however, the order of all the equalizers is chosen equal to the channel order in the simulations.

Channel 1 is considered first. The SER achieved by the LBE after 500, 1500 and 2500 received symbols is presented as a function of the SNR in Fig. 4. For a 16-QAM constellation, the eye can be considered open below a SER of 0.04 (c.f. [13]). The number of symbols needed to achieve this limit by the different equalizers is plotted in Fig. 5. SNRs of 15–30 dB were used in the simulations with a grid of 2.5 dB. At the SNR of 15 dB and 17.5 dB, no equalizer opens the eye within 2500 received symbols. The LBE is the first algorithm to do so at higher SNRs. Although the block versions of the LBE and the CRLS are equivalent, the adaptive versions behave differently. One major difference is the CRLS-algorithm's sensitivity to initialization. Additionally, the performance of the LBE is expected to degrade faster at low SNRs, because $\mathbf{h}(0)$ is estimated from the noisy data. The SBE does not function well at low SNRs, but its performance improves rapidly as the noise power decreases.

The corresponding plot for channel 2 is shown in Fig. 6. Now the CRLS and the FS-CMA are able to open the eye already at the SNR of 17.5 dB. In general, the FS-CMA outperforms the other methods. The convergence of the adaptive equalizers in the MSE at the SNR of 30 dB is plotted in Fig. 7. No major difference can be noted after the effects of initialization have died down. In [4], the robustness of the LBE to inaccurate channel order estimation in the MSE-criterion was noted. On the other hand, the SER of all the adaptive equalizers increases with the channel order mismatch. The FS-CMA seems to be the most sensitive of the three algorithms. This can be seen from Fig. 8, where the equalizer orders of 6–16 were used and the SER was evaluated after 1000 received symbols at the SNR of 30 dB. The channel order is 7.

The LBE and the CRLS function well even for channel 3, as seen in Fig. 9. Similar results were obtained in [14]. Also the FS-CMA opens the eye, as opposed to the SBE. The mixed-phase channel with common subchannel zeros (channel 4) cannot be equalized by any of the algorithms. The LBE and the CRLS converge in the MSE-criterion, but the eye remains closed.

5. CONCLUSIONS

The prediction-based adaptive LBE-algorithm using second order cyclostationary statistics was studied in simulations. The results

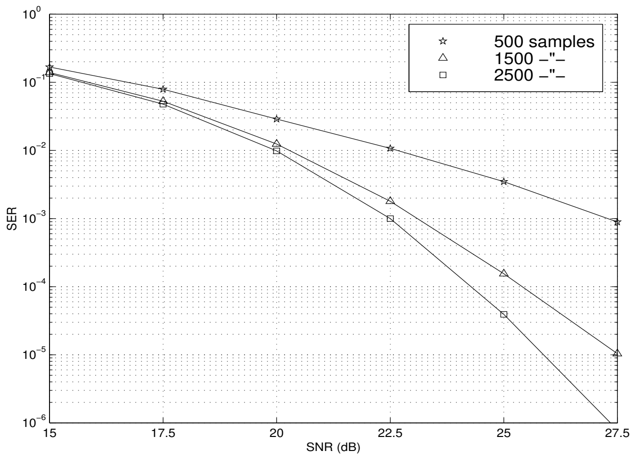

Figure 4: SER after 500, 1500 and 2500 symbols, channel 1.

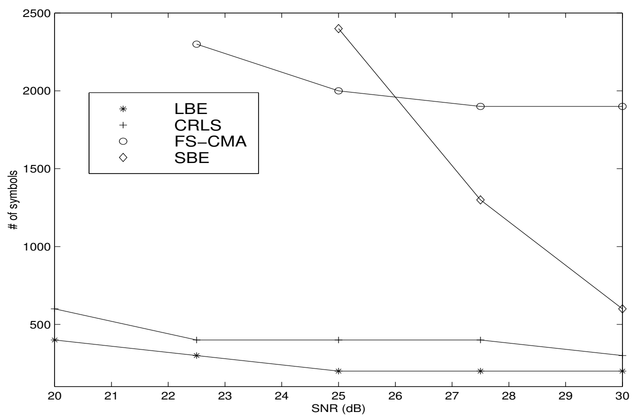

Figure 5: Open eye for channel 1.

indicate that the proposed method provides a good overall performance in a variety of situations. The LBE and the CRLS behave in a similar fashion, as expected. For channel 1, they perform significantly better than the FS-CMA. For channel 2, the FS-CMA is the fastest to open the eye, but is more affected under channel order mismatch. For the minimum-phase channel with equal subchannel zeros, the LBE and CRLS once again cope better than the FS-CMA. All the techniques considered in this study fail to equalize the corresponding mixed-phase channel.

When complexity is considered, the LBE has an advantage over the CRLS, because it does not have a need for a block initialization period and its implementation enjoys the benefits of the lattice structure. Also the problem of obtaining the first channel coefficient is solved implicitly in the LBE. The extreme simplicity of the FS-CMA makes it very attractive for implementation. However, the complexity of the LBE is also low when a small number of samples per symbol is taken.

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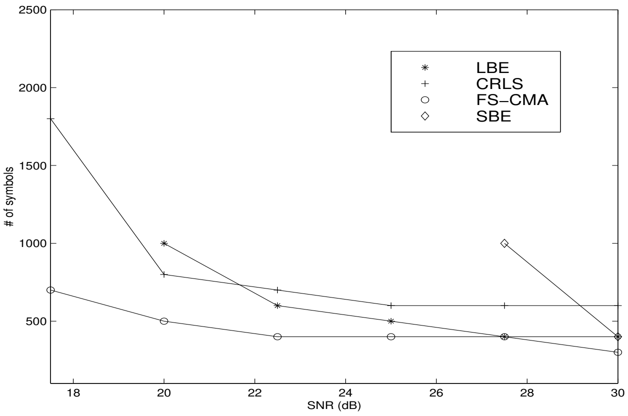

Figure 6: Open eye for channel 2.

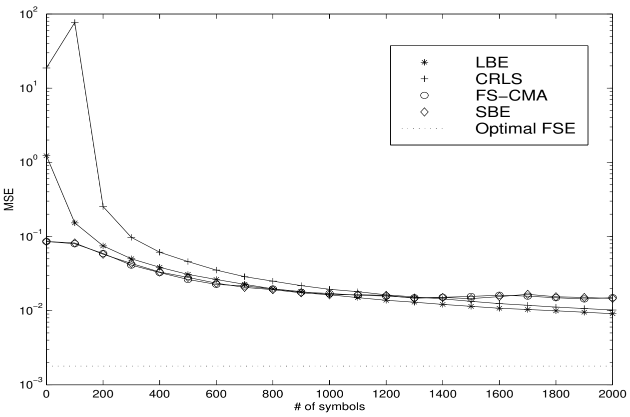

Figure 7: MSE convergence for channel 2, SNR 30 dB.

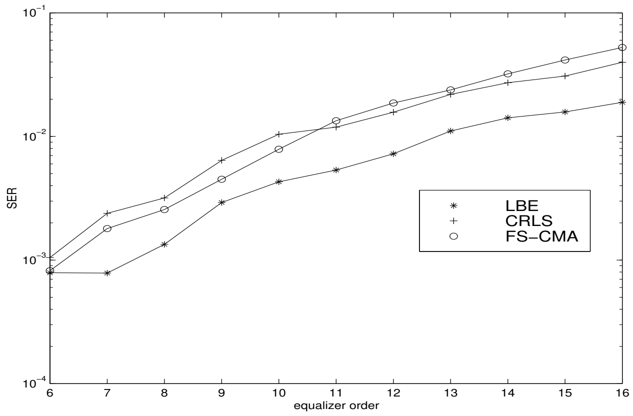

Figure 8: Effect of the equalizer order for channel 2, SER after 1000 received symbols at the SNR of 30 dB.

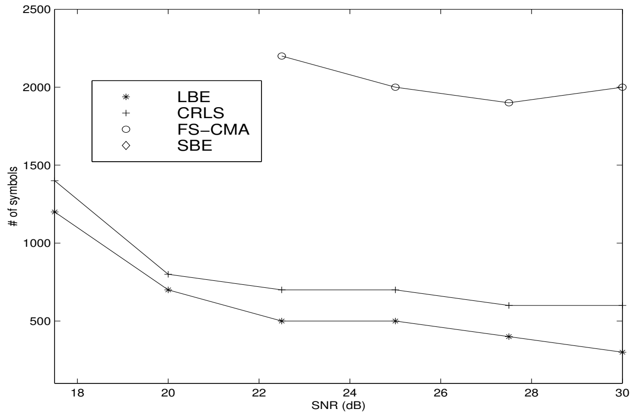

Figure 9: Open eye for channel 3.

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