PREDICTION-BASED BLIND EQUALIZATION WITH ENCODED INPUT SEQUENCES

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ABSTRACT

A blind equalizer attempts to compensate the inter-symbol interference caused by a communication channel without the knowledge of the transmitted sequence. Linear prediction-error filters (PEF) can be used to obtain the equalized symbols. These equalizers are derived assuming a white information sequence. In real communication systems, channel encoding is commonly used to enable the detection and correction of symbol errors. It is interesting to study the effect of coding on the performance of the equalizers. A linear block code and a convolutional code are used in simulations and the results are compared to the results obtained for an unencoded white information sequence. Both the mean square error (MSE) and the symbol error rate (SER) are considered.

1. INTRODUCTION

Inter-symbol interference (ISI) caused by multipath propagation of the transmitted information sequence must be removed in the receiver before symbol detection. Reversing the effects of the communication channel can be performed by the means of linear equalization [6]. Conventionally, a training sequence is periodically transmitted and the equalizer parameters are computed according to the resulting error signal. In blind equalization, no training is needed and the equalizer is obtained only with the aid of the received symbols. Consequently, valuable communication bandwidth can be saved. Fractional sampling resulting in a single-input multiple-output (SIMO) system model is employed to enable the equalization of nonminimum phase channels [9].

Linear multichannel predictors can be used to obtain blind equalizers [7, 8]. The approach is based on whitening the received sequence, assuming whiteness of the transmitted sequence. In real communication systems, redundancy is added to the source sequence in a channel encoder in order to be able to detect and correct channel noise induced symbol errors. It is not obvious how channel encoding will affect the assumption of whiteness. The performance of the prediction-based equalizer is studied in simulation using linear block encoding and convolutional encoding. The MSE and the SER resulting from the use of encoded and unencoded sequences are compared. No error detection/correction is considered when evaluating the SER.

In section 2 the employed SIMO communication system is introduced. In section 3, prediction-based blind equalization is briefly considered. Linear block codes and convolutional codes are briefly described in section 4. Finally, section 5 contains the simulation results.

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2. SIMO SYSTEM MODEL

Let $x(k)$ denote the symbol emitted by a random digital source at time kT, T being the symbol period. The channel is considered to be linear and time-invariant (LTI) during a time window sufficient to allow its estimation. The continuous time received signal is (c.f. Fig. 1)

$$y(t) = \sum_{k=-\infty}^{\infty} x(k)h(t-kT) + v(t) \tag{1}$$

where $h(t)$ is the complex baseband channel impulse response, $x(k)$ is the sequence of complex information symbols and $v(t)$ is additive white noise with variance σ_v^2. At this point, let us assume the input sequence to be white.

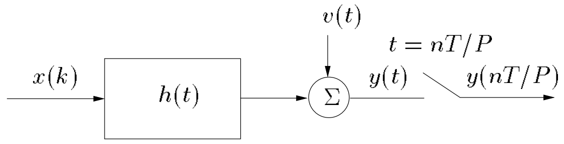

Figure 1: Communication system model.

When $P \geq 2$ samples of $y(t)$ per symbol interval are available at the receiver, the system can be modeled as consisting of P subchannels. The ith subchannel response is defined as $h_i(n) \triangleq h(t_0+iT/P+nT)$, $n = 0, \ldots, L_h-1$, where L_h is the subchannel length. Its output sequence $y_i(n) \triangleq y(t_0 + iT/P + nT)$ is

$$y_i(n) = \sum_{k=0}^{L_h-1} x(n-k)h_i(k) + v_i(n) \tag{2}$$

where $i = 0, \ldots, P-1$, $-\infty < n < \infty$, $v_i(n)$ are samples of $v(t)$ corresponding to $y_i(n)$ and the symbol period is normalized to $T = 1$. In each symbol interval, a vector $\mathbf{y}(n) = [y_0(n), \ldots, y_{P-1}(n)]^T$ of length P is received, $[\ ]^T$ denoting transposition. The channel impulse response can also be represented in vector form as $\mathbf{h}(n) = [h_0(n), \ldots, h_{P-1}(n)]^T$ and the noise as $\mathbf{v}(n) = [v_0(n), \ldots, v_{P-1}(n)]^T$. We consider a finite symbol sequence of length M and define a matrix $\mathcal{H}_M$ with dimension $MP \times (L_h+M-1)$ consisting of the channel coefficient vectors,

$$\mathcal{H}_M = \begin{bmatrix} \mathbf{h}(0) & \ldots & \mathbf{h}(L_h-1) & & \mathbf{0}_{P\times(M-1)} \\ & \ddots & & \ddots & \\ \mathbf{0}_{P\times(M-1)} & & \mathbf{h}(0) & \ldots & \mathbf{h}(L_h-1) \end{bmatrix} \tag{3}$$

Stacking M received vector samples into an $(MP \times 1)$-vector $\mathbf{y}_M(n) = [\mathbf{y}^T(n), \ldots, \mathbf{y}^T(n-M+1)]^T$, we can write a matrix equation for the system as

$$\mathbf{y}_M(n) = \mathcal{H}_M \mathbf{x}_{L_h+M-1}(n) + \mathbf{v}_M(n) \tag{4}$$

where the input vector is $\mathbf{x}_{L_h+M-1}(n) = [x(n), \ldots, x(n-L_h-M+2)]^T$ [10]. We denote the correlation matrix of $\mathbf{y}_M(n)$ as

$$\mathbf{R}_M = \mathrm{E}[\mathbf{y}_M(n)\mathbf{y}_M^H(n)] \tag{5}$$

The following assumptions will be used in this paper:

A1. The additive noise $v(t)$ is zero-mean and white with variance σ_v^2.

A2. The sequences $x(t)$ and $v(t)$ are uncorrelated.

A3. The matrix $\mathcal{H}_M$ has full column rank, i.e. the subchannels $h_i(n)$ have no common zeros.

3. PREDICTION-BASED LINEAR BLIND EQUALIZATION

In the receiver, ZF- or MMSE-equalizers can be used to estimate the transmitted symbol. The ith subchannel $h_i(n)$ is equalized by the filter $g_i(n)$, as shown in Fig. 2 for $P = 2$. The equalizer impulse response is in vector form,

$$\mathbf{g}(n) = [g_0(n), g_1(n), \ldots, g_{P-1}(n)]^T \tag{6}$$

where $n = 0, \ldots, L_g - 1$ and L_g is the length of the longest branch of the equalizer.

Equalization introduces a delay of d symbols relative to the transmitted sequence. Setting $L_g = M + 1$, a d-delay equalizer vector of length $(M+1)P$ is constructed as

$$\mathbf{g}_d = [\mathbf{g}^T(0), \ldots, \mathbf{g}^T(M)] \tag{7}$$

and the symbol estimate is obtained from

$$\hat{x}(n-d) = \mathbf{g}_d \mathbf{y}_{M+1} \tag{8}$$

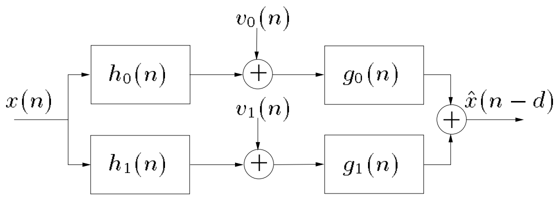

Figure 2: A single-input multiple-output (SIMO) communication system model.

Linear one-step forward prediction uses the samples $\mathbf{y}_M(n-1)$ to estimate the sample $\mathbf{y}(n)$ [4]. The forward prediction error of order M is defined as

$$\mathbf{f}_M(n) = \mathbf{A}_M \mathbf{y}_{M+1}(n) \tag{9}$$

where $\mathbf{A}_M = [\mathbf{I}, \mathbf{A}_{M,1}, \ldots, \mathbf{A}_{M,M}]$ and the $\mathbf{A}_{M,k}$ are $P \times P$ coefficient matrices of a forward prediction-error filter (PEF) of order M. The forward PEF coefficient matrix can be obtained from the equation [5]

$$\mathbf{A}_M \mathbf{R}_{M+1} = [\mathbf{P}_{f,M} \; \mathbf{0}] \tag{10}$$

where $\mathbf{P}_{f,M}$ is the covariance matrix of $\mathbf{f}_M$.

An MMSE-optimized prediction error filter can be used as a blind ZF- or MMSE-equalizer as shown for the forward case in [7] and [8]. One-step forward prediction results in zero-delay equalization. Multi-step prediction has been suggested as a solution to the arbitrary-delay equalization problem [8, 2]. For the case of simplicity, we will consider zero-delay MMSE-equalization. A zero-delay MMSE-equalizer $\mathbf{g}_0^{\mathrm{MMSE}}$ can be obtained as [8]

$$\mathbf{g}_0^{\mathrm{MMSE}} = \sigma_x^2 \mathbf{h}^H(0) \mathbf{P}_{f,M}^{-1} \mathbf{A}_M \tag{11}$$

where σ_x^2 is the variance of the transmitted symbol.

4. BLOCK AND CONVOLUTIONAL CHANNEL CODING

4.1. Linear block codes

In binary linear block encoding, a set of k bits from the source sequence $x(n)$ is represented by a code word of length n [6]. The code words are then used to construct the eventually transmitted sequence $\tilde{x}(n)$. A consistent code consists of two parts, the k information bits unchanged appended with a parity check pattern that is computed from the k bits. The code word can be generated by the linear shift register of Fig. 3. After the information bits have been fed through as they are, the switch is turned to the parity generating part and the parity bits are then concatenated to the information bits.

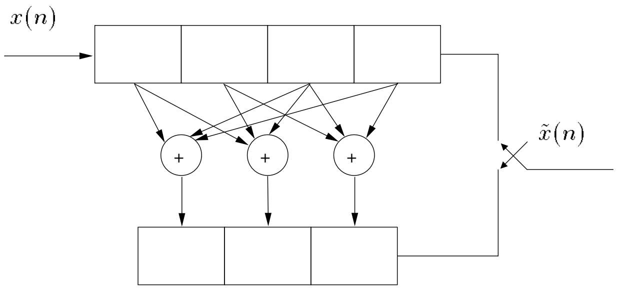

Figure 3: A shift register to produce a linear block code [6].

An easy way to represent a linear block code is by the means of a generator matrix. In this paper, we employ a $k = 4$ and $n = 7$ code. The encoded sequence $\tilde{\mathbf{x}}_N$ is obtained from the information sequence $\mathbf{x}_N$ as

$$\tilde{\mathbf{x}}_7 = \mathbf{x}_4 \mathbf{G} \tag{12}$$

where

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \tag{13}$$

The code is clearly systematic, with G consisting of an identity matrix and a parity sequence generating part.

4.2. Convolutional codes

A convolutional code is generated with the aid of a linear finite-state shift register [6]. Fig. 4 shows the generator used in this paper to obtain a $k = 1$ and $n = 3$ convolutional code. One bit at a time is clocked into the shift registers. The three outputs are read from top to bottom for each input source bit and then concatenated into the encoded sequence. A generator matrix of a convolutional code is in general semi-infinite. The code can be described with the aid of the vectors $\mathbf{g}_1 = [100]$, $\mathbf{g}_1 = [101]$ and $\mathbf{g}_1 = [111]$ being the generators of the three outputs in Fig. 4. The transfer function of the code can be given in octal form as $(4, 5, 7)$.

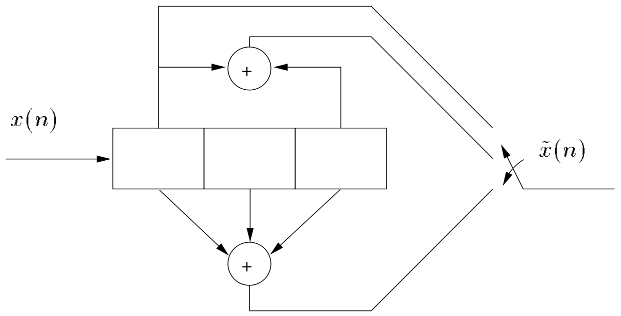

Figure 4: A shift register to produce a convolutional code [6].

5. SIMULATION RESULTS

In this section, the performance of the blind equalizer (11) is evaluated in simulation for unencoded and encoded channel inputs. The unencoded sequence is white 4-QAM with unit variance. For simplicity, the I- and Q-components of the sequence are independently encoded. More efficient coding schemes for M-ary modulation types are given in [6]. The additive noise is white and Gaussian. The SNR is defined as

$$\mathrm{SNR} := \frac{\mathrm{E}[\Sigma_{j=0}^{P-1} |y'_j(n)|^2]}{\mathrm{E}[\Sigma_{k=0}^{P-1} |v_k(n)|^2]} \quad (14)$$

where $y'_j(n)$ is the signal component of $y_j(n)$. The employed communication channel (Fig. 5) was obtained from the web-site http://spib.rice.edu/spib/microwave.html and was further time-decimated to length 16 as explained in [1]. At each evaluated point, 5000 test symbols were used to compute the estimate of the MSE and the SER and the procedure was repeated 100 times. The SER is evaluated before any error detection/correction has been performed.

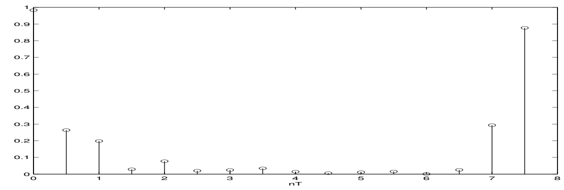

Figure 5: Magnitude of the channel impulse response.

Fig. 6 shows the MSE of the PEF-based blind equalizer (11) for the three different input sequences at different SNRs and a sample size of 1000. No major difference in the performance can be noted for the white, linear block encoded and the convolutionally encoded sequences. The same observation can be made from Fig. 7, where the MSE is plotted for the SNR of 15 dB as a function of the sample size. Further confirmation is obtained from Fig. 8, where the SER is similarly plotted.

Correlation coefficients of the transmitted sequence are defined as $r(i-j) = \mathrm{E}[s(n-j)s^*(n-i)]$. The coefficients estimated from a set of 10000 samples are printed in Table 1 for the three sequences. No significant difference between the coefficients can be seen.

Table 1: Correlation coefficients

	$r(0)$	$r(1)$	$r(2)$	$r(3)$	$r(4)$
white	1.000	0.007	0.009	0.017	0.011
block	1.000	0.008	0.004	0.007	0.006
convol	1.000	0.010	0.004	0.009	0.002

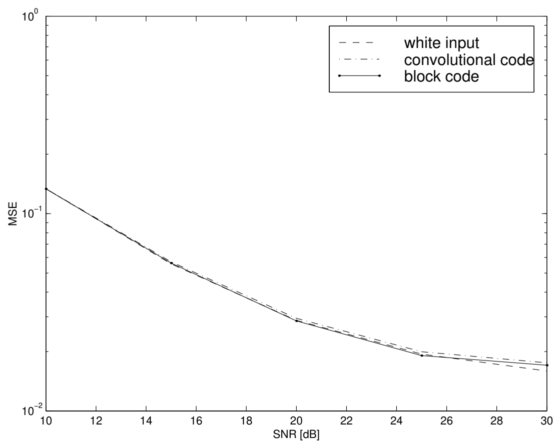

Figure 6: The MSE of the PEF-based blind equalizer for different SNRs with the sample size of 1000.

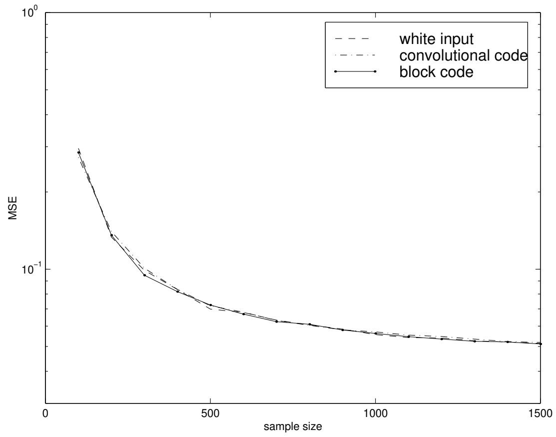

Figure 7: Convergence of the MSE of the PEF-based blind equalizer with the SNR of 15 dB.

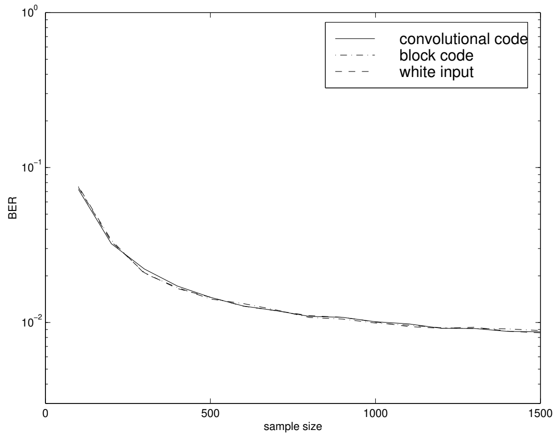

Figure 8: Convergence of the SER of the PEF-based blind equalizer with the SNR of 10 dB.

6. CONCLUSIONS

The effect of channel encoding on the performance of a PEF-based blind equalizer was studied in simulation. The theory behind the PEF-equalizer relies on whiteness of the transmitted sequence. It is interesting to note from the results of this paper that the performance of the equalizer does not degrade with the introduction of linear block encoding or convolutional encoding.

7. REFERENCES

[1] T. J. Endres, S. D. Halford, C. R. Johnson, and G. B. Giannakis. Blind adaptive channel equalization using fractionally-spaced receivers: A comparison study. In Proc. 30th Conf. Inform. Sci. Syst., Princeton, NJ, March 1996.

[2] D. Gesbert and P. Duhamel. Robust blind channel identification and equalization based on multi-step predictors. In Proc. of Int'l Conf. on Acoust. Speech and Signal Proc., pages 3621–3624, 1997.

[3] G. B. Giannakis and S. D. Halford. Blind fractionally spaced equalization of noisy fir channels: Direct and adaptive solutions. IEEE Trans. Signal Proc., 45(9):2277–2292, September 1997.

[4] S. Haykin. Adaptive Filter Theory. Prentice-Hall, Englewood Cliffs, NJ, 3rd edition, 1996.

[5] T. Kailath. A view of three decades of linear filtering theory. IEEE Trans. Info. Theory, 20(2):146–181, March 1974.

[6] J. G. Proakis. Digital Communications. McGraw-Hill, 3rd edition, 1995.

[7] D. T. Slock. Blind fractionally-spaced equalization, perfect reconstruction filter banks and multichannel linear prediction. In Proc. of Int'l Conf. on Acoust. Speech and Signal Proc., pages IV585–IV588, Adelaide, Australia, April 1994.

[8] D. T. Slock and C. B. Papadias. Further results on blind identification and equalization of multiple fir channels. In Proc. of Int'l Conf. on Acoust. Speech and Signal Proc., pages 1964–1967, Detroit, MI, May 1995.

[9] L. Tong, G. Xu, and T. Kailath. Blind identification and equalization based on second-order statistics: A time-domain approach. IEEE Trans. Info. Theory, 40(2):340–349, March 1994.

[10] L. Tong, G. Xu, and T. Kailath. Blind identification based on second-order statistics: A frequency-domain approach. IEEE Trans. Info. Theory, 41(1):329–334, January 1995.