Sensitivity of SIMO blind equalizers to the channel precursor magnitude

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Abstract – The performance of one blind channel identification algorithm and three blind equalizers is studied. The main concern is in investigating their behavior when the communication channel has little energy in the beginning of its response. In our previous experiments, the equalizers' performance has not been satisfactory for such channels. All considered equalizers are based on a pseudo-inversion of a block-Toeplitz matrix. We study the effect of the channel precursor magnitude on the singular values of the matrix and consequently on the equalizer performance. For each algorithm, scatter plots are used to evaluate the success of the equalization under various conditions. We provide an explanation to the success or failure of the equalizers, based on the magnitude of the singular values.

I. Introduction

Direct channel equalization or channel identification combined with e.g. a Viterbi decoder or is a necessity in communication systems with multipath propagation [6]. Transmission capacity of the system may be increased if the inter-symbol interference (ISI) resulting from such channels can be mitigated without any training sequences. This blind channel identification/equalization problem can be approached from a stochastic perspective, as in [4, 1, 5]. All these algorithms are based on a single-input multiple-output (SIMO) model resulting from fractional sampling. To all of them, computationally feasible adaptive algorithms can be applied, making them interesting for practical applications. This work is motivated by our previous work on these algorithms (c.f. [3]). In our experience, the algorithms perform unexpectedly for certain type of channels. We study the behavior of these algorithms, with our main concern in their robustness to channels that have little energy in the beginning of their response.

The considered algorithms are based on computations on the block-Toeplitz correlation matrix of the received sequence. The subspace-based channel identification algorithm (SSI) of [4] exploits the rank-deficient nature of the correlation matrix. An estimate of the channel is found by minimizing a projection to the noise subspace of the matrix. A direct equalizer (SSE) can then be computed from the channel estimate. The correlation matching (CM) algorithm of [1] is based on the pseudo-inversion of the correlation matrix. The linear prediction based equalizer of [5] (FB-PEF) employs similar pseudo-inversion, but uses it to compute a multi-step forward prediction error filter (PEF) of the received sequence. The filter output is then used in a one-step backward PEF to produce the equalized symbol.

The SSI channel identification algorithm is studied for three trivial example channels. The three direct equalizers are then employed for the same channels and scatter plots are presented. The work in this paper is based on the numerical computation of the singular value decomposition (SVD) of the correlation matrix. We demonstrate how a small channel precursor results in small singular values that are a part of the signal subspace. This may cause severe problems for the various equalization procedures. When the correlation matrix is full rank, the pseudo-inversion can be replaced by regular matrix inversion, but the SVD-based analysis is still feasible. This paper is organized as follows. In section 2, we present the employed SIMO system model and briefly introduce the employed algorithms in section 3. The performance of the blind algorithms is studied in section 4. Section 5 concludes the paper.

II. System model

The output of a linear communication channel at time t is

$$y(t) = \sum_{k=-\infty}^{\infty} x(k)h(t-kT) + v(t) \qquad (1)$$

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where $h(t)$ is the complex baseband channel impulse response, $x(k)$ the sequence of complex information symbols and $v(t)$ additive noise. When P samples per symbol are available at the receiver, we can consider a multichannel model, where at time nT a vector $\mathbf{y}(n) = [y_0(n), \ldots, y_{P-1}(n)]^T$ is received, $()^T$ denoting transposition. The output $y_i(n)$ of each subchannel $h_i(n), n = 0, \ldots, p-1$ can be written as

$$y_i(n) = \sum_{k=0}^{L_h-1} x(n-k)h_i(k) + v_i(n) \qquad (2)$$

where $v_i(n)$ are samples of $v(t)$ corresponding to $y_i(n)$. The channel impulse response in vector form is $\mathbf{h}(n) = [h_0(n), \ldots, h_{P-1}(n)]^T$ and the noise $\mathbf{v}(n) = [v_0(n), \ldots, v_{P-1}(n)]^T$.
Defining a matrix $\mathcal{H}_M$ with dimension $MP \times (L_h + M - 1)$,

$$\mathcal{H}_M = \begin{bmatrix} \mathbf{h}(0) & \ldots & \mathbf{h}(L_h-1) & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \mathbf{h}(0) & \ldots & \mathbf{h}(L_h-1) \end{bmatrix} \qquad (3)$$

we can write an equation for the $(MP \times 1)$-vector $\mathbf{y}_M(n) = [\mathbf{y}^T(n), \ldots, \mathbf{y}^T(n-M+1)]^T$,

$$\mathbf{y}_M(n) = \mathcal{H}_M \mathbf{x}_{L_h+M-1}(n) + \mathbf{v}_M(n) \qquad (4)$$

where the input vector is $\mathbf{x}_{L_h+M-1}(n) = [x(n), \ldots, x(n-L_h-M+2)]^T$ [8].
The commonly used *channel length and zero condition* requires that the matrix $\mathcal{H}_M$ has full column rank, i.e. the subchannels $h_i(n)$ have no common zeros. Also, $h_i(0) \neq 0$ for some i and $h_i(L_h-1) \neq 0$ for some i.
The block-Toeplitz correlation matrix $\mathbf{R}_M$ of $\mathbf{y}_M(n)$ is

$$\mathbf{R}_M = \begin{bmatrix} \mathbf{r}(0) & \ldots & \mathbf{r}(M-1) \\ \vdots & \ddots & \vdots \\ \mathbf{r}^H(M-1) & \ldots & \mathbf{r}(0) \end{bmatrix} \qquad (5)$$

where $\mathbf{r}(i-j) = \mathrm{E}[\mathbf{y}(i)\mathbf{y}^H(j)]$.
Equalizer $g_i(n)$ is used in cascade with the subchannel $h_i(n)$. The equalizer coefficients are collected in a vector $\mathbf{g}_d = [\mathbf{g}^T(0), \ldots, \mathbf{g}^T(M)]$, where $\mathbf{g}(n) = [g_0(n), \ldots, g_{P-1}(n)]^T$ and d denotes the equalizer delay. The output of the d-delay equalizer is then $\hat{x}(n-d) = \mathbf{g}_d \mathbf{y}_{M+1}$.

III. Stochastic blind channel identification and equalization

In this section, the considered blind identification and equalization algorithms are briefly introduced. All the equalizer algorithms require computation of a matrix pseudo-inverse. The pseudo-inverse of matrix $\mathbf{A}$ is based on a singular-value decomposition, so that $\mathbf{U}\mathbf{A}\mathbf{V}^H = \mathbf{\Sigma}$, where the unitary matrices $\mathbf{U}$ and $\mathbf{V}$ contain the right and left singular vectors corresponding to the singular values λ_i on the diagonal of the diagonal matrix $\mathbf{\Sigma}$ [2]. The pseudo-inverse of $\mathbf{A}$ is obtained as $\mathbf{A}^\dagger = \mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^H$, i.e only the inversion of scalars is needed. An orthogonalization process is used to obtain the singular vectors and values. The computation of the vectors proceeds in the order of decreasing singular values.

III.1. Subspace-based identification and equalization

The SSI channel identification algorithm and the SSE equalizer are based on the subspace decomposition of $\mathbf{R}_M$. The dimension of the correlation matrix must obey $M \geq L_h$. This requirement ensures that $\mathbf{R}_M$ is rank deficient and contains a noise subspace in addition to the signal subspace. The eigen vectors $\mathbf{u}_i, i = 0, \ldots, M + L_h - 1$ corresponding to the $M + L_h$ largest eigen values $\lambda_0 > \lambda_1 > \cdots > \lambda_{M+L_h-1}$ constitute the signal subspace. The remaining eigen vectors span the noise subspace that is orthogonal to the channel vector $\mathcal{H}_M$. The dimension of the signal subspace must be estimated and in this paper we use the minimum description length (MDL) criterion [9]. Thus, channel order estimation is a crucial part of the algorithm. The channel response estimate $\tilde{\mathbf{h}}(n)$ is found by minimizing a projection to the noise subspace under a linear or quadratic constraint, as explained in detail in [4].
The SSE direct equalizer exploits the channel estimate of the SSI. The equalizer can be found from the matrix

$$\mathbf{E} = \tilde{\mathcal{H}}_M^H (\tilde{\mathcal{H}}_M \tilde{\mathcal{H}}_M^H)^\dagger \qquad (6)$$

where $\tilde{\mathcal{H}}_M$ is constructed by using $\tilde{\mathbf{h}}(n)$ in (3). The ith column of matrix $\mathbf{E}$ contains a direct equalizer $\mathbf{g}_{i-1}$ corresponding to a specific delay. Note that the channel matrix $\tilde{\mathcal{H}}_M$ is of different form than in [4], but the only difference is in the ordering of the rows and the theory is the same for both cases. The channel estimate may also be used in other ways to obtain the symbol estimate (c.f. [6]).

III.2. Prediction-based equalization

The FB-PEF blind equalizer uses a cascade of a k-step forward prediction-error filter and a one-step backward prediction-error filter. The prediction step parameter k can be used to control the delay of the equalizer. A k-step forward predictor of order M produces an estimate $\hat{\mathbf{y}}(n)$ of the received symbol $\mathbf{y}(n)$ based on the M previous symbols $\mathbf{y}_M(n-k)$. The crucial part of the algorithm is computing the forward predictor coefficients from the Wiener-Hopf equations

$$\mathbf{w}_{M,k} = \mathbf{r}_k \mathbf{R}_M^{\dagger} \qquad (7)$$

where $\mathbf{r}_k = [\mathbf{r}(k), \ldots, \mathbf{r}(k+M-1)]$ and $\mathbf{R}_M^{\dagger}$ is the pseudo-inverse of $\mathbf{R}_M$. The backward predictor coefficients are obtained similarly from the correlation coefficients of the forward PEF output. The prediction-based equalizers are generally considered robust to channel order over-determination [5]. The criterion for the prediction order is $M \geq M_0 = L_h - 1$, when the number of subchannels $p = 2$. For details of the algorithm, refer to [7, 5].

III.3. Correlation matching equalizer

The CM-algorithm is also based on the pseudo-inversion of $\mathbf{R}_M$. First, we define a time-shifted correlation matrix $\mathbf{R}_{M,d} = \mathrm{E}[\mathbf{y}_M(n)\mathbf{y}_M^H(n-d)]$. The d-delay CM-equalizer is then obtained as

$$\mathbf{g}_d = \mathbf{H}^H(0)\mathbf{R}_M^{\dagger}\mathbf{R}_{M,d}\mathbf{R}_M^{\dagger} \qquad (8)$$

where $\mathbf{H}(0)$ is the first column of $\mathcal{H}_M$. The first channel coefficient $\mathbf{h}(0)$ is assumed to be known. We will not consider its estimation in this paper. The requirement for the equalizer length is the same as for the FB-PEF, $M \geq M_0 = L_h - 1$, when the number of subchannels $p = 2$. For further details, c.f. [1].

IV. Performance of the blind algorithms

The behavior of the blind algorithms is studied in simulation. The coefficients of the $T/2$ fractionally spaced ($P = 2$) communication channel A are $[-0.35+0.24j,\ 0.35+0.12j,\ 0.12+0.12j,\ -0.47+0.35j,\ -0.24+0.12j,\ 0.12+0.47j]$. We construct channel B by introducing two small coefficients $0.006+0.024j$ and $0.006+0.008j$ in the beginning of channel A. Thus the first coefficient of each subchannel of channel B is small in magnitude. Further, channel C contains coefficients $0.004+0.001j$ and $0.004+0.007j$ appended in the beginning of channel B, so that the two first coefficients of each subchannel are small.

The real part of each channel is shown in Fig. 1. A white 16-QAM sequence is fed through the channels and a sample size of 5000 is used to estimate the correlation matrix. A scatter plot of 500 test symbols at the output of each equalizer is presented using a suitable delay for each channel. The unknown complex gain of the algorithms is manually corrected by rotating the constellation plots. Since we are interested in the general behavior of the algorithms under the described circumstances and not presenting any numerical results, suffice it to show one realization of each case.

IV.1. SSI and SSE

The SSI algorithm is used to identify channels A,B and C. The MDL criterion is used to determine the dimension of the signal subspace. Fig. 1 shows the real part of each channel estimate in the same plot with the original coefficients. Clearly, the algorithm works well for all the channels in the identification problem.

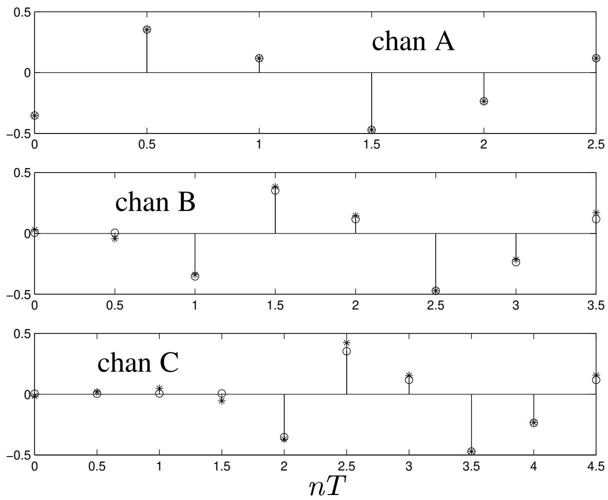
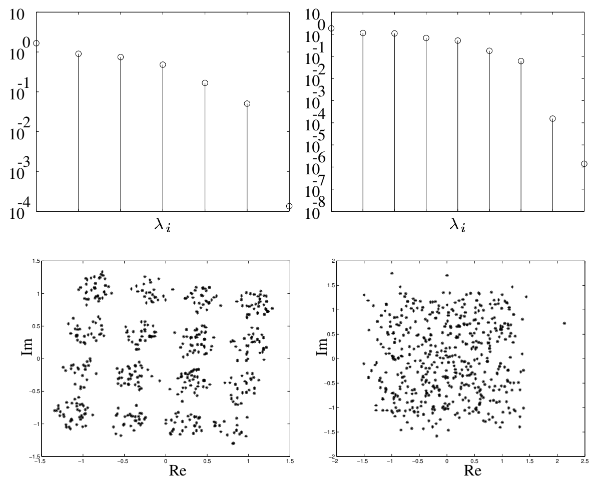


Figure 1: SSI channel estimates, 'o' true coefficients, '*' estimate. Only the real part is shown.

Next, direct equalization is considered applying (6) and the presented channel estimates. Since the matrix $\tilde{\mathcal{H}}_M\tilde{\mathcal{H}}_M^H$ is full rank, all the singular vectors belong to the signal subspace. For channel A and B, the equalization is successful, whereas for channel C it fails. The scatter plots of the equalizer output for channels B and C together with the corresponding singular values of $\tilde{\mathcal{H}}_M\tilde{\mathcal{H}}_M^H$ are shown in Fig. 2. We offer the following explanation for the results: For channel B, two singular values of $\tilde{\mathcal{H}}_M\tilde{\mathcal{H}}_M^H$ are very small. Since the pseudo-inversion uses the reciprocal of these values to multiply the corresponding singular vectors, even small perturbations in the vectors cause sever damage to the equalizer performance. A natural source for such perturbations is the variance in the sample statistics. Furthermore, the perturbations in the orthogonalization process accumulate, which could explain why for channel B, the eye is closed, whereas having only one small singular value is tolerated, as for channel A. We would like to note that although none of the channels have common subchannel roots, due to the small precursor channels B and C may be interpreted as approaching the violation of the channel length and zero condition, presented in section 2. However, channels with a small precursor cannot be ruled out in practical situations.

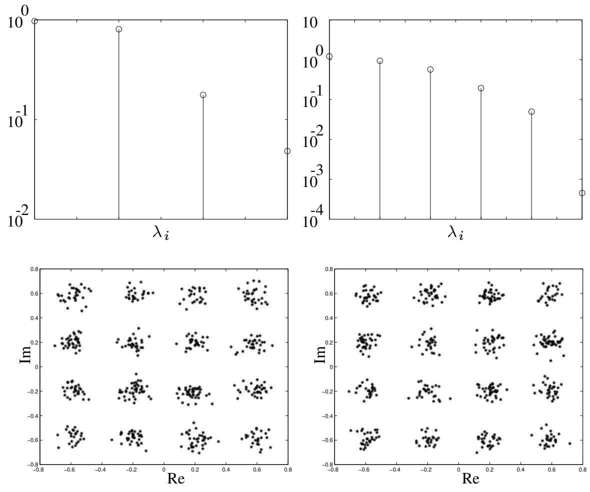


(a) Channel B (b) Channel C

Figure 2: Scatter plots for SSE, channel B $M = 2$ $d = 2$, channel C $M = 3$ $d = 3$ with MDL.

IV.2. FB-PEF and CM-equalizer

The prediction-based FB-PEF is studied next. The presented singular values are now from $\mathbf{R}_M$ that is potentially rank deficient, depending on the choice of M. For channel A using the prediction order $M = M_0 = 2$, the eye pattern is open, as seen from Fig. 3. The singular values all belonging to the signal subspace are relatively large and no dramatic sensitivity to the singular vector perturbation can be observed. As expected, increasing the prediction order to $M = M_0 + 1 = 3$ does not affect the performance negatively. Even though there is one small singular value, it belongs to the noise subspace and causes no problems.



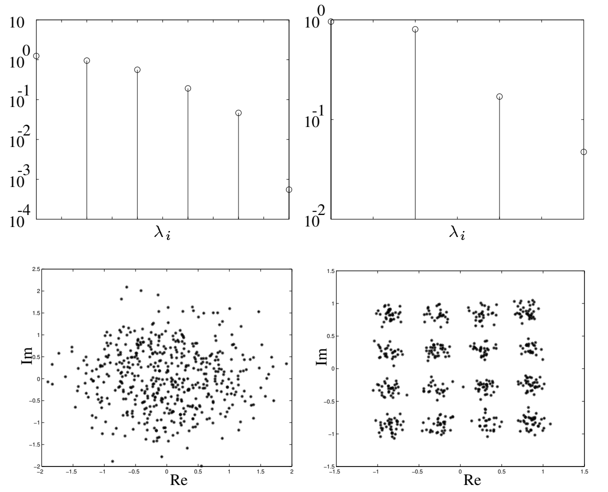
(a) $M = M_0 = 2$ (b) $M = M_0 + 1 = 3$

Figure 3: Scatter plots for the FB-PEF, channel A $d = 0$.

For channel B, however, the order $M = M_0 = 3$ FB-PEF equalizer fails to open the eye, as seen from the left side plots of Fig. 4. The small singular value belongs now to the signal subspace and is needed in finding the equalizer. Inverting the small singular value amplifies any perturbation in the corresponding singular vector to the extent that the equalization fails. The FB-PEF is more sensitive than the SSE in that only one small singular value is enough to cause the failure. On the right side of Fig. 4, we have decreased the prediction order to $M = M_0 - 1 = 2$ and now the smallest singular value is not present and the eye is opened. Similar analysis on channel C in Fig. 5, reveals that also in this case the prediction order needs to be decreased to $M = 2$ to get rid of any overly small singular values belonging to the signal subspace.

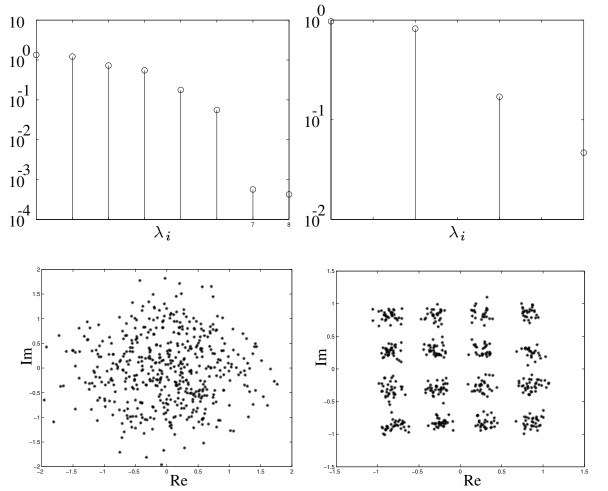
Neglecting the smallest singular values and vectors of the signal subspace results in some residual ISI in the asymptotic sense, but the price is justified by the robustness to the variance the sample statistics introduce to the singular vectors. However, the FB-PEF robustness to channel order over-determination does not seem to hold in these cases.

All the observations we have made concerning the FB-PEF hold for the CM-equalizer as well. The singular values are computed from the same matrix $\mathbf{R}_M$ as in FB-PEF and the two algorithms behave similarly in all the described situations. In Fig. 6, the scatter plots for channel B with $M = M_0$ and $M = M_0 - 1$ are presented (c.f. Fig. 4 for an identical situation).



(a) $M = M_0 = 3$ (b) $M = M_0 - 1 = 2$

Figure 4: Scatter plots for the FB-PEF, channel B $d = 2$.



(a) $M = M_0 = 4$ (b) $M = M_0 - 1 = 2$

Figure 5: Scatter plots for the FB-PEF, channel C $d = 2$.

V. Conclusions

The behavior of blind channel identification and equalization algorithms has been studied. It has been observed that the pseudo-inverse required in computing the direct equalizers is sensitive to the variance in the sample statistics, when one or more of the singular values belonging to the signal subspace is very small. We can therefore conclude that the robustness of the FB-PEF and CM-equalizer to the channel order mismatch seems to be limited to channels with significant energy in the precursor. However, the eye pattern can be opened when M is chosen sufficiently small.

VI. References

[1] G. B. Giannakis and S. D. Halford. Blind fractionally spaced equalization of noisy FIR channels: Direct and adaptive solutions. *IEEE Tr. Signal Proc.*, 45(9):2277–2292, September 1997.

[2] S. Haykin. *Adaptive Filter Theory*. Prentice-Hall, Englewood Cliffs, NJ, 3rd edition, 1996.

[3] J. Mannerkoski and V. Koivunen. Performance bounds for multi-step prediction based blind equalization. *IEEE Tr. Comm.*, under review 1999.

[4] E. Moulines, P. Duhamel, J.-F. Cardoso, and S. Mayrargue. Subspace methods for the blind identification of multichannel FIR filters. *IEEE Tr. Signal Proc.*, 43(2):516–525, February 1995.

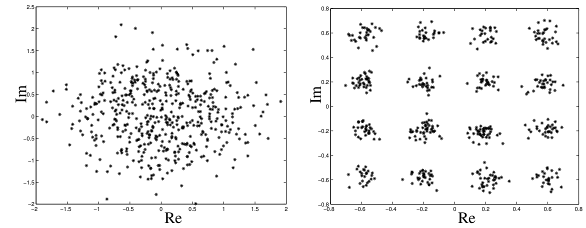
[5] C. B. Papadias and D. T. M. Slock. Fractionally spaced equalization of linear polyphase channels and related blind techniques based on multichannel linear prediction. *IEEE Tr. Signal Proc.*, 47(3):641–653, March 1999.

[6] J. G. Proakis. *Digital Communications*. McGraw-Hill, 3rd edition, 1995.

[7] D. T. Slock and C. B. Papadias. Further results on blind identification and equalization of multiple FIR channels. In *IEEE ICASSP*, pages 1964–1967, Detroit, MI, May 1995.

[8] L. Tong, G. Xu, and T. Kailath. Blind identification based on second-order statistics: A frequency-domain approach. *IEEE Tr. Inf. Theory*, 41(1):329–334, January 1995.

[9] M. Wax and T. Kailath. Detection of signals by information theoretic criteria. *IEEE Tr. ASSP*, 33(2):387–392, December 1985.



(a) $M = M_0 = 3$ (b) $M = M_0 - 1 = 2$

Figure 6: Scatter plots for the CM-equalizer, channel B $d = 3$.