Robust Subspace DOA Estimation for Wireless Communications

Samuli Visuri[1]* *Hannu Oja*[2] *Visa Koivunen*[1]

[1]Laboratory of Signal Processing and Computer Technology
Helsinki Univ. of Technology
P.O. Box 3000, FIN-02015 HUT
Finland
email: samuli.visuri@hut.fi and visa.koivunen@hut.fi

[2]Dept. of Statistics
University of Jyväskylä
P.O. Box 35, FIN-40351 Jyväskylä
Finland
email: hannuoja@jyu.fi

Abstract – This paper is concerned with array signal processing in non-Gaussian noise typical in urban and indoor radio channels. Robust and fully nonparametric high resolution algorithms for Direction of Arrival (DOA) estimation are presented. The algorithms are based on multivariate spatial sign and rank concepts. The performance of the algorithms is studied using simulations. The results show that almost optimal performance is obtained in wide variety of noise conditions.

I. Introduction

Direction of Arrival (DOA) estimation is an important task in smart antennas for wireless communications. It is needed e.g. in signal separation, interference suppression and determining the location of the mobile with high accuracy. Subspace methods provide high resolution DOA estimates. They exploit the eigendecomposition of the sample covariance matrix.

The first step in most of the algorithms is to estimate the covariance matrix of the M sensor array output from the data vectors (or snapshots) $\mathbf{x}_1, \dots, \mathbf{x}_n$. A standard estimator for the covariance matrix is the sample covariance matrix $\frac{1}{n}\sum_{i=1}^{n}(\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})^H$ where $\bar{\mathbf{x}}$ is the sample mean vector and superscript H denotes hermitian transpose. The sample covariance matrix is an optimal estimator for an unknown covariance matrix if the data comes from Gaussian distribution. If this is not true, the sample covariance matrix may perform poorly and unreliable DOA estimates may result.

It has been observed through experimental measurements that the ambient noise in indoor and urban radio channels is decidedly non-Gaussian (c.f. [5]). Consequently, there has been a growing interest towards algorithms which work properly also in non-Gaussian noise environments. Robust algorithms based on M-estimation have been proposed for DOA estimation [12, 11, 3, 4]. The problem of DOA estimation in impulsive noise coming from α-stable distribution have been studied in [8, 9]. Kozick and Sadler [2] model the noise as a finite mixture of Gaussian random variables and use the SAGE algorithm for DOA estimation.

All the DOA estimation methods for non-Gaussian noise environment mentioned above are either parametric or "semiparametric" in the sense that knowledge on the pdf, number of mixtures, user-defined threshold values or weighting function need to be determined. In this paper, we introduce new fully nonparametric and robust high resolution DOA estimation algorithms. The algorithms are based on estimating the signal or noise subspace from the spatial rank covariance matrices. The reliable performance of the methods is shown using simulations in Gaussian and non-Gaussian noise conditions.

The plan is as follows. In section II, we define the theoretical concepts from nonparametric statistics needed in this paper. The signal model and the algorithms are introduced in section III. Section IV introduces the simulation results. Finally, section V concludes the paper.

II. Spatial Rank Covariance Matrices

We begin by giving definitions for multivariate spatial sign and rank concepts used in this article. Spatial sign and rank vectors are natural multivariate generalizations of the univariate sign and rank. They are derived using an objective function related to the multivariate spatial median [6]. In this paper we deal with complex data and therefore the earlier definitions have been slightly modified. However, if the data are real valued, the definitions given in this paper and the earlier definitions for real data coincide. For a p-variate data set $\mathbf{x}_1, \dots, \mathbf{x}_n$, the *spatial rank*

* This work was funded by the Academy of Finland

function is

$$\mathbf{r}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n}\mathbf{s}(\mathbf{x}-\mathbf{x}_i)$$

where $\mathbf{s}$ is the *spatial sign function*

$$\mathbf{s}(\mathbf{x}) = \begin{cases} \frac{\mathbf{x}}{||\mathbf{x}||}, & \mathbf{x} \neq \mathbf{0} \\ \mathbf{0} & \mathbf{x} = \mathbf{0} \end{cases}$$

with $||\mathbf{x}|| = (\mathbf{x}^H\mathbf{x})^{1/2}$. Using these concepts, the spatial Kendall's Tau Covariance matrix (TCM) and the spatial Rank Covariance Matrix (RCM) may be defined as

$$R_{TCM} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\mathbf{s}(\mathbf{x}_i-\mathbf{x}_j)\mathbf{s}^H(\mathbf{x}_i-\mathbf{x}_j)$$

and

$$R_{RCM} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{r}(\mathbf{x}_i)\mathbf{r}^H(\mathbf{x}_i),$$

respectively. To define the corresponding theoretical concepts, let $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ be i.i.d. p-variate random variables with distribution F. Then the theoretical TCM and RCM for the distribution F are

$$\Sigma_{TCM} = E_F\{\mathbf{s}(\mathbf{x}_1-\mathbf{x}_2)\mathbf{s}^H(\mathbf{x}_1-\mathbf{x}_2)\}$$

and

$$\Sigma_{RCM} = E_F\{\mathbf{s}(\mathbf{x}_1-\mathbf{x}_2)\mathbf{s}^H(\mathbf{x}_1-\mathbf{x}_3)\},$$

respectively. In [10] we show that the TCM and RCM are reasonable estimators for the theoretical TCM and RCM.

III. Subspace DOA estimation algorithms

III.1. Signal Model

Suppose K incoherent plane waves are incident on a linear uniform array of M sensors. Then the received signal vector $\mathbf{x}(t)$ is an $M \times 1$ complex vector given by

$$\mathbf{x}(t) = A\mathbf{u}(t) + \mathbf{n}(t) \qquad (1)$$

where A is an $M \times K$ matrix such that

$$A = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K)]$$

with $\mathbf{a}(\theta_k)$ being the $M \times 1$ array steering vector corresponding to the DOA of the k^{th} signal, given by

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi(d/\lambda)\cos(\theta)} \\ \vdots \\ e^{-j2\pi(M-1)(d/\lambda)\cos(\theta)} \end{bmatrix}$$

where d denotes the interelement spacing and λ denotes the wavelength. The K-vector

$$\mathbf{u}(t) = [u_1(t), u_2(t), \ldots, u_k(t)]^T$$

is the vector of incident signals and $\mathbf{n}(t)$ is the $M \times 1$ complex noise vector. For simplicity, we will drop the time argument from $\mathbf{x}$, $\mathbf{u}$ and $\mathbf{n}$ from this point onward.

In this paper we assume that the distribution of the noise is spherically symmetric about the origin, i.e.,

$$G\mathbf{n} \sim \mathbf{n}$$

for any unitary matrix G ($G\mathbf{n}$ and $\mathbf{n}$ have the same distribution). For the complex valued spherically symmetric distributions see for example [1]. The signals are assumed to be zero-mean and with finite second order moments.

It follows from the signal model and the assumptions that the covariance matrix of the received array output vector (if it exists) is

$$\Sigma = E[\mathbf{x}\mathbf{x}^H] = A\Sigma_s A^H + \sigma_n^2 I$$

where Σ_s is the signal covariance matrix. As a result, $M-K$ smallest eigenvalues of Σ are equal to σ^2 and the corresponding eigenvectors are orthogonal to the columns of the matrix A. This property is used in many conventional subspace based DOA estimation algorithms. For example, in the MUSIC algorithm, the DOA estimates are chosen to be the K largest peaks in the pseudospectrum

$$V(\theta) = \frac{1}{\mathbf{a}^H(\theta)\hat{P}\hat{P}^H\mathbf{a}(\theta)}.$$

where $\hat{P} = [\hat{\mathbf{p}}_{K+1}, \ldots, \hat{\mathbf{p}}_M]$ is the matrix of the eigenvectors corresponding to the $M-K$ smallest eigenvalues of the sample covariance matrix.

III.2. Subspaces in TCM and RCM

The following result is proven in [10].

Result 1 *Let the covariance matrix of the signal model (1) be decomposed as*

$$\Sigma = P\Lambda P^H$$

where $\Lambda = diag\{\lambda_1, \ldots, \lambda_M\}$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$, *and the columns of the matrix* P *are the corresponding eigenvectors* $\mathbf{p}_1, \ldots, \mathbf{p}_M$. *Partition* P *as* $P = (P^1\ P^2)$ *where* $P^1 = (\mathbf{p}_1, \cdots, \mathbf{p}_K)$ *and* $P^2 = (\mathbf{p}_{K+1}, \cdots, \mathbf{p}_M)$. *Then*

$$\Sigma_{TCM} = P_{TCM}\Lambda_{TCM}P_{TCM}^H$$

and

$$\Sigma_{RCM} = P_{RCM}\Lambda_{RCM}P_{RCM}^H$$

where

$$P_{TCM} = (P_{TCM}^1 \; P^2),$$

$$P_{RCM} = (P_{RCM}^1 \; P^2),$$

$$\Lambda_{TCM} = diag\{\lambda_1^{TCM}, \dots, \lambda_K^{TCM}, c_1, \dots, c_1\}$$

and

$$\Lambda_{RCM} = diag\{\lambda_1^{RCM}, \dots, \lambda_K^{RCM}, c_2, \dots, c_2\}.$$

Moreover, the columns of P_{TCM}^1 and P_{RCM}^1 are linear combinations of the columns of P^1.

This result implies that the noise subspace eigenvectors can be estimated from the eigenvectors of TCM or RCM and therefore we can estimate the DOAs by using these eigenvectors in any noise subspace algorithm. Furthermore, because the K eigenvectors of the theoretical TCM and RCM are linear combinations of so called signal subspace eigenvectors, TCM and RCM may also be used for example in the ESPRIT algorithm.
Note that the result does not state that the noise subspace eigenvalues correspond to the smallest eigenvalue of TCM or RCM. In practical simulations, however, this has always been the case. This correspondence remains to be shown.
When constructing algorithms based on the TCM or RCM, the following result is useful.

Result 2 *For the transformed random variables*

$$\mathbf{x}' = P_{TCM}^H\mathbf{x} \quad \textit{and} \quad \mathbf{x}'' = P_{RCM}^H\mathbf{x}$$

it is true that

$$E\{|x_i'|^2\} > E\{|x_j'|^2\} = \sigma^2$$

$$\forall i \in \{1, \dots, K\},\; j \in \{K+1, \dots, M\}$$

and

$$E\{|x_i''|^2\} > E\{|x_j''|^2\} = \sigma^2$$

$$\forall i \in \{1, \dots, K\},\; j \in \{K+1, \dots, M\}.$$

This result now implies that we can divide the eigenvectors of the calculated TCM or RCM to the signal and noise subspace eigenvectors by using some robust estimator for variance. In the algorithms introduced in this paper we use an estimator based on Median Absolute Deviation (MAD). For a real data set $X = \{x_1, \dots, x_n\}$, the MAD is defined as

$$\text{MAD}(X) = c \underset{i=1,\dots,n}{\text{med}} \{|x_i - \text{med}\{x_1, \dots, x_n\}|\}$$

where c is a constant ensuring the consistency of the estimator. For the complex data, the used estimate for the variance is the sum of the squared MADs for the real and imaginary part. Note that when "ordering" the eigenvectors, the result does not depend on the choice of the constant c.

III.3. DOA Algorithms

We are now ready to give two algorithms illustrating the usage of the TCM and RCM in DOA estimation. The algorithms are presented for the TCM only but naturally the TCM can be replaced by the RCM. In practice, the behavior of the TCM and RCM based algorithms is almost identical.
The first algorithm is a MUSIC type noise subspace algorithm.
Algorithm 1:

1. Calculate the R_{TCM} for the snapshots $X = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$. Denote its eigenvecor matrix by $\hat{P}_T$.

2. Estimate the marginal variances of transformed observations $\hat{P}_T^H X$ by using the method described above.

3. Choose the DOA estimates to be the K highest peaks in the pseudospectrum

$$V(\theta) = \frac{1}{a^H(\theta)\hat{P}\hat{P}^H a(\theta)}$$

where $\hat{P}$ is the matrix of the eigenvectors of R_{TCM} corresponding to $M - K$ smallest estimated variances of $\hat{P}_T^H X$.

The second algorithm is based on estimating the signal subspace from the eigenvectors of the TCM.
Algorithm 2

1. Calculate the R_{TCM} for the snapshots $X = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$. Denote its eigenvecor matrix by $\hat{P}_T$.

2. Estimate the marginal variances of transformed observations $\hat{P}_T^H X$ by using the method described above.

3. Apply the TLS-ESPRIT [7] algorithm to the eigenvectors corresponding to K largest estimated variances.

IV. Simulation results

In this section we compare the performance of TCM based algorithms to the standard MUSIC and ESPRIT algorithms in different noise environments. The noise model considered is complex isotropic symmetric α-stable ($S\alpha S$) distribution [9]. The characteristic function of such $S\alpha S$ distribution is

$$\rho(\omega) = \exp(-\gamma|\omega|^{\alpha}).$$

The smaller the characteristic exponent $\alpha \in [0, 2]$, the heavier the tails of the density (the case $\alpha = 2$ corresponds to the Gaussian distribution). The positive valued scalar γ is the dispersion of the distribution. The dispersion plays a role analogous to the role that the variance for second order processes.

In our first simulation we use an 6 element ULA. Two 4-QAM communication signals of power 100 come to the array from directions $\theta_1 = 85°$ and $\theta_2 = 95°$. We assume the number of signals to be known. The conventional MUSIC and the algorithm 1 from the previous section are used to estimate the DOAs.

The performance of the algorithms is compared in the case of different complex isotropic $S\alpha S$ distributions. The values used for the characteristic exponent are $\alpha = 2$, $\alpha = 1.4$ and $\alpha = 1$. The value for the dispersion is $\gamma = 0.25$ (in the Gaussian case the SNR is 20 dB). The number of snapshots used is 300. Five realizations of the estimation results are presented in figure 1. In the Gaussian case, both algorithms perform almost similarly. When the characteristic exponent is 1.4, the behavior of the conventional MUSIC degrades and in the case of extremely heavy tailed noise ($\alpha = 1$), the MUSIC algorithm totally fails to estimate the DOAs. On the other hand, the algorithm 1 performs reliably also in these noise conditions.

In our second simulation, the DOA of the first signal was changed to $\theta_1 = 90°$. All the other parameter values were the same as in the first simulation. Five realizations of the estimation results are presented in figure 2. The results are similar to the first simulation and imply that the algorithm 1 is able to perform high resolution estimation also in extremely heavy tailed noise conditions.

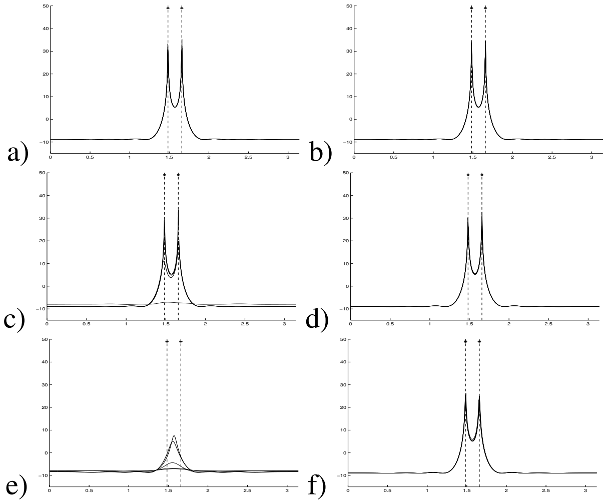

Figure 1: Five realizations of DOA estimation results for α-stable noise conditions a) MUSIC, $\alpha = 2$; b) TCM based MUSIC, $\alpha = 2$; c) MUSIC, $\alpha = 1.4$; d) TCM based MUSIC, $\alpha = 1.4$; e) MUSIC, $\alpha = 1$ and f) TCM based MUSIC, $\alpha = 1$. The size of the ULA is 8. The DOAs are $85°$ and $95°$.

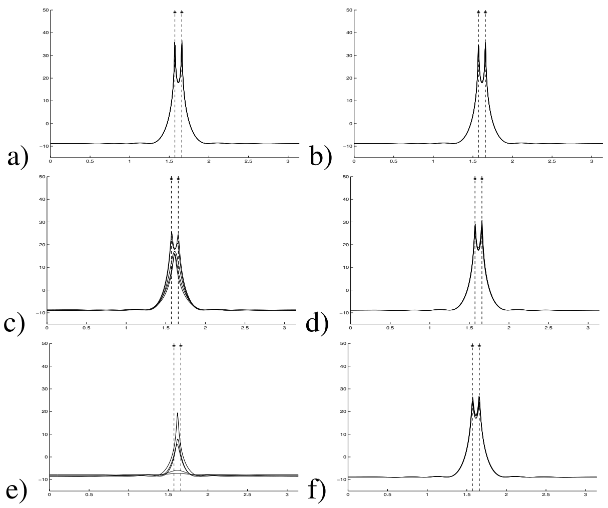

Figure 2: Five realizations of DOA estimation results for α-stable noise conditions a) MUSIC, $\alpha = 2$; b) TCM based MUSIC, $\alpha = 2$; c) MUSIC, $\alpha = 1.4$; d) TCM based MUSIC, $\alpha = 1.4$; e) MUSIC, $\alpha = 1$ and f) TCM based MUSIC, $\alpha = 1$. The size of the ULA is 8. The DOAs are $90°$ and $95°$.

The third simulation compares the performance of the conventional TLS-ESPRIT [7] algorithm and the algorithm 2 introduced in the previous section. All the parameter values are the same as in the second simulation. The even sensors formed the first subarray and the second subarray was formed by the odd sensors. Figure 3 shows five realizations of the estimation results. The results are similar to the second simulation.

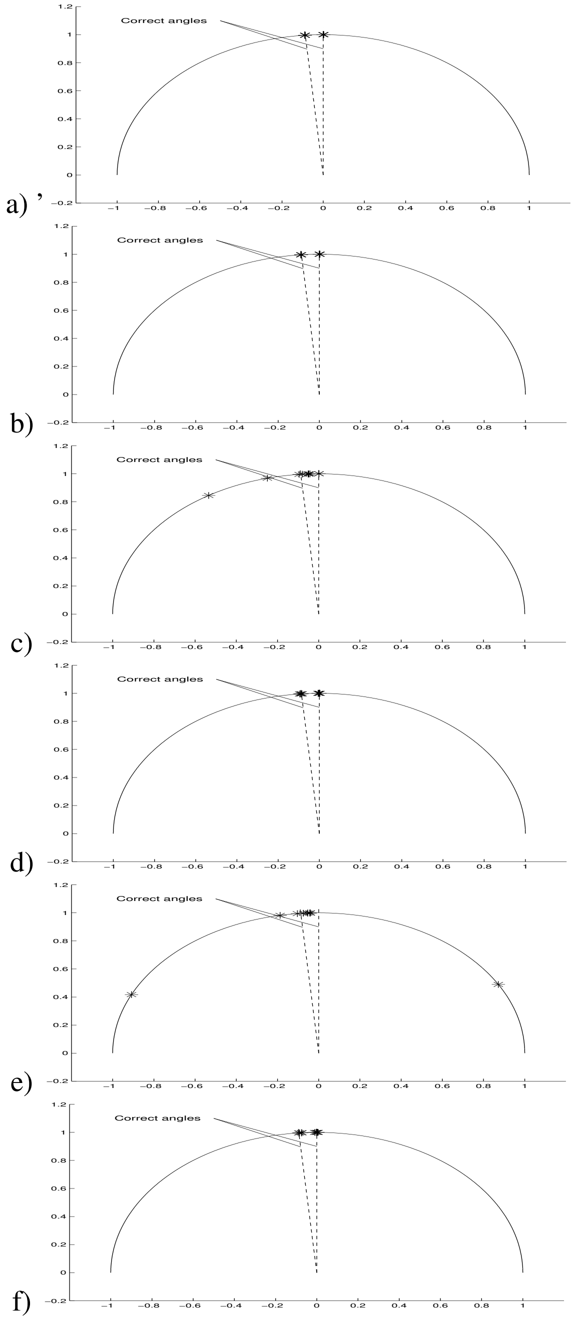

Figure 3: Five realizations of DOA estimation results for α-stable noise conditions a) TLS-ESPRIT, $\alpha = 2$; b) TCM based TLS-ESPRIT, $\alpha = 2$; c) TLS-ESPRIT, $\alpha = 1.4$; d) TCM based TLS-ESPRIT, $\alpha = 1.4$; e) TLS-ESPRIT, $\alpha = 1$ and f) TCM based TLS-ESPRIT, $\alpha = 1$. The size of the ULA is 8. The DOAs are 90° and 95°.

V. Conclusion

In this paper, spatial rank covariance matrices are introduced and new high resolution DOA estimation algorithms are derived based on these nonparametric statistics. The algorithms are shown to perform almost optimally in Gaussian noise and have highly reliable performance in non-Gaussian noise. The major difference to the existing DOA methods is that the nonparametric statistics allow for relaxing assumptions on noise distribution and need no user-defined parameters. In the future, the algorithms will be extended to deal also with completely coherent signals.

VI. References

[1] K.-T. Fang, S. Kotz, and K. W. Ng. *Symmetric multivariate and related distributions*. Chapman and Hall, London, 1990.

[2] R. J. Kozick and B. M. Sadler. Robust maximum likelihood bearing estimation in contaminated gaussian noise. submitted to *IEEE Transactions on Signal Processing*, 1999.

[3] D. D. Lee and R. L. Kashyap. Robust maximum likelihood bearing estimation in contaminated gaussian noise. *IEEE Transactions on Signal Processing*, 40(8):1983–1986, 1992.

[4] D. D. Lee, R. L. Kashyap, and R. N. Madan. Robust decentralized direction-of-arrival estimation in contaminated noise. *IEEE Transactions on Signal Processing*, 38(3):496–505, 1990.

[5] D. Middleton. Man-made noise in urban environments and transportation systems: Models and measurements. *IEEE Transactions on Communications*, 21:1232–1241, 1973.

[6] J. Möttönen and H. Oja. Multivariate spatial sign and rank methods. *Nonparametric Statistics*, 5:201–213, 1995.

[7] R. Roy and T. Kailath. ESPRIT– estimation of signal parameters via rotational invariance techniques. *IEEE Transactions on Acoustics Speech and Signal Processing*, 37(7):984–995, 1989.

[8] P. Tsakalides and C. L. Nikias. Maximum likelihood localization of sources in noise modeled as a stable process. *IEEE Transactions on Signal Processing*, 43(11):2700–2713, 1995.

[9] P. Tsakalides and C. L. Nikias. The robust covariation-based music (roc-music) algorithm for bearing estimation in impulsive noise environments. *IEEE Transactions on Signal Processing*, 44(7):1623–1633, 1996.

[10] S. Visuri, H. Oja, and V. Koivunen. Direction of arrival estimation based on nonparametric statistics. *IEEE Transactions on Signal Processing.* Submitted for publication, 2000.

[11] X. Wang and V. Poor. Robust adaptive array for wireless communications. In *IEEE ICC'98*, volume 3, pages 1541–1545, 1998.

[12] D. B. Williams and D. H. Johnson. Robust estimation of structured covariance matrices. *IEEE Transactions on Signal Processing*, 41(9):2891–2906, 1994.